Holographic QCD in light-front quantization and superconformal algebra: An overview

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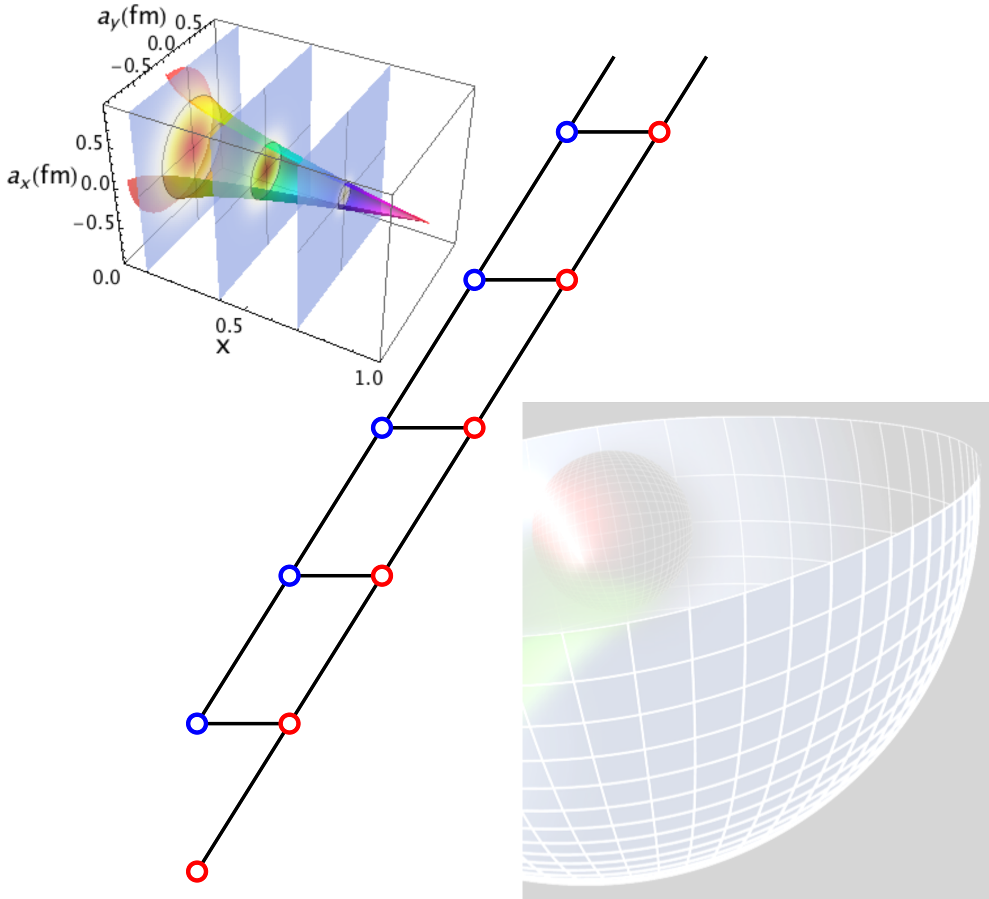

Applications of gauge topology,

holography and string models to QCD

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SIMONS CENTER
FOR GEOMETRY AND PHYSICS

In collaboration with Stan Brodsky, Hans G. Dosch, Alexandre Deur, Tianbo Liu and Raza Sabbir Sufian

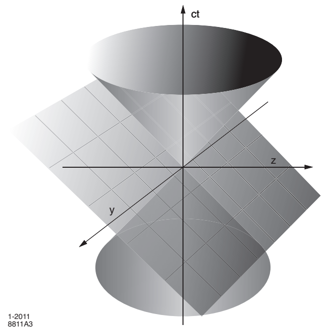
Complexity of QCD

- The QCD Lagrangian in the limit of massless quarks has no scale: still confinement and a mass scale should emerge from the quantum theory built upon the classical QCD conformal theory
- Description of the dynamics is vastly complex and understanding the mechanism of confinement is an unsolved problem
- As a first step we would require a semiclassical approximation which captures essential aspects of the nonperturbative confinement dynamics, which are not obvious from the QCD Lagrangian
- Recent analytical insights into the nonperturbative structure of QCD based on light-front quantization and its holographic embedding have lead to effective semiclassical bound-state equations for arbitrary spin where the confinement potential is determined by an underlying superconformal algebra
- This approach leads to unsuspected connections across the entire mass spectrum and incorporates the structure of Veneziano amplitudes useful to describe form factors and parton distributions

Contents

Reviews: S. J. Brodsky, GdT, H.G. Dosch, J. Erlich, Phys. Rept. **584**, 1 (2015) and arXiv:2004.07756 [hep-ph]

1 Semiclassical approximation to QCD in the light front



[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Start with $SU(3)_C$ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Express the hadron 4-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, in terms of dynamical fields $\psi_+ = \Lambda_+ \psi$ and $\mathbf{A}_\perp$ $(\Lambda_\pm = \gamma^0\gamma^\pm)$ quantized in the null plane $x^+ = x^0 + x^3 = 0$

$$\begin{aligned} P^- &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \,\gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi_+ + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+ \psi_+ \\ \mathbf{P}_\perp &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\nabla_\perp \psi_+ \end{aligned}$$

- Construct LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$ from mass-shell relation

$$P^2|\psi(P)\rangle = M^2|\psi(P)\rangle, \qquad |\psi\rangle = \sum_n \psi_n |n\rangle$$

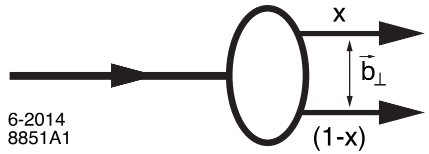
- Simple structure of LF vacuum allows a quantum-mechanical probabilistic interpretation of hadronic states in terms of wave functions: $\psi_n = \langle n|\psi\rangle$

- The mass spectrum for a two-parton bound state is computed from the hadron matrix element

$$\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = M^2\langle\psi(P')|\psi(P)\rangle$$

- We factor out the longitudinal $X(x)$ and orbital $e^{iL\varphi}$ kinematical dependence from the LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\varphi}X(x)\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



with invariant impact "radial" LF variable $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$ and $L = L^z$

Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple

$$M^2 = \int d\zeta\,\phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\,\phi^*(\zeta)\,U(\zeta)\,\phi(\zeta)$$

where the effective potential U includes all interactions, including those from higher Fock states

- The LF Hamiltonian equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ becomes a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)}$$

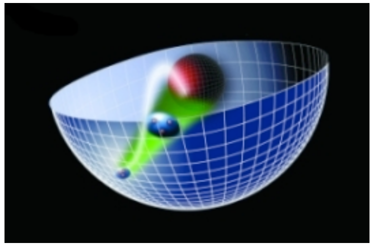
- Critical valuel $L=0$ corresponds to the lowest possible stable solution

- Relativistic and frame-independent semiclassical WE: It has identical structure of AdS WE

2 Integer-spin wave equations in AdS and LF holographic embedding

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- AdS$_5$ is a 5-dim space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu dx^\mu - dz^2\right)$$

- Isomorphism of $SO(4,2)$ conformal group with the group of isometries of AdS$_5$
- We start from the AdS action for a rank-J tensor field $\Phi_{N_1 \dots N_J}$ with AdS mass μ and a dilaton profile φ which breaks the maximal symmetry of AdS (the conformality in the dual theory)

$$S = \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{M M'} D_M \Phi^*_{N_1 \dots N_J} \, D_{M'} \Phi_{N_1' \dots N_J'} \\ - \mu^2 \, \Phi^*_{N_1 \dots N_J} \, \Phi_{N_1' \dots N_J'} + \cdots \Big)$$

where $\sqrt{g} = (R/z)^{d+1}$ and the covariant derivative D_M includes the affine connection

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics

- In holographic QCD a hadron is described by a z-dependent wave function $\Phi_J(z)$ and a plane wave in physical spacetime with polarization indices ν along Minkowski coordinates

$$\Phi_{\nu_1 \cdots \nu_J}(x, z) = e^{iP \cdot x}\, \Phi_J(z)\, \epsilon_{\nu_1 \cdots \nu_J}(P)$$

with invariant mass $P_\mu P^\mu = M^2$

- From $\delta S / \delta \Phi = 0$ follows the eigenvalue equation $(m = m(\mu, \varphi) = const)$

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{mR}{z} \right)^2 \right] \Phi_J = M^2 \Phi_J$$

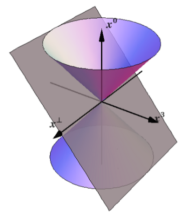
plus kinematical constraints to eliminate lower spin from the symmetric tensor $\Phi_{N_1 \ldots N_J}$

- Upon the substitution

$$\Phi_J(z) = z^{(d-1)/2 - J} e^{-\varphi(z)/2}\, \phi_J(z)$$

we find the semiclassical QCD LFWE for $d = 4$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$



with the holographic variable z

$$z^2 \to \zeta^2 = x(1-x) b_\perp^2$$

identified with the LF invariant separation between two quarks and $(mR)^2 = -(2-J)^2 + L^2$

- The effective LF potential U

$$U(\zeta, J) = \tfrac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta)$$

  is determined in terms of the AdS dilaton profile φ (the IR modification of AdS space)

- Non-trivial geometry of AdS encodes the higher-spin kinematics, including the constraints required to eliminate lower spin from the symmetric J-tensor field $\Phi_{N_1 \dots N_J}$

$$\eta^{\mu\nu}\partial_\mu \Phi_{\nu\nu_2\cdots \quad \nu_J} = 0, \quad \eta^{\mu\nu}\Phi_{\mu\nu\nu_3\cdots\nu_J} = 0$$

- Additional IR deformations of AdS encode the dynamics, including confinement
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ is equivalent to LF QM stability condition $L^2 \geq 0$
- Question: how can we determine the effective LF potential U, equivalently, the dilaton field φ?
- Important clues from the description of baryons in AdS . . .

3 Half-integer-spin wave equations in AdS and LF holographic embedding

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

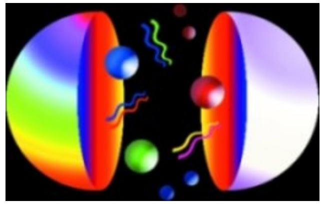
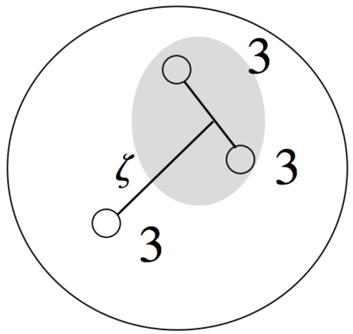


Image credit: N. Evans

- Extension of holographic ideas to higher half-integral spin-J hadrons by considering wave equations for Rarita-Schwinger spinor fields in AdS space and their mapping to LF physics

- The invariant LF impact variable ζ is the weighted distribution of the spectator diquark cluster relative to the active quark

$$\zeta = x(1-x)^{-1} a_\perp^2, \quad \mathbf{a} = \sum_{j=1}^{N-1} x_j \mathbf{b}_{\perp j}$$



- LF cluster decomposition follows from mapping of EM form factor in AdS to the light front [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

- Quark-diquark approximation for baryons: no internal degrees of freedom in the spectator cluster

- "Missing resonances" problem for higher mass from spectator cluster excitations?

- Effective AdS action for half-integer spin $J = T + \frac{1}{2}$ RS spinor $\left[\Psi_{N_1 \cdots N_T}\right]_\alpha$ with AdS mass μ and effective potential $\rho(z)$ (no dynamical dilaton)

$$S_{\mathit{eff}} = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, g^{N_1 \, N_1'} \cdots g^{N_T \, N_T'} \\ \left[\overline{\Psi}_{N_1 \cdots N_T} \Big(i \, \Gamma^A \, e_A^M \, D_M - \mu - \rho(z) \Big) \Psi_{N_1' \cdots N_T'} + h.c. \right]$$

where the covariant derivative D_M includes the affine connection and the spin connection

- e_M^A is the vielbein and Γ^A tangent space Dirac matrices $\left\{\Gamma^A, \Gamma^B\right\} = \eta^{AB}$
- Factoring out the four-dimensional plane-wave and spinor dependence

$$\Psi_\pm^T(x, z) = e^{iP \cdot x} u^\pm_{\nu_1 \cdots \nu_T}(P) z^{d/2-T} \psi_\pm^T(z)$$

we find the coupled equations for the chiral components $\psi_\pm$ $\left(|\nu R| = \mu + \frac{1}{2}\right)$

$$\begin{aligned} -\frac{d}{dz}\psi_- - \frac{\nu + \frac{1}{2}}{z}\psi_- - V(z)\psi_- &= M\psi_+ \\ \frac{d}{dz}\psi_+ - \frac{\nu + \frac{1}{2}}{z}\psi_+ - V(z)\psi_+ &= M\psi_- \end{aligned}$$

where $\psi_\pm \equiv \psi_\pm^T$ and $V(z) = \frac{R}{z}\rho(z)$ are J-independent $\left(T = J - \frac{1}{2}\right)$

- Mapping to the light front $z \to \zeta$, system of linear eqs in AdS is equivalent to the second order eqs:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U^+(\zeta)\right)\psi_+ \;=\; M^2\psi_+$$
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + U^-(\zeta)\right)\psi_- \;=\; M^2\psi_-$$

the semiclassical QCD LF WE with ψ_+ and ψ_- corresponding to LF orbital L and $L+1$ with

$$U^{\pm}(\zeta) = V^2(\zeta) \pm V'(\zeta) + \frac{1+2L}{\zeta}V(\zeta), \quad L = \mu R - \tfrac{1}{2},$$

a J-independent potential – No spin-orbit coupling along a given trajectory !

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

- Example: For internal spin $S = \frac{3}{2}$ and $L = 2$ the $\Delta^{\frac{1}{2}}, \Delta^{\frac{3}{2}}, \Delta^{\frac{5}{2}}$ and $\Delta^{\frac{7}{2}}$ quartet should not depend on the value of J: its mass should be fully degenerate independent of the specific form of $V(\zeta)$

- How can we determine the form of $V(\zeta)$? Actually $V(\zeta)$ can be identified with the superpotential in SUSY QM: If extended to superconformal QM it is fully determined and a mass scale is introduced !

4 Superconformal algebraic structure in LFHQCD

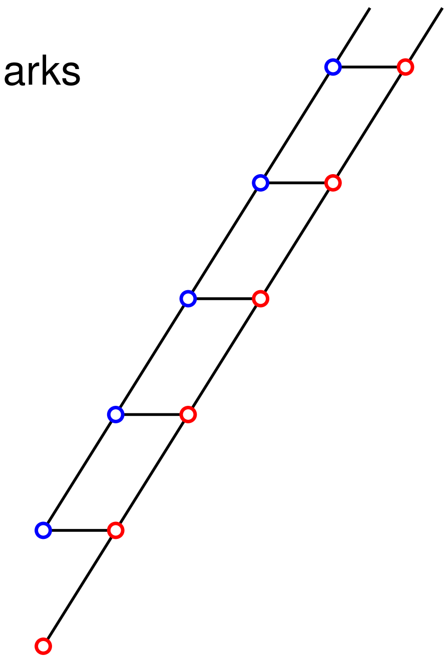
[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

- Superconformal algebra underlies in LFHQCD the scale invariance of the QCD Lagrangian. It leads to the introduction of a scale in the Hamiltonian maintaining the action conformal invariant

- It also leads to a specific connection between mesons, baryons and tetraquarks underlying the $SU(3)_C$ representation properties: $\overline{3} \to 3 \times 3$



- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H [E. Witten, NPB **188**, 513 (1981)]

$$\tfrac{1}{2}\{Q, Q^\dagger\} = H$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \quad [Q, H] = [Q^\dagger, H] = 0$$

which closes under the graded algebra $sl(1/1)$

- Since $[Q^\dagger, H] = 0$, the states $|E\rangle$ and $Q^\dagger|E\rangle$ for $E \neq 0$ are degenerate, but for $E = 0$ we can have the trivial solution $Q^\dagger|E = 0\rangle = 0$ (the pion ?)

- Matrix representation of SUSY generators $Q, Q^\dagger$ and H

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \qquad H = \frac{1}{2}\begin{pmatrix} q\,q^\dagger & 0 \\ 0 & q^\dagger q \end{pmatrix}$$

- For a conformal theory (f dimensionless)

$$q = -\frac{d}{dx} + \frac{f}{x}, \qquad q^\dagger = \frac{d}{dx} + \frac{f}{x}$$

- Superconformal QM: Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to the generator of conformal transformations K

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \qquad S^\dagger = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}$$

- It leads to the conformal enlarged algebra [Haag, Lopuszanski and Sohnius (1974)]

$$\begin{aligned} \tfrac{1}{2}\{Q, Q^\dagger\} &= H, & \tfrac{1}{2}\{S, S^\dagger\} &= K, \\ \{Q, S^\dagger\} &= f - B + 2iD, & \{Q^\dagger, S\} &= f - B - 2iD \end{aligned}$$

where $B = \frac{1}{2}\sigma_3$ is a baryon number operator and H, D and K are the generators of translation, dilatation and the special conformal transformation

- H, D and K

$$H = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 + 2Bf}{x^2}\right), \qquad D = \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right), \qquad K = \tfrac{1}{2}x^2$$

satisfy the conformal algebra $[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$

- Following F&R we define the fermionic generator $R = Q + \lambda\, S$ with anticommutation relations $\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0$. It generates the new Hamiltonian $G_\lambda = \{R_\lambda, R_\lambda^\dagger\}$ which also closes under the graded algebra $sl(1/1)$: $[R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$

- The Hamiltonian G_λ is given by

$$G_\lambda = 2H + 2\lambda^2 K + 2\lambda\,(f - \sigma_3)$$

and leads to the eigenvalue equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2\, x^2 + 2\lambda\, f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2}\right)\phi_1 = E\,\phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2\, x^2 + 2\lambda\, f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2}\right)\phi_2 = E\,\phi_2$$

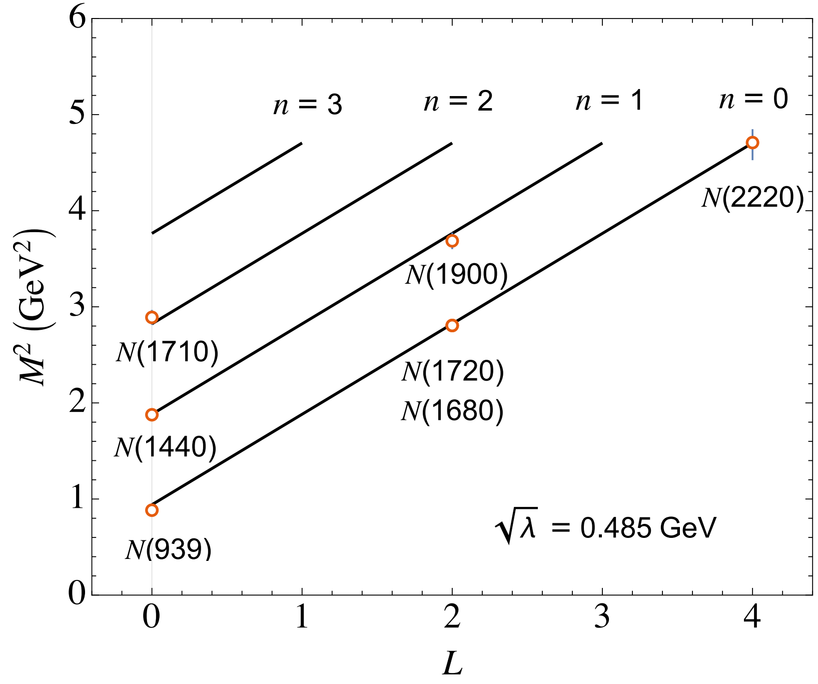
5 Light-front mapping and baryons

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

- Upon the substitutions in slide 14

$$
\begin{aligned}
x &\mapsto \zeta \\
E &\mapsto M^2 \\
f &\mapsto L + \tfrac{1}{2} \\
\phi_1 &\mapsto \psi_-, \quad \phi_2 \mapsto \psi_+
\end{aligned}
$$

we find the LF semiclassical bound-state equations



$$
\begin{aligned}
\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L+1)\right)\psi_+ &= M^2\psi_+ \\
\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L\right)\psi_- &= M^2\psi_-
\end{aligned}
$$

- Eigenvalues

$$M^2 = 4\lambda(n+L+1)$$

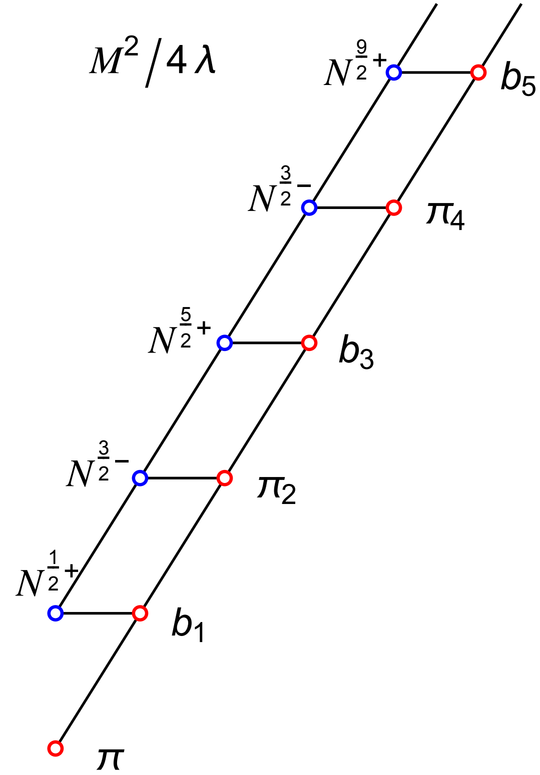
- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2), \quad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$

6 Superconformal meson-baryon-tetraquark symmetry

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]



- Upon the substitutions in slide 14

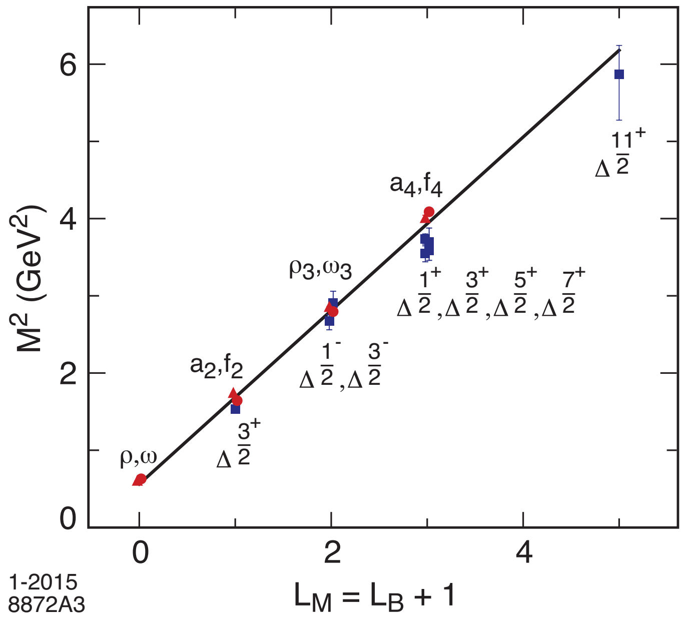
$$\begin{aligned} x &\mapsto \zeta \\ E &\mapsto M^2 \\ \lambda &\mapsto \lambda_B = \lambda_M \\ f &\mapsto L_M - \tfrac{1}{2} = L_B + \tfrac{1}{2} \\ \phi_1 &\mapsto \phi_M, \quad \phi_2 \mapsto \phi_B \end{aligned}$$

we find the LF bound-state equations

$$\begin{aligned} \left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M(L_M - 1)\right)\phi_M &= M^2\,\phi_M \\ \left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_N + 1)\right)\phi_B &= M^2\,\phi_B \end{aligned}$$

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $\boxed{L_M = L_B + 1}$

- L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator cluster in the baryon



- Special role of the pion as a unique state of zero energy

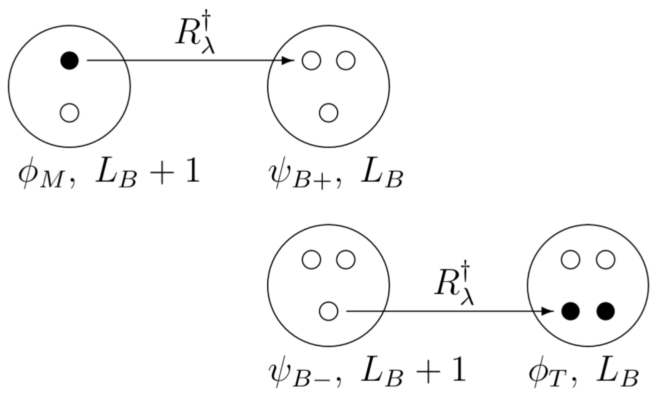
$$R^\dagger|M, L\rangle = |B, L-1\rangle, \quad R^\dagger|M, L=0\rangle = 0$$

- Hadron quantum numbers determined from the pion Q N

- Spin-dependent Hamiltonian for mesons and baryons with internal spin S

$$\boxed{G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S} \qquad S = 0, 1$$

- Supersymmetric 4-plet: quark-antiquark (M), quark-diquark (B) , diquark-antidiquark (T)



$$\begin{aligned} M_M^2 &= 4\lambda\,(n + L_M) + 2\lambda S \\ M_B^2 &= 4\lambda\,(n + L_B + 1) + 2\lambda S \\ M_T^2 &= 4\lambda\,(n + L_T + 1) + 2\lambda S \end{aligned}$$

- $\sqrt{\lambda} = 0.523 \pm 0.024$ GeV from the light hadron spectrum including radial and orbital excitations

$$\bar{\mathbf{3}} \to \mathbf{3} \times \mathbf{3} \qquad \mathbf{3} \to \bar{\mathbf{3}} \times \bar{\mathbf{3}}$$

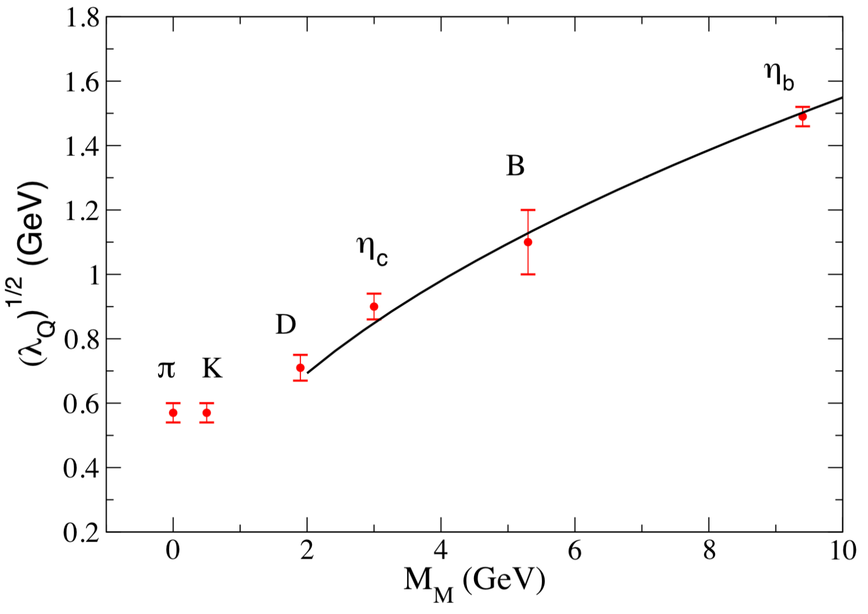
7 Heavy-light and heavy-heavy sectors

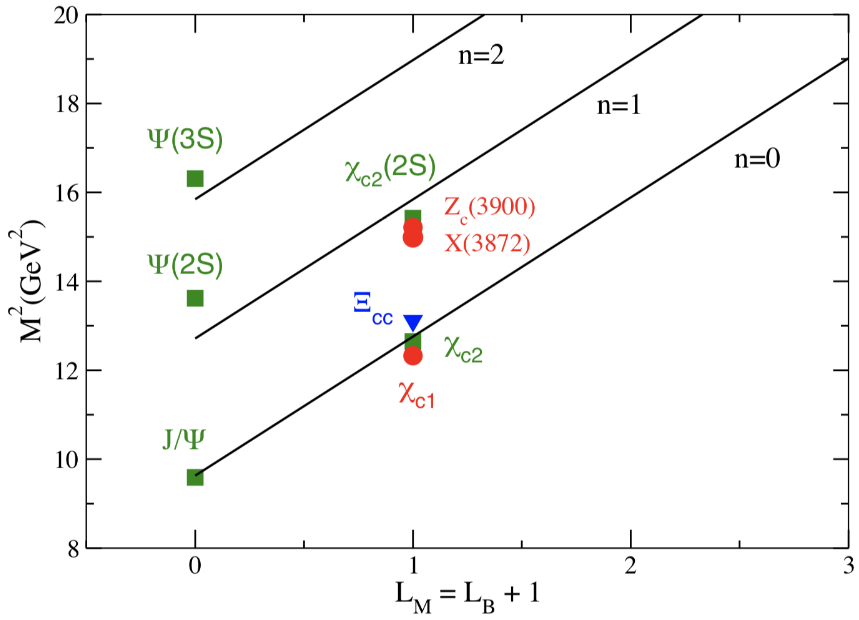
- Scale dependence of hadronic scale λ from HQET
- Extension to the heavy-light hadronic sector:

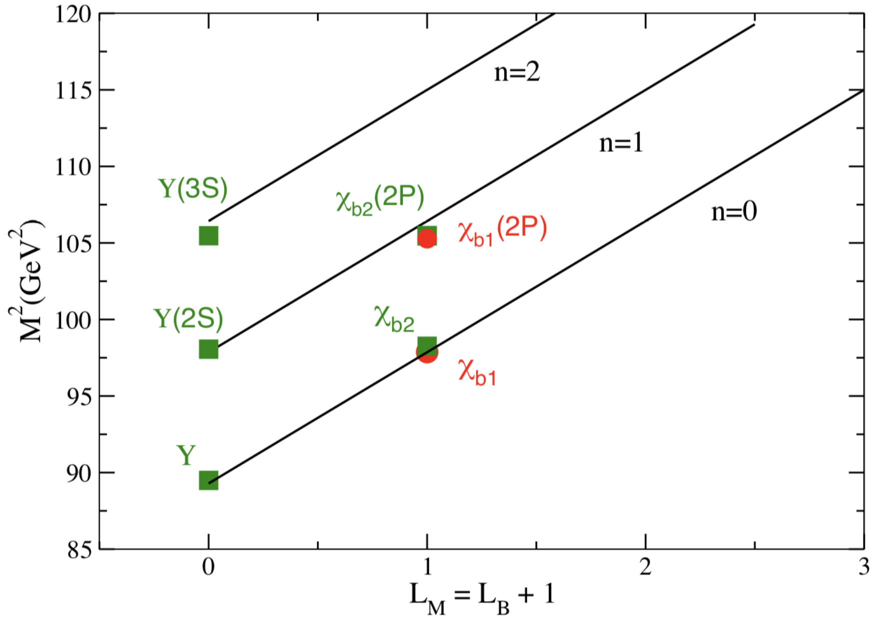
  [H. G. Dosch, GdT, S. J. Brodsky, (2015, 2017)]
- Extension to the double-heavy hadronic sector:

  [M. Nielsen, S. J. Brodsky *et al.* (2018)]
- Extension to the isoscalar hadronic sector:

  [L. Zou, H. G. Dosch, GdT, S. J. Brodsky (2018)]







8 Form factors, parton distributions and intrinsic quark sea

- Recent study of form factors, polarized and unpolarized quark distributions by extending the LF holographic QCD framework to incorporate the analytic structure of Veneziano amplitudes

- Recent study of sea quark content of the proton using combined lattice QCD and holographic methods

- Nucleon Form Factors

  [R. S. Sufian, GdT, S. J. Brodsky, A . Deur, H. G. Dosch (2017)]
- Generalized parton distributions

  [GdT, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, A. Deur (2018)]
- Strange-quark sea in the nucleon

  [R. S. Sufian, T Liu, GdT, H. G. Dosch, S. J. Brodsky, A. Deur, M. T. Islam, B-Q. Ma (2018)]
- Unified description of polarized and unpolarized quark distributions

  [T Liu, R. S. Sufian, GdT, H. G. Dosch, S. J. Brodsky, A. Deur (2019)]
- Intrinsic-charm content of the proton

  [R. S. Sufian, T. Liu, A. Alexandru, S. J. Brodsky, GdT, H. G. Dosch, T. Draper, K. F. Liu and Y. B. Yang (2020)]

- Form factor expressed as a sum from the Fock expansion of states

$$F(t) = \sum_\tau c_\tau F_\tau(t)$$

where the c_τ are spin-flavor twist-expansion coefficients

- $F_\tau(t)$ in LFHQCD has the Euler's Beta form structure

$$F_\tau(t) = \frac{1}{N_\tau} B\big(\tau - 1, 1 - \alpha(t)\big)$$

found by Ademollo and Del Giudice and Landshoff and Polkinghorne in the pre-QCD era, extending the Veneziano duality model (1968)

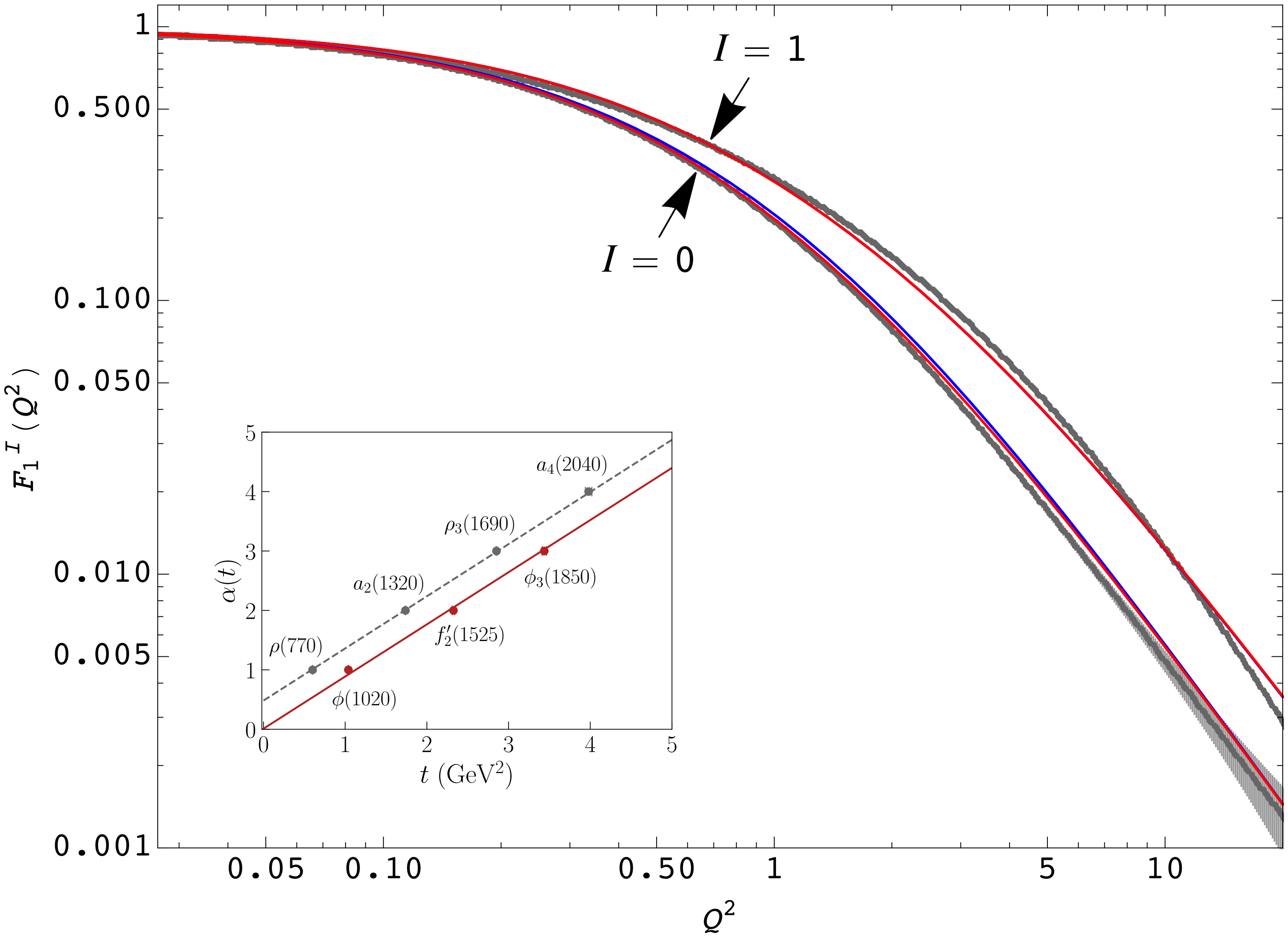
- $\alpha(t)$ is the Regge trajectory of the VM which couples to the quark EM current in the hadron
- For $\tau = N$, the number of constituents in a Fock component, the FF is an $N - 1$ product of poles

$$F_\tau(Q^2) = \frac{1}{\Big(1 + \frac{Q^2}{M^2_{n=0}}\Big)\Big(1 + \frac{Q^2}{M^2_{n=1}}\Big) \cdots \Big(1 + \frac{Q^2}{M^2_{n=\tau-2}}\Big)}$$

located at

$$-Q^2 = M_n^2 = \frac{1}{\alpha'}\big(n + 1 - \alpha(0)\big)$$

which generates the radial excitation spectrum of the exchanged VM particles in the t-channel



Nucleon isospin form factors $F^{I=0,1}(t) = F_p(t) \pm F_n(t)$

Liu et al. (2020): Valence contribution only (This work I)

Liu et al. (2020): Including $u\overline{u}$ and $d\overline{d}$ (This work II)

Ye et al. (2018): z-expansion data analysis

- Using integral representation of Beta function FF is expressed in a reparametrization invariant form

$$F(t)_\tau = \frac{1}{N_\tau} \int_0^1 dx w'(x) w(x)^{-\alpha(t)} \left[1 - w(x)\right]^{\tau-2}$$

with $w(0) = 0, \quad w(1) = 1, \quad w'(x) \geq 0$

- Flavor FF is given in terms of the valence GPD $H_\tau^q(x, \xi = 0, t)$ at zero skewness

$$F_\tau^q(t) = \int_0^1 dx H_\tau^q(x, t) = \int_0^1 dx\, q_\tau(x) \exp[t f(x)]$$

with the profile function $f(x)$ and PDF $q(x)$ determined by $w(x)$

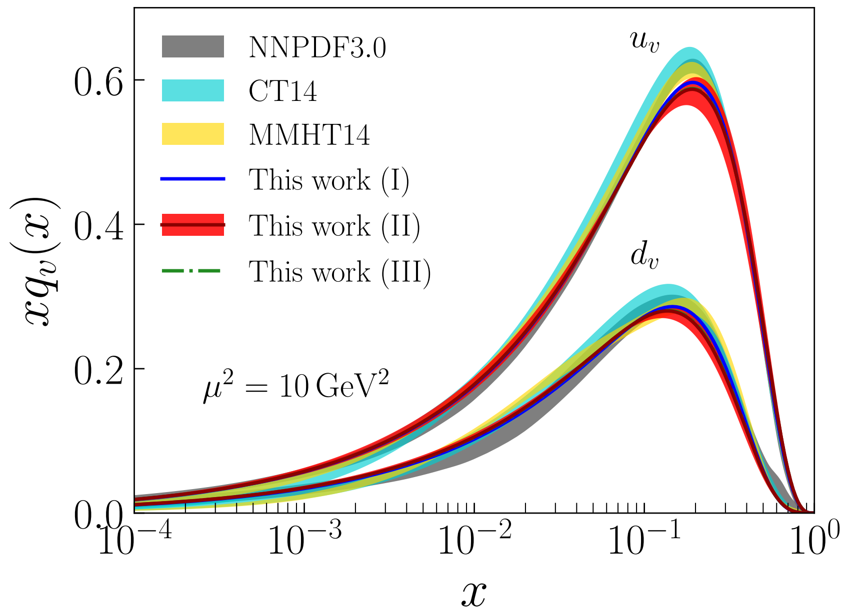
$$\begin{aligned} f(x) &= \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right) \\ q_\tau(x) &= \frac{1}{N_\tau} [1 - w(x)]^{\tau-2} w(x)^{-\alpha(0)} w'(x) \end{aligned}$$

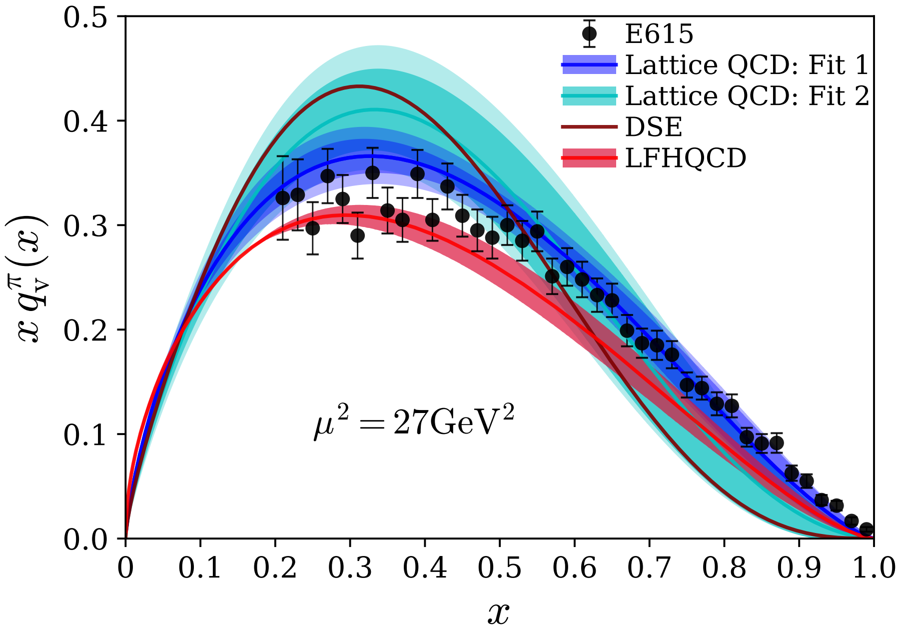
- Boundary conditions: At $x \to 0$, $w(x) \sim x$ from Regge behavior, $q(x) \sim x^{-\alpha(0)}$, and $w'(1) = 0$ to recover Drell-Yan counting rules at $x \to 1$, $q_\tau(x) \sim (1 - x)^{2\tau - 3}$ (inclusive-exclusive connection)

- If $w(x)$ is fixed by the nucleon PDFs then the pion PDF is a prediction. Example:

$$w(x) = x^{1-x} e^{-a(1-x)^2}$$

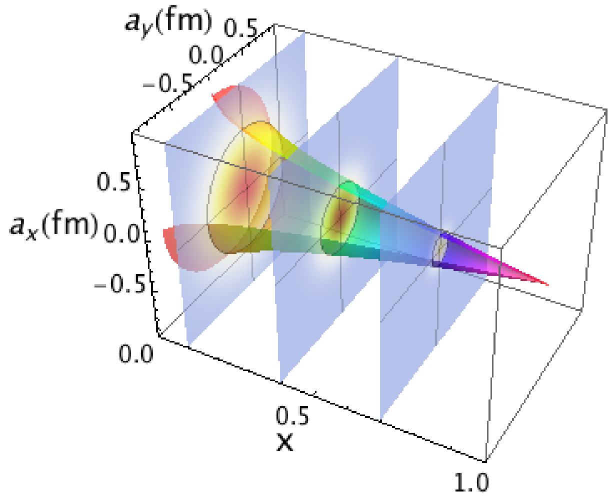
Unpolarized GPDs and PDFs





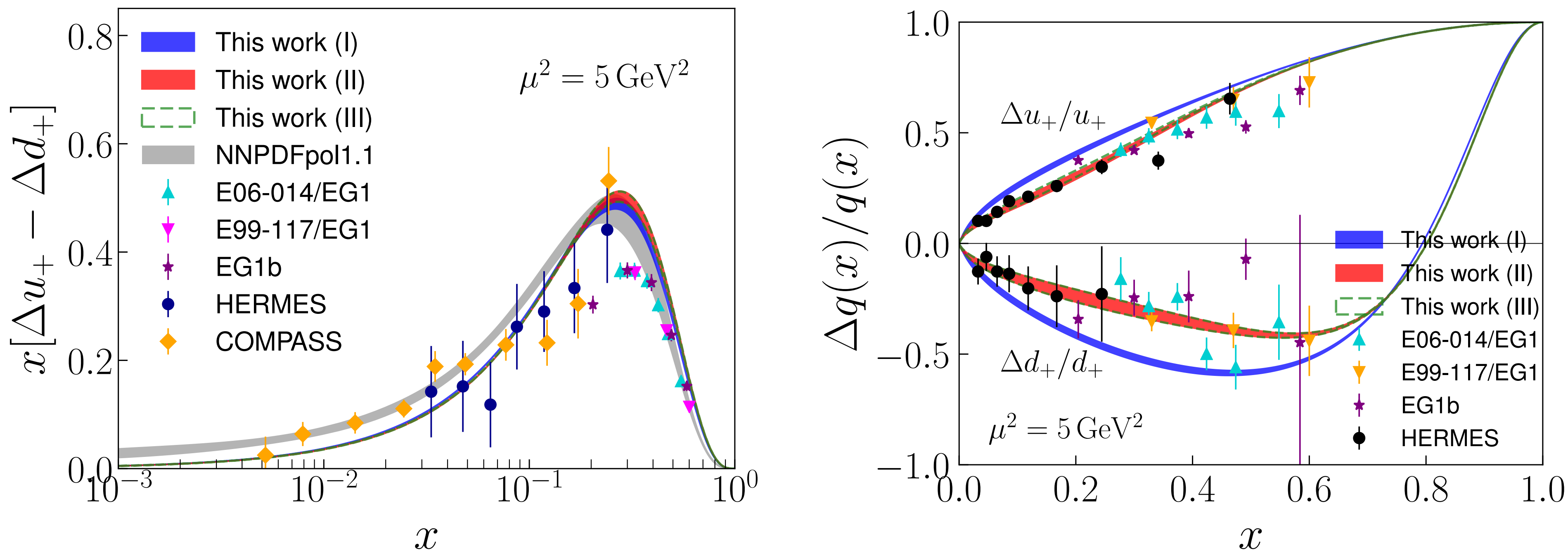
- Transverse impact parameter quark distribution

$$u(x, \mathbf{a}_\perp) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{a}_\perp \cdot \mathbf{q}_\perp} H^u(x, \mathbf{q}_\perp^2)$$



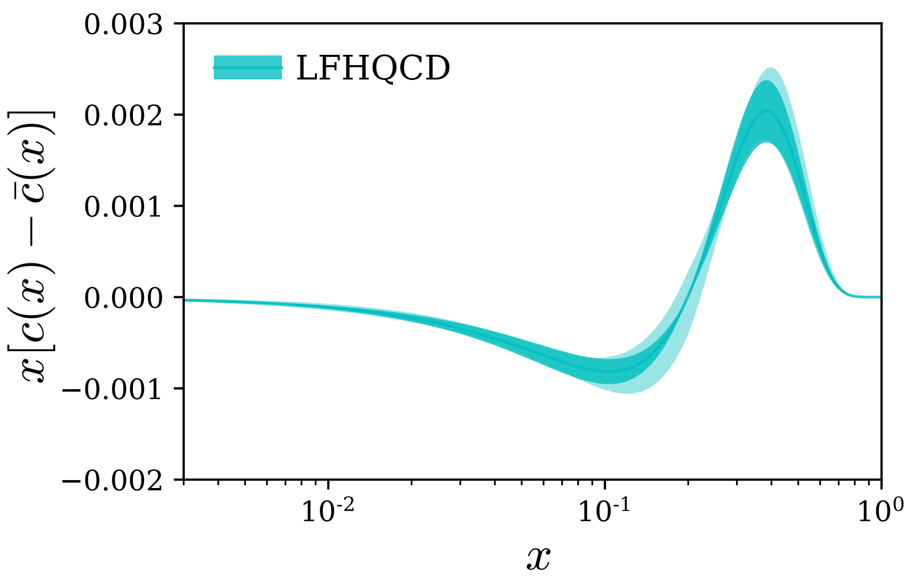
Polarized GPDs and PDFs

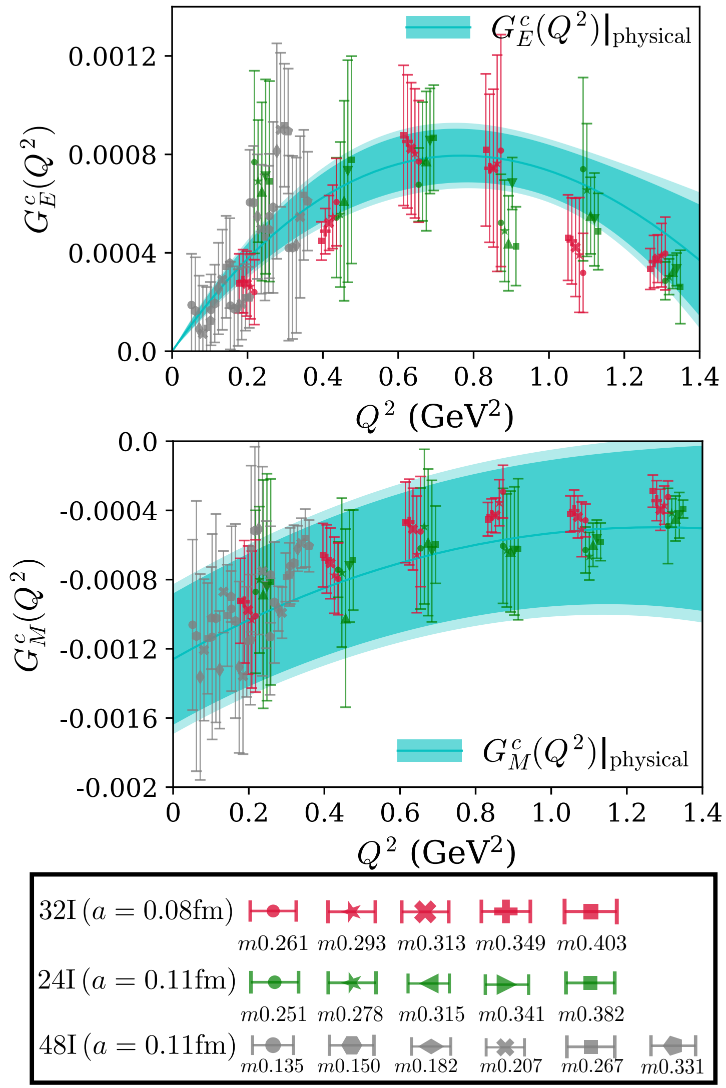
- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients c_τ are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction): $\lim_{x\to 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron: $\lim_{x\to 0} \frac{\Delta q(x)}{q(x)} = 0$



Intrinsic charm in the proton

- S. J. Brodsky, P. Hoyer, C. Peterson, N. Sakai, The intrinsic charm of the proton (1980)
- First lattice QCD computation of the charm quark EM form factors with three gauge ensembles (one at the physical pion mass)
- Nonperturbative intrinsic charm asymmetry $c(x) - \bar{c}(x)$ computed from LFHQCD analysis





9 Outlook

- Classical equations of motion derived from the 5-dim theory have identical form of the semiclassical bound-state equations for massless constituents in LF quantization
- Implementation of superconformal algebra determines uniquely the form of the confining interaction and thus the modification of the AdS action, both for mesons and nucleons
- Approach incorporates basic nonperturbative properties which are not apparent from the chiral QCD Lagrangian, such as the emergence of a mass scale and the connection between mesons and baryons
- Prediction of massless pion in chiral limit is a consequence of the superconformal algebraic structure and not of the Goldstone mechanism
- Structural framework of LFHQCD also provides nontrivial connection between the structure of form factors and polarized and unpolarized quark distributions with pre-QCD nonperturbative results such as Regge theory and the Veneziano model