4d physics from 2d chiral correlators

Natalie Paquette, work w/ Kevin Costello

Strings 2022, Vienna

U.S. DEPARTMENT OF ENERGY | Office of Science

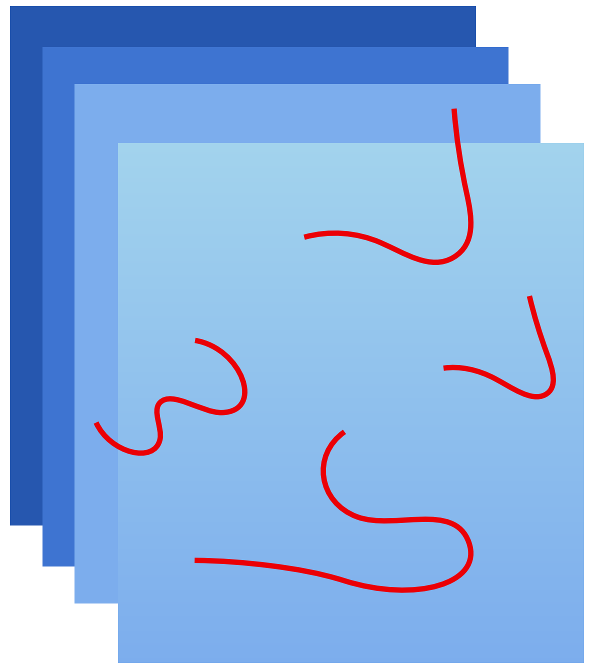
Early Career Research Program

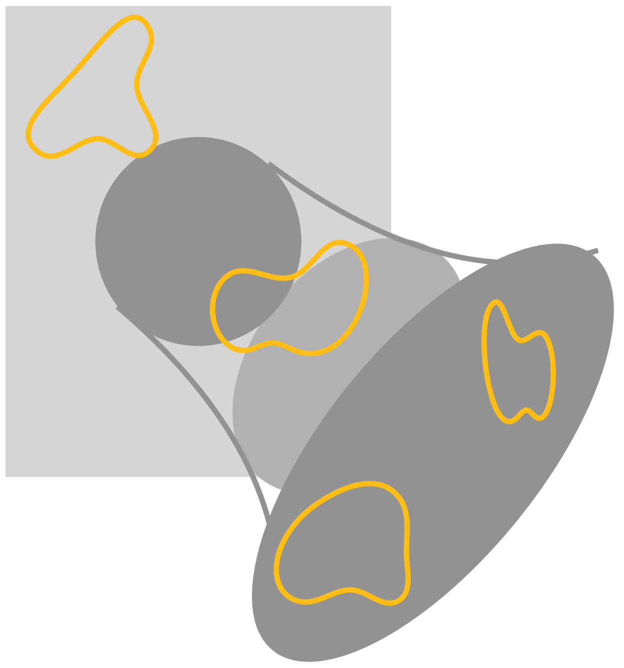
Based on 2204.05301, 2201.02595 + WIP

A confluence of progress in a few different subfields

points of contact: symmetry, universality

Twisted holography





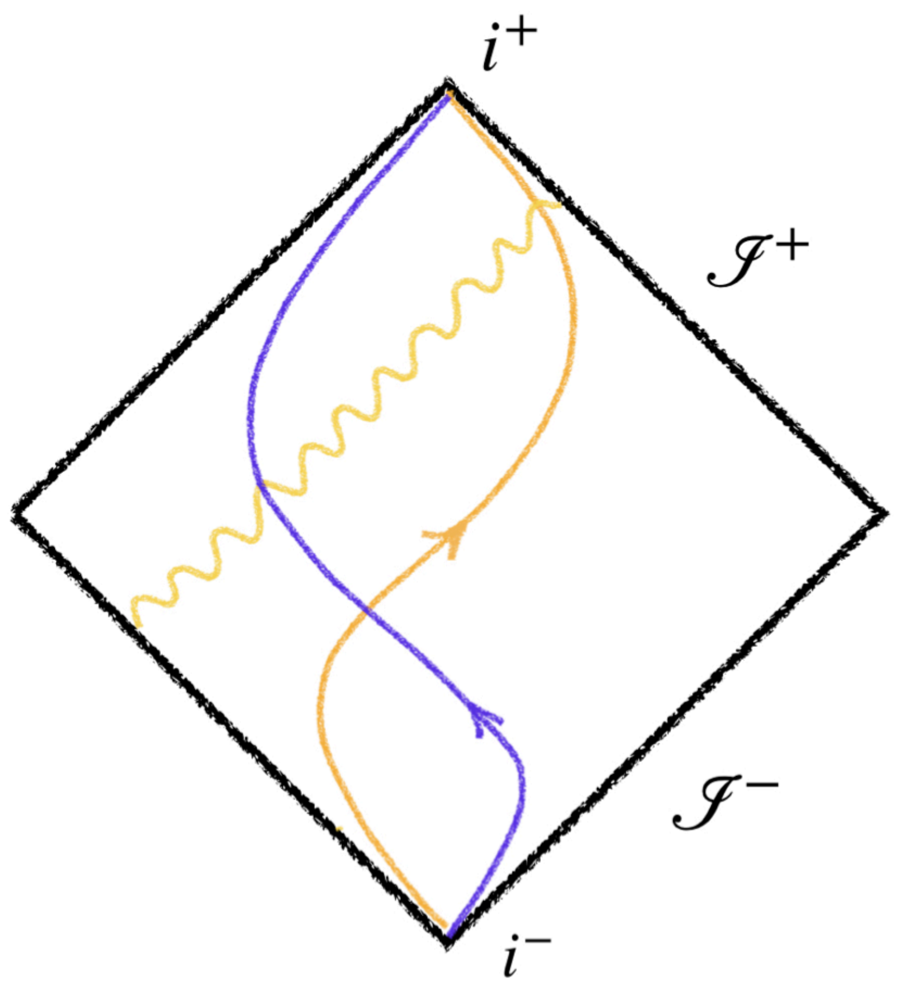
$$A = A_{\bar{z}} d\bar{z} + A_{\bar{w}_1} d\bar{w}_1 + A_{\bar{w}_2} d\bar{w}_2$$

$$\int \Omega \wedge CS(A)$$

$$\eta \in \Omega^{2,1}(M)$$

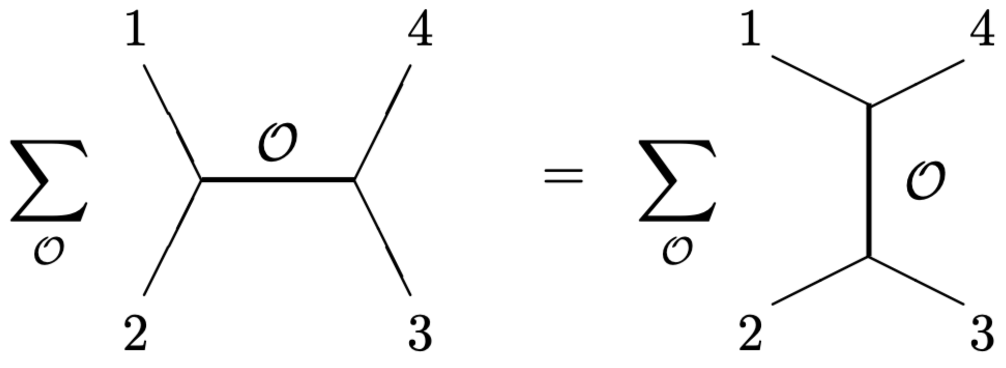
$$\frac{1}{2}\int (\partial^{-1}\eta)(\bar{\partial}\eta) + \frac{1}{6}\int \eta^3$$

Celestial holography

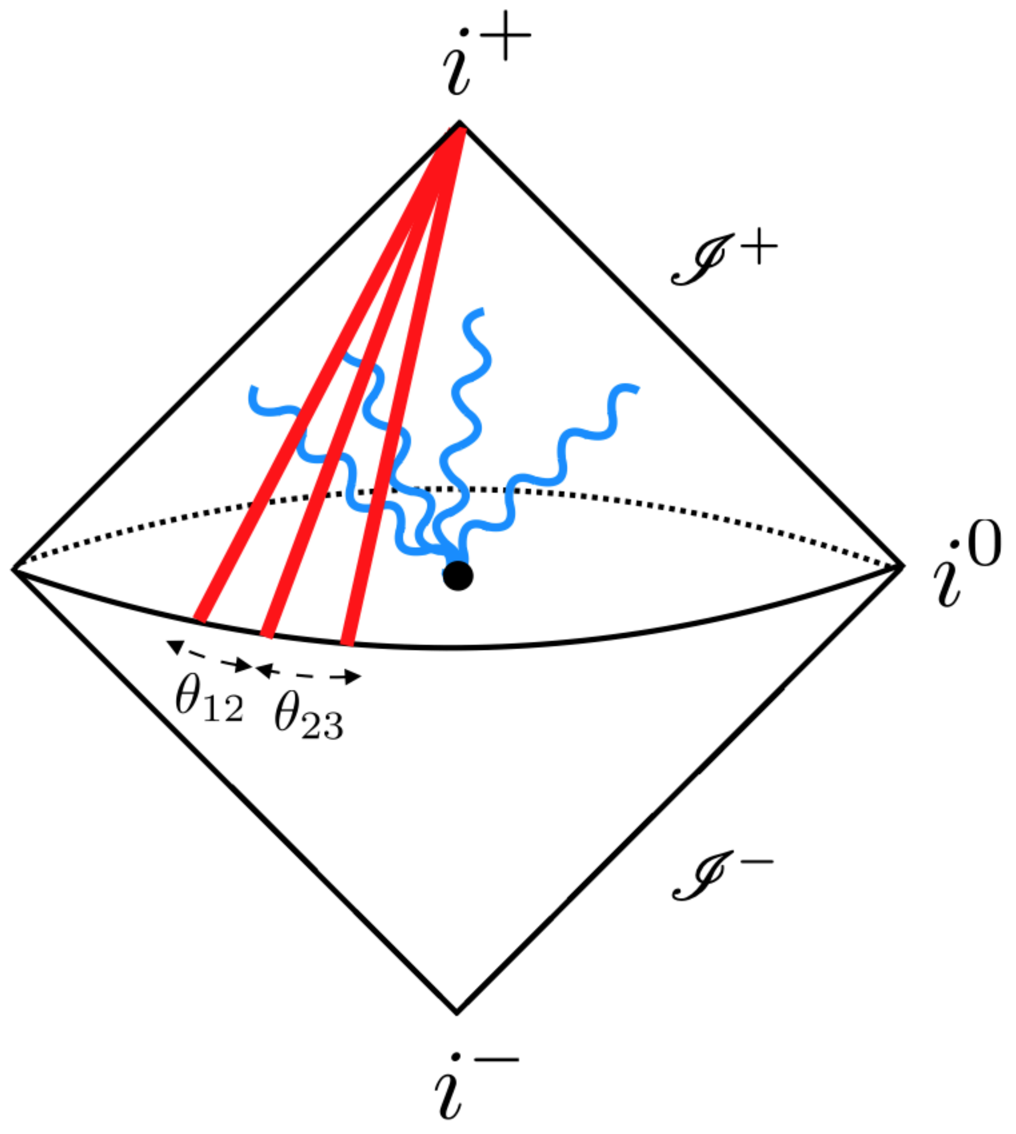


$$SO^+(1,3) \simeq SL(2,\mathbb{C})/\mathbb{Z}_2$$

Bootstrap (CFTs, S-matrix,...)

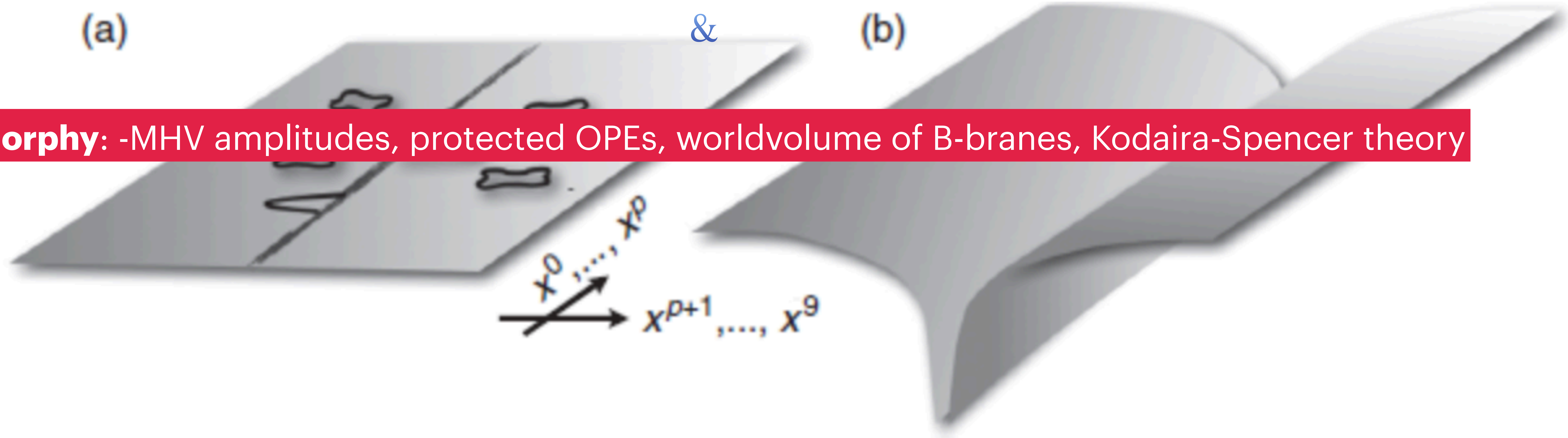


$$\mathcal{O}_1(z,\bar{z})\,\mathcal{O}_2(0,0) = \sum_{k_{\text{Schur}}} \frac{\lambda_{12k}}{z^{h_1+h_2-h_k}}\,\mathcal{O}_k(0) + \{\mathbb{Q},\ldots]$$



In all these arenas we see (hints of):

Chiral algebras: -asymptotic symmetries in flat space, protected subsectors of (SUSY) CFTs, twisted string theory,...

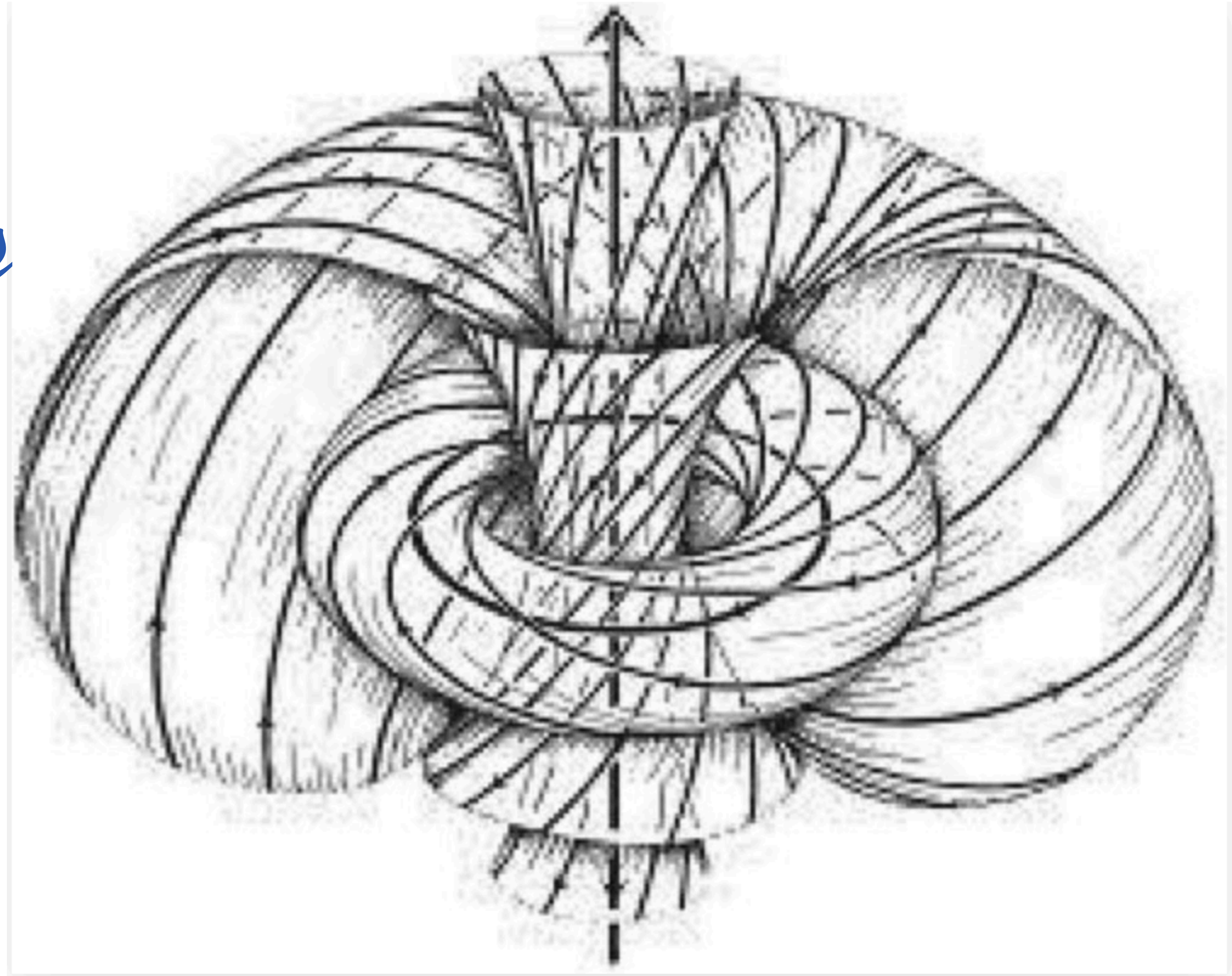


Holomorphy: -MHV amplitudes, protected OPEs, worldvolume of B-branes, Kodaira-Spencer theory

One may hope to tie these structures together
(at the level of universal/symmetry-governed/soft/conformal... physics)
in a fruitful and clarifying way

Today: we will start to flesh out some of those connections

Holomorphic theories on Twistor space



6d bridge between 4d & 2d massless physics

To *any* local twistor thy:

1. **a 4d (non-unitary) CFT [Penrose transform]**
2. **a 2d chiral algebra [Koszul duality]**

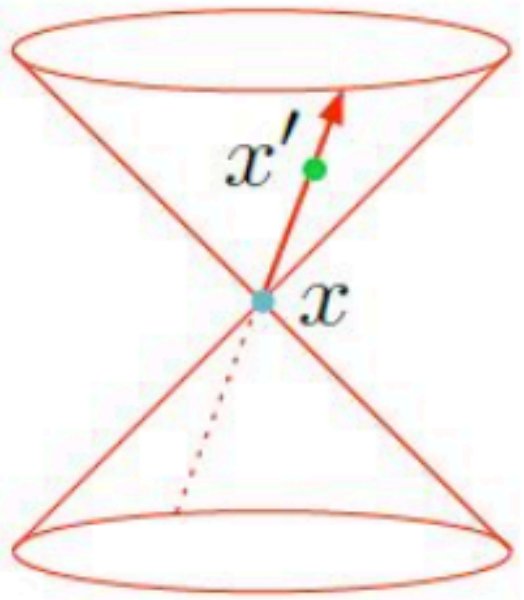
From this perspective:

form factors which reproduce some 4d QCD scattering amplitudes	⟷	correlators of a certain 2d chiral algebra

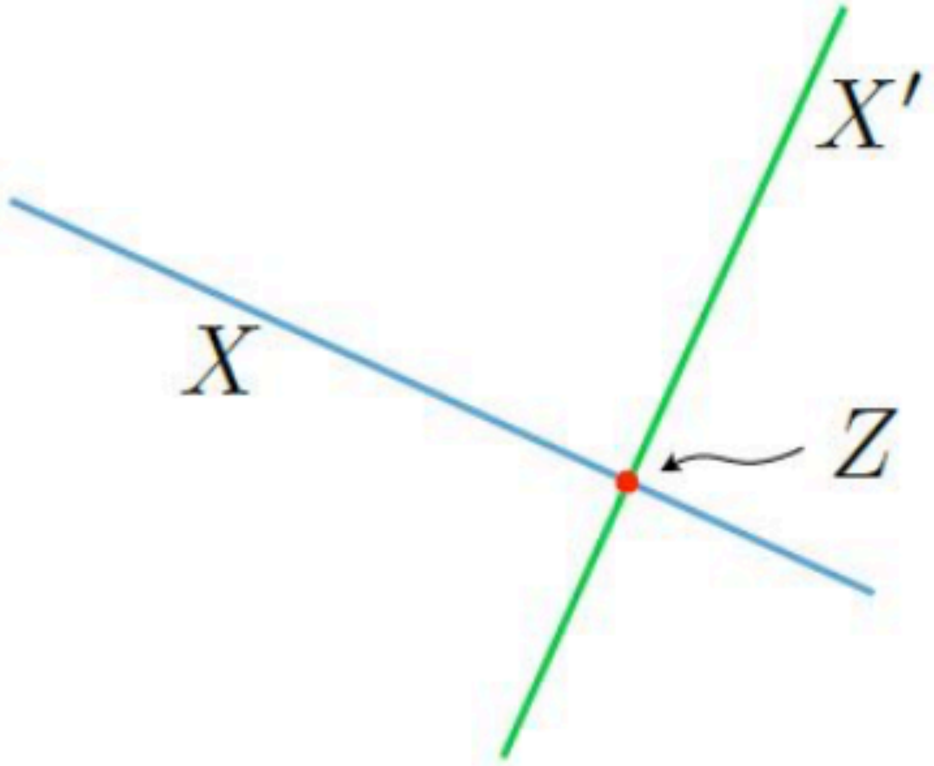
[Costello: 2111.08879]

See Costello's Strings `21 talk

What's nice about 4d theories w/ twistorial uplifts?



$x \in \mathbb{C}^4$



$\mathbb{CP}^1_x \in \mathbb{PT}$

$$\mathbb{PT} = \mathcal{O}(1) \oplus \mathcal{O}(1) \simeq \mathbb{R}^4 \times \mathbb{CP}^1$$

$$\downarrow$$

$z \in \mathbb{CP}^1$ $\qquad \mathbb{CP}^1$

Penrose transform $\quad H^{0,1}(\mathbb{PT}, \mathcal{O}(2h-2))$

hol'c massless fields on $\mathbb{C}^4$

solutions to massless field equations

harmonic functions

entire analytic functions, can pass to any signature

- $\mathbb{CP}^1_x, \mathbb{CP}^1_y$ intersect iff $|x-y|^2 = 0$
- ∴ correlation fns entire analytic (rational) fns
- singularities on lightcone
- no anomalous dimensions
- integrability (cf. motivations)

[Penrose, Atiyah, Ward, Woodhouse, Mason, Lebrun,...]

The Setting for Today's Talk

full (twisted) string embedding?

cf. [Witten, Mason-Skinner,...]

WIP w/ Costello, Sharma

↑ lift to 10d?

a local hol'c theory on $\mathbb{PT}$

``universal defect" on $\mathbb{CP}^1$ ↙

inspired by twisted AdS/CFT program [Costello-Gaiotto, Costello-NP]

↘ shrink $\mathbb{CP}^1$

A 2d chiral algebra (hol'c "half"of CFT) on celestial sphere

Mathematically: Koszul dual [Costello, Costello-NP, NP-Williams,...]

non-unitary 4d CFT (SDYM + "axion")

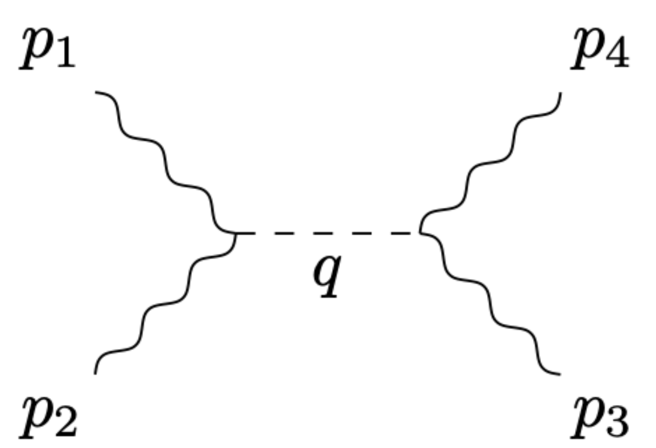
A local hol'c theory on $\mathbb{PT}$

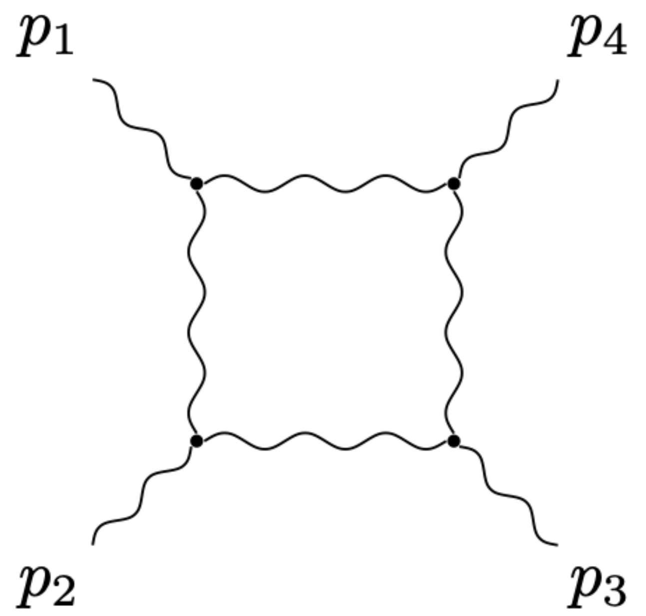
Classically
[Penrose, Ward]

$$\int_{\mathbb{PT}} \mathrm{Tr}(\mathscr{B}\mathscr{F}^{(0,2)}(\mathscr{A})) \mapsto \int_{\mathbb{R}^4} \mathrm{Tr}(BF(A)_-)$$

$\mathscr{B} \in \Omega^{3,1}(\mathbb{PT}, \mathfrak{g})$
$\mathscr{A} \in \Omega^{0,1}(\mathbb{PT}, \mathfrak{g})$

$B \in \Omega^2_-(\mathbb{R}^4, \mathfrak{g})$





[Costello-Li: 1905.09269]
[Costello: 2111.08879]

6d: free "closed string" (BCOV) sector (twist of 6d $\mathcal{N} = (1,0)$ gauge + 1 tensor)

$$\frac{1}{2}\int (\partial^{-1}\eta)(\bar{\partial}\eta) + k\hat{\lambda}_g \int \eta \mathrm{tr}(\mathscr{A}\partial\mathscr{A})$$

4d: scalar field w/ quartic kinetic term cf. Fradkin/Tseytlin, Komargodski/Schwimmer, Riegert,...

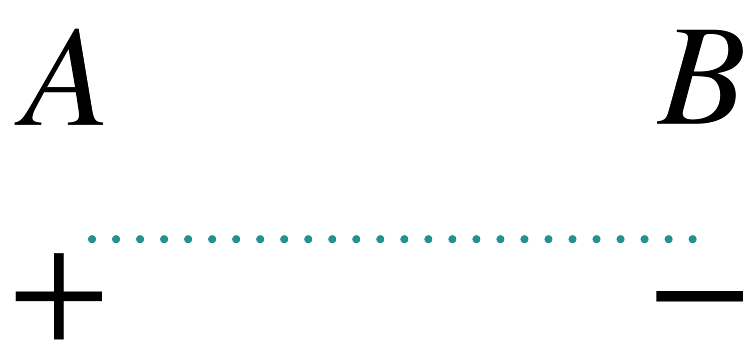
$\mathfrak{g} = su(2), su(3), so(8), e_{6,7,8}$

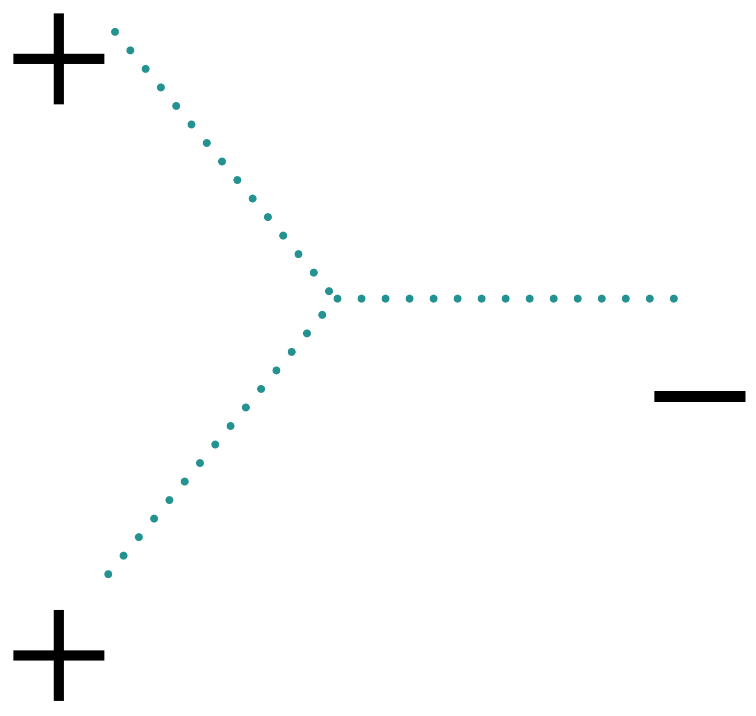
$su(N_c) +$ matter $(N_f = N_c)$

non-unitary 4d CFT
(SDYM + "axion")

$$\frac{1}{2}\int (\Delta\rho)^2 + k'\hat{\lambda}_g \int \rho(F \wedge F)$$

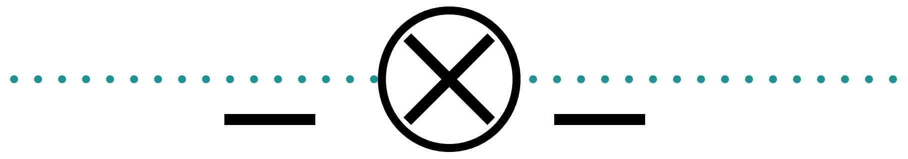
Self-dual YM





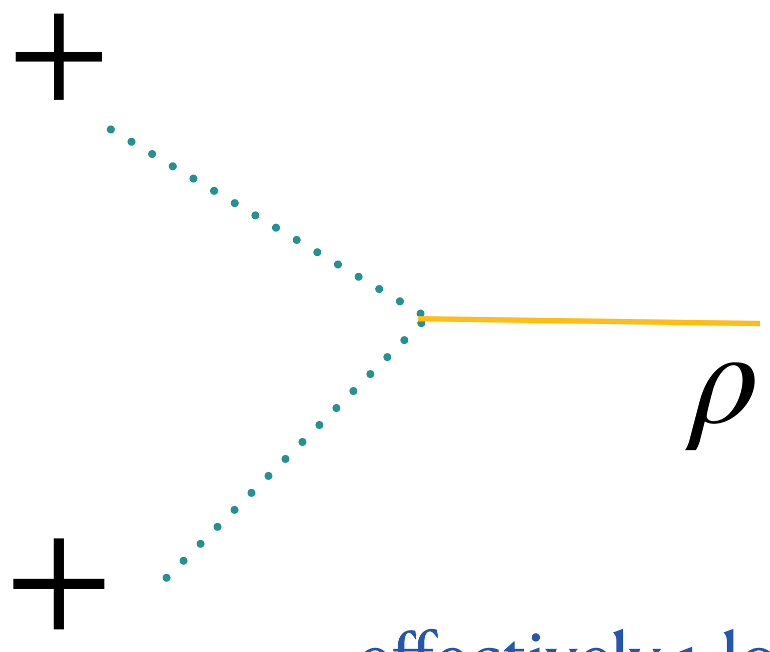
form factors:

$\mathrm{tr}(B^2)(x)$



L loops, N insertions →
N-L+1 (-) helicity, arbitrary (+) helicity gluons
in QCD (integrand)

+ axion



effectively 1-loop
by Green-Schwarz

4d OPE:

$$\mathrm{Tr}B^2(0)\mathrm{Tr}B^2(x_1)\ldots\mathrm{Tr}B^2(x_{n-1}) \sim \sum_i F_i(x_1,\ldots,x_{n-1})\mathcal{O}^i(0)$$

rational,
constrained by associativity

$$\mathrm{tr}(B^2)(0)\,\mathrm{tr}(B^2)(x) \sim \frac{1}{\|x\|^2} B^a_{\alpha_1\beta_1} B^b_{\alpha_2\beta_2} B^c_{\alpha_3\beta_3} f_{abc}\epsilon^{\beta_1\alpha_2}\epsilon^{\beta_2\alpha_3}\epsilon^{\beta_3\alpha_1}.$$

4d form factors as computed by 2d chiral correlators

$P^{\alpha\dot{\alpha}} =: \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$

$\lambda^{\alpha} \equiv (1, z)$

tree-level

$\langle tr(B^2)\,|\,\tilde{J}(z_1)\tilde{J}(z_2)J(z_3)\ldots J(z_n)\rangle \leftrightarrow$ color ordered MHV amps [Parke-Taylor]

$\frac{1}{|x|^2}\langle tr(B^3)\,|\,\tilde{J}(z_1)\tilde{J}(z_2)\tilde{J}(z_3)J(z_4)\ldots J(z_n)\rangle \leftrightarrow$ NMHV amps in CSW form [Cachazo-Svrcek-Witten]

1-loop (axion comes in)

$\langle(\Delta\rho)^2\,|\,J_{a_1}(\tilde{\lambda}_1, z_1)\ldots J_{a_n}(\tilde{\lambda}_n, z_n)\rangle \leftrightarrow$ all-(+) one-loop in SDYM/QCD [Mahlon, Bern et. al.,...]

$$\langle tr(B^2)\,|\,\tilde{J}_{a_1}(\tilde{\lambda}_1, z_n)J_{a_2}(\tilde{\lambda}_2, z_2)\ldots J_{a_n}(\tilde{\lambda}_n, z_n)\rangle =$$

$$\frac{1}{192\pi^2}\sum_{2\le i<j\le n}\frac{[ij]\langle 1i\rangle^2\langle 1j\rangle^2}{\langle ij\rangle\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}\mathrm{Tr}(t_1\ldots t_n) + \text{ perms in } S_{n-1}$$

[Costello-NP]

cf. [Dixon-Glover-Khoze] for QCD

Whence a chiral algebra?
(1) Generators

conformal primary states on twistor space of neg. weight
(on-shell gauge theory states)

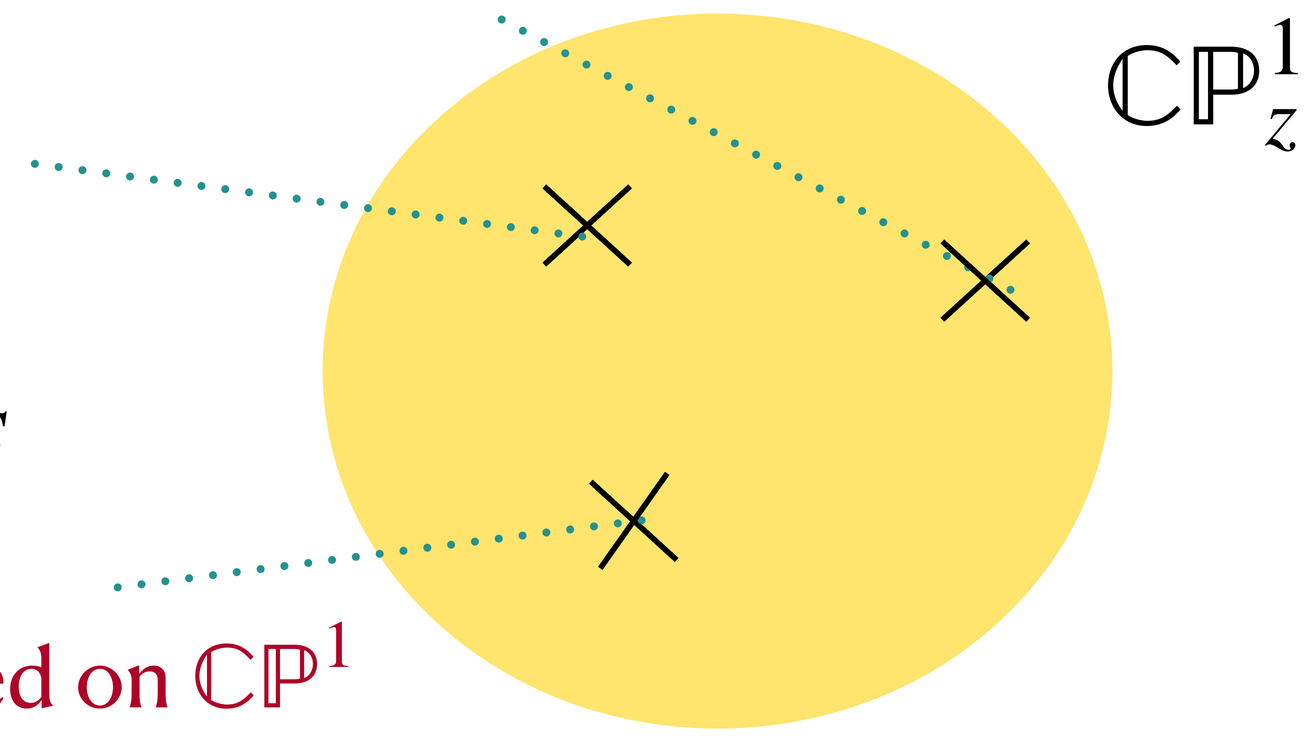
Penrose transform

4d basis of conformal primary states w/ neg. weight

[Pasterski-Shao-Strominger]

$$J[r,s](z_i) \leftrightarrow \mathscr{A} = \delta_{z=z_i}(\tilde{\lambda}^{\dot{1}})^r(\tilde{\lambda}^{\dot{2}})^s$$

state in vacuum module = on-shell background field localized on $\mathbb{CP}^1$



$$\langle ij \rangle = z_i - z_j$$

Momentum eigenbasis:

$$J(\tilde{\lambda}, z) = \sum_{r,s} \omega^{r+s} \frac{(\tilde{\lambda}^{\dot{1}})^r(\tilde{\lambda}^{\dot{2}})^s}{r!s!} J[r,s](z)$$

Generator	Spin	Weight	$SU(2)_+$ representation	Field	Dimension
$J[m,n]$, $m,n \geq 0$	$1-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	A	$-m-n$
$\tilde{J}[m,n]$, $m,n \geq 0$	$-1-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	B	$-m-n-2$
$E[m,n]$, $m+n > 0$	$-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	ρ	$-m-n$
$F[m,n]$, $m,n \geq 0$	$-(m+n)/2$	$(m-n)/2$	$(m+n)/2$	ρ	$-m-n-2$

Table 1: The generators of our 2d chiral algebra and their quantum numbers. Dimension refers to the charge under scaling of $\mathbb{R}^4$.

Whence a chiral algebra?
(2) Conformal blocks

$$\mathbb{PT} \setminus \mathbb{CP}^1_0 = S^3 \times \mathbb{CP}^1 \times \mathbb{R}_{>0}$$

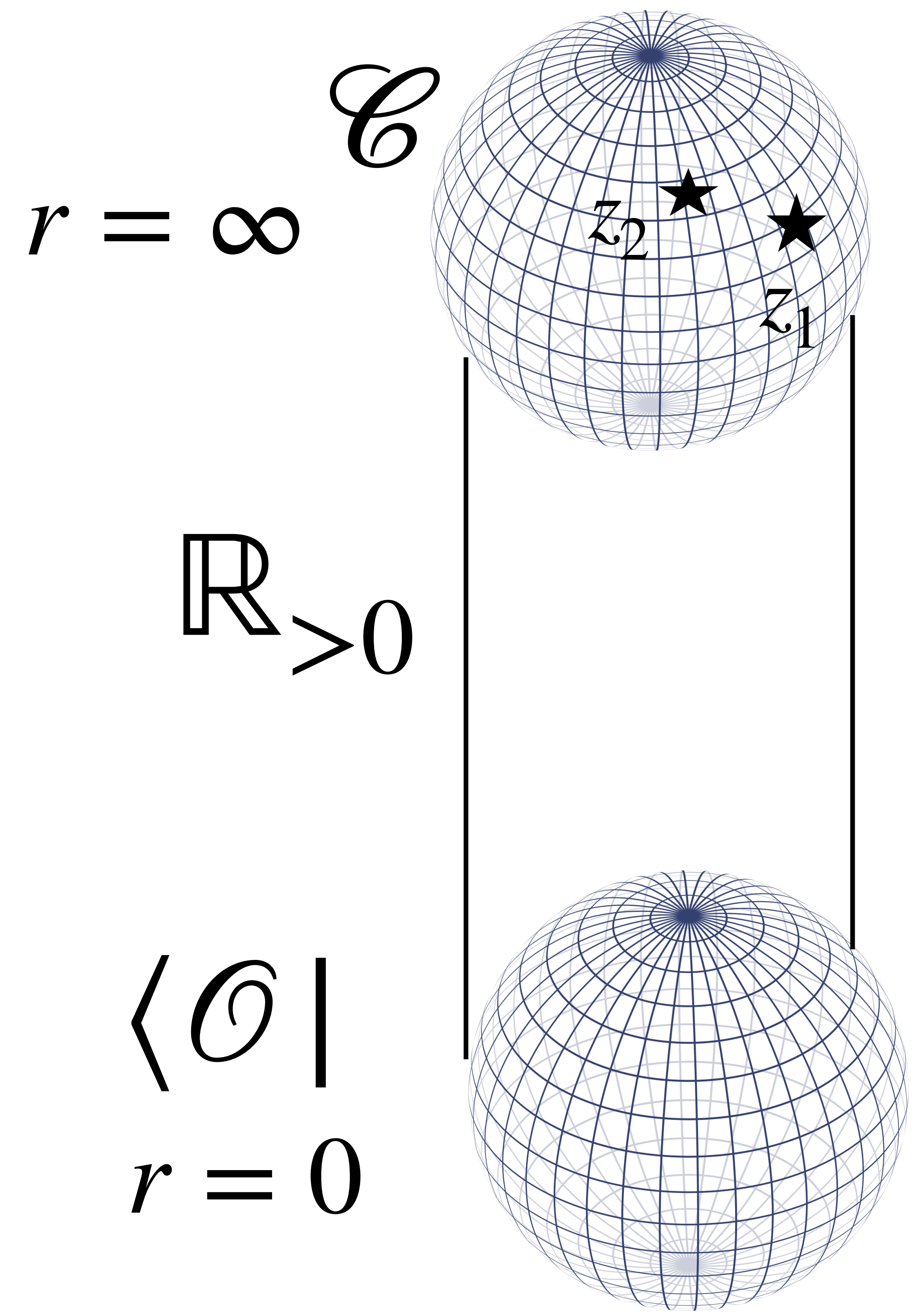
include all KK modes!

$$\mathbb{R}^4 \setminus 0 = \mathbb{R}_{>0} \times S^3 \qquad\qquad \mathbb{R}_{>0} \times \mathbb{CP}^1_z$$

$\mathscr{H}(S^3) :=$ space of local operators

$\mathscr{H}(\mathbb{CP}^1) :=$ space of conformal blocks

$$\mathcal{O}(0) \leftrightarrow \langle \mathcal{O} |$$



$$\langle \mathcal{O} | J(z_2) J(z_1) \rangle$$

Chiral algebra as 3d bdy algebra:

hol'c-top'l theory (twist of 3d $\mathcal{N} = 2$)

- chiral algebra a boundary condition at ∞
- conformal block a state at 0
- correlator in 3d bulk/boundary system

Whence a chiral algebra?
(3) OPEs

$$\sum_{r,s\geq 0}\int_{\mathbb{CP}^1_z}(\partial^r_{\tilde\lambda^{\dot 1}}\partial^s_{\tilde\lambda^{\dot 2}}\mathcal{B}^a_{\bar z})\tilde J_a[r,s](z)$$

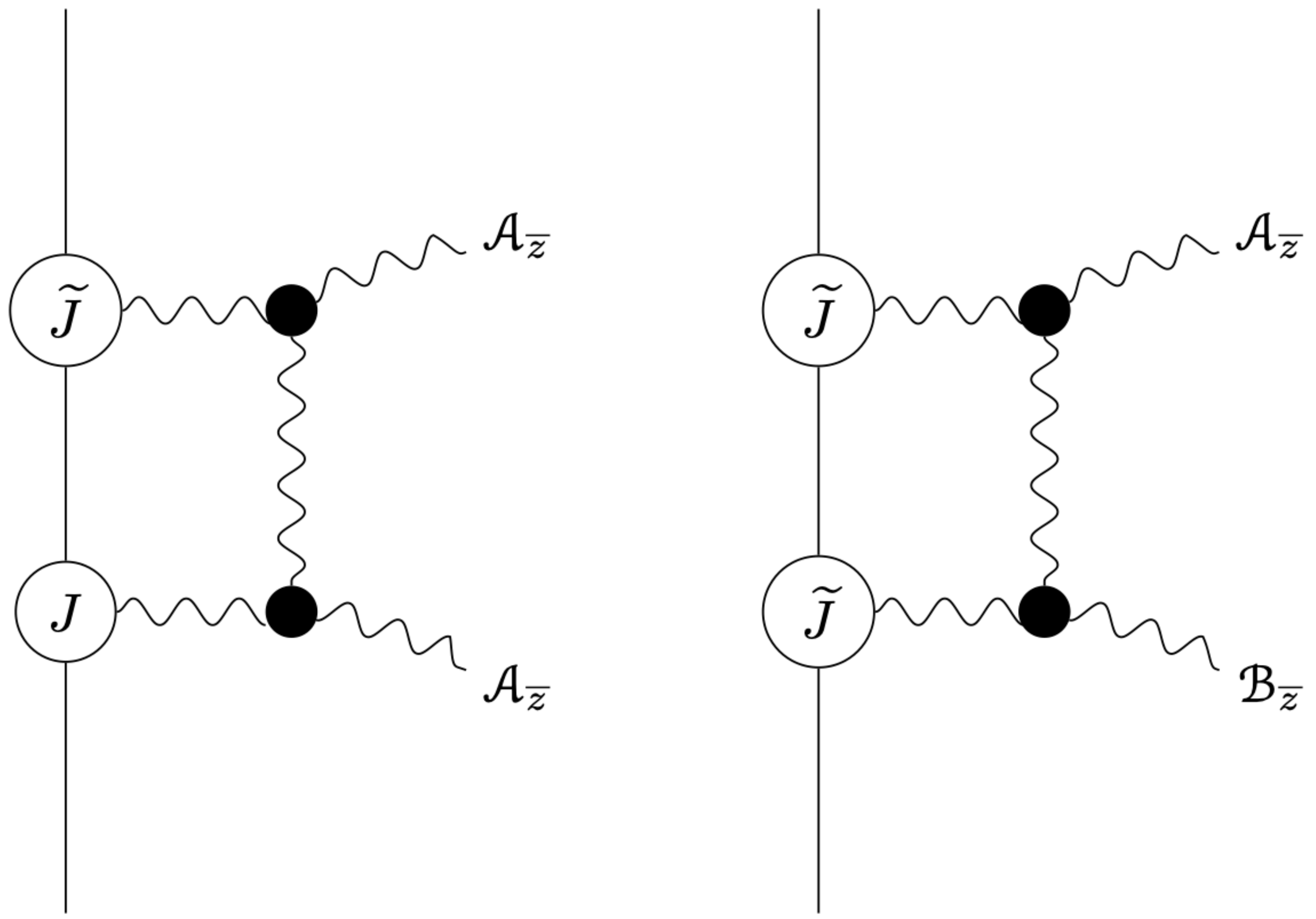
$$\sum_{r,s\geq 0}\int_{\mathbb{CP}^1_z}(\partial^r_{\tilde\lambda^{\dot 1}}\partial^s_{\tilde\lambda^{\dot 2}}\mathcal{A}^a_{\bar z})J_a[r,s](z)$$

OPEs among currents on defect by imposing gauge (BV-BRST) invariance

→ universal or "Koszul dual" algebra
Tree level: recover current algebra for gauge symmetry

gauge inv't couplings to arbitrary defect
↔ Hom from Koszul dual algebra into defect algebra

Quantum deformations:



cf. e.g. integrating out d.o.f. to obtain Wilson lines
[Gomis-Passerini]

Koszul duality:

In twisted holography [Costello, Costello-NP] + WIP

In field theory ***(includes math review for physicists!)*** [NP-Williams]

$$J^a[r,s](0)J^b[t,u](z) \sim \frac{1}{z} f^{ab}_c J^c[r+t,s+u](0)$$

$$J^a[r,s](0)\widetilde{J}^b[t,u](z) \sim \frac{1}{z} f^{ab}_c \widetilde{J}^c[r+t,s+u](0)$$

$$J^a[r,s](0)E[t,u](z) \sim \frac{1}{z}\frac{(ts-ur)}{t+u}\widetilde{J}^a[t+r-1,s+u-1](0)$$

$$J^a[r,s](0)F[t,u](z) \sim -\frac{1}{z}\partial_z \widetilde{J}^a[r+t,s+u](0) - \frac{1}{z^2}(1+\frac{r+s}{t+u+2})\widetilde{J}^a[r+t,s+u](0)$$

$$J^a[r,s](0)J^b[t,u](z) \sim \frac{1}{z}K^{ab}(ru-st)F[r+t-1,s+u-1](0)$$
$$-\frac{1}{z}K^{ab}(t+u)\partial_z E[r+t,s+u](0) - \frac{1}{z^2}K^{ab}(r+s+t+u)E[r+t,s+u](0).$$

Tree-level

Includes level-0 Kac-Moody algebra for $\mathrm{Maps}(\mathbb{C}^2, \mathfrak{g})$

[Guevara-Himwich-Pate-Strominger]

[Strominger]

Failure of associativity in pure SDYM theory in one-loop
Axion field necessary for its restoration

[Bern-Dixon-Kosower]

$$\mathrm{Split}^{[1]}_{+}(a^+, b^+) = -\frac{N_c}{96\pi^2}\frac{[ab]}{\langle ab\rangle^2}$$

$$J_a[1,0](0)J_b[0,1](z)$$
$$= -\frac{1}{2\pi i z} C K^{fe}(f^c_{ae}f^d_{bf} + f^d_{ae}f^c_{bf}) : J_c[0,0]\widetilde{J}_d[0,0] :$$
$$+ \frac{1}{2\pi i z}\tfrac{1}{2} D f^c_{ab}\partial_z \widetilde{J}_c(0) + \frac{1}{2\pi i z^2} D f^c_{ab}\widetilde{J}_c(0).$$

Quantum deformation

C, D fixed by
anomaly coefficient in 6d

Main Theorems [Costello-Paquette: 2201.02595]

2d chiral algebra	4d theory
conf. primary generators	conf. primary states (boost eigenbasis)
OPEs	collinear limits
conformal blocks (cf. CS/WZW)	local operators
correlation functions	form factors

- Hol'c defect on $\mathbb{CP}^1_0$ in 6d, integrate out 2d fields → state/operator at $0 \in \mathbb{R}^4$
- **Exchange of 6d fields doesn't contribute to amplitude: *localization to 2d physics***
- 6d conformal primaries at different points on sphere *talk to each other by exchange of 2d defect fields only*

Final thoughts & future directions

- Top-down topological string model of celestial holography (WIP w/ Costello & Sharma)
- Magnetic monopoles/spectral flow in chiral algebra (WIP w/ Garner)
- Self-dual gravity! (Anomaly cancellation: Bittleston-Sharma-Skinner)
- Compute more QCD (+ matter?) amplitudes: via multi-point axion exchange? via anomaly-cancelling theories for special choices of matter?

Thank you!

Example: the Parke-Taylor Formula

$$\langle tr(B^2)\,|\,\tilde{J}(z_1)\tilde{J}(z_2)J(z_3)\ldots J(z_n)\rangle \leftrightarrow \text{ color ordered MHV amps}$$

1. $\langle tr(B^2)\,|\,\tilde{J}^a(z_1)\tilde{J}^b(z_2)\rangle = K^{ab}(z_1 - z_2)^2$ direct computation from 6d, or fixed by symmetry arguments

2. $\langle tr(B^2)\,|\,\tilde{J}^a(z_1)\tilde{J}^b(z_2)J^c(z_3)\rangle = \quad f^{cb}_d \frac{1}{z_{23}}\langle \mathrm{tr}(B^2)\,|\,\tilde{J}^a(z_1)\tilde{J}^d(z_2)\rangle + f^{ca}_d \frac{1}{z_{13}}\langle \mathrm{tr}(B^2)\,|\,\tilde{J}^d(z_1)\tilde{J}^b(z_2)\rangle$ use OPE

$$= \frac{z_{12}^3}{z_{13}z_{23}} f^{abc}$$ identity 1.

+ elementary induction:

$$\langle tr(B^2)\,|\,J^{a_1}(z_1)\ldots\tilde{J}^{a_2}(z_i)\ldots\tilde{J}^{a_j}(z_j)\ldots J^{a_n}(z_n)\rangle = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle} tr(t^{a_1}\ldots t^{a_n}) + \text{ permutations}$$