Holographic Complexity and de Sitter Space

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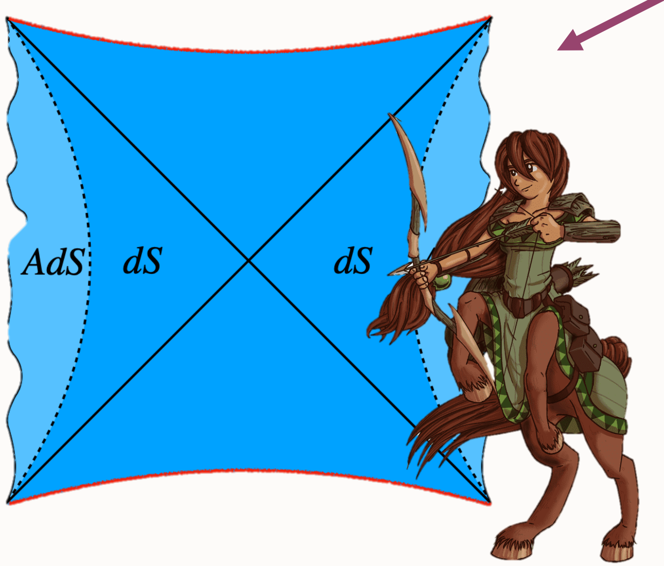
JHEP 02 (2022) 198 - with E.D. Kramer, D.A. Galante

Work in progress with S. Baiguera, R. Berman

Poster session!

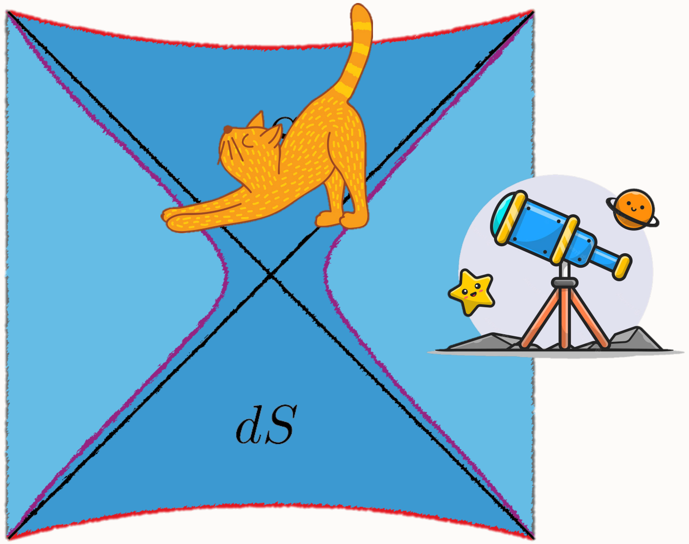
Goal and Outline

Can we use holography to probe the de-sitter horizon?

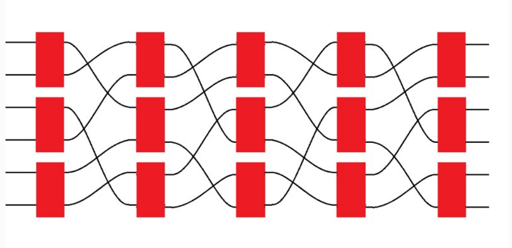


Centaur geometries – embedding dS in AdS

Holography at the stretched horizon



What does holographic complexity tell us about the dual theory?

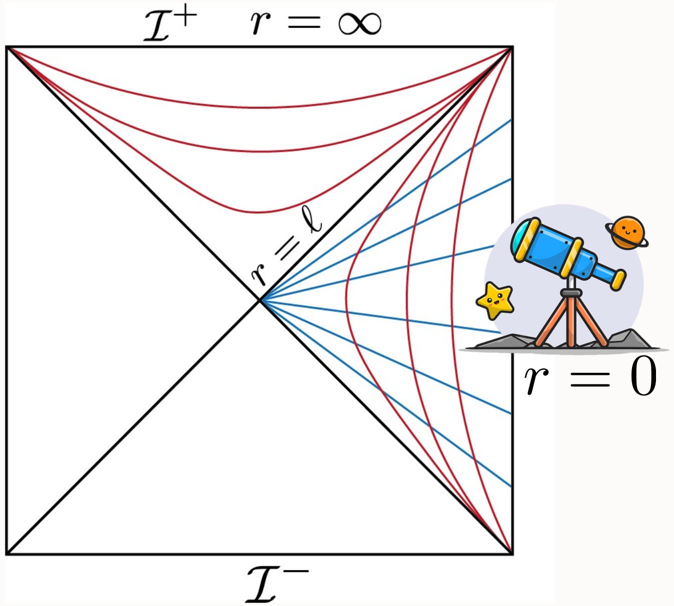


Quick Recap of de Sitter Space

- Maximally symmetric solution to Einstein's equations with positive cosmological constant

$$\Lambda = \frac{(d-1)(d-2)}{2\ell^2} > 0$$

- Describes early universe (inflation) and its far future.



$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-2}$$

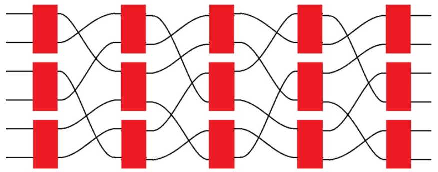
$$f(r) = 1 - \frac{r^2}{\ell^2}$$

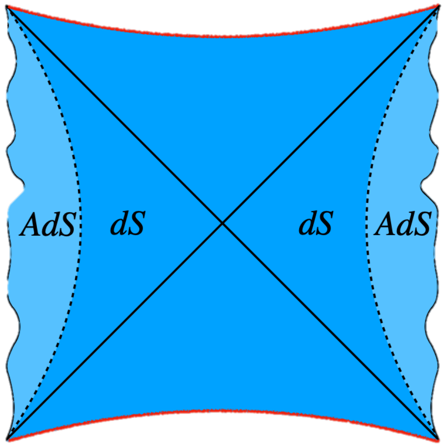
Holography in dS Space

- **Why holography in dS space?** We hope to extend the success it had in AdS! (hydrodynamics, entanglement, thermalization and chaos, quantum computation, BH evaporation and the Page curve)
- **Why is this hard?** No asymptotic timelike boundary within the static patch where we can put the field theory.
- **Different approaches:**
  - dS-CFT at future infinity [Strominger; Witten; Maldacena; ….]
  - Embed dS inside AdS [Anninos, Galante, Hofman; Freivogel, Hubeny, Maloney, Myers, Rangamani, Shenker;…]
  - Holography at the stretched horizon? [Banks, Fischler, Fiol, Morisse; Susskind, Shaghoulian, Lin; Jørstada, Myers, Ruan, …]
  - QM models with finite degrees of freedom [Banks, Fischler, Fiol, Morisse; Parikh, Verlinde; Bousso; Balasubramanian, Horava, Minic; …)
  - dS/dS correspondence and $T\bar{T} + \Lambda_2$ deformation [Alishahiha, Karch, Silverstein, Tong; Dong, Horn, Silverstein, Torroba; Gorbenko, Silverstein, Torroba; Lewkowycz, Liu, Silverstein, Torroba; Shyam; Coleman, Mazenc, Shyam, Silverstein, Soni, Torroba, Yang, …]
  - And Many others…

First Approach

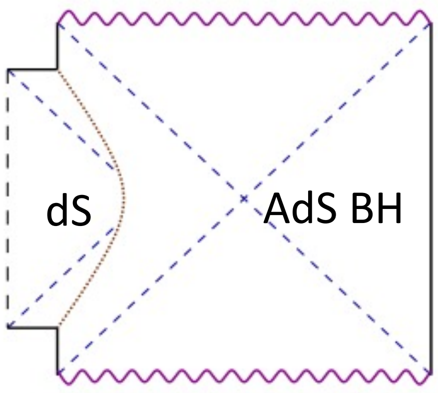
Holographic Complexity in Centaur Geometries





Centaur (Flow) Geometries

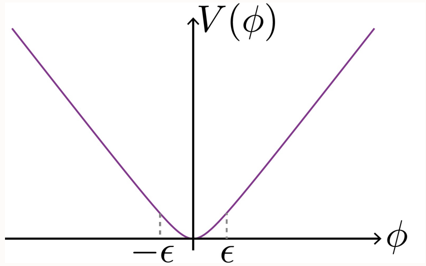
- Embed a portion of dS inside AdS.
- Attempted in the past, but inflating region (and dS horizon) always hidden behind a black hole horizon from the AdS boundary.
  [Freivogel, Hubeny, Maloney, Myers, Rangamani, Shenker, '05]
- Obstruction: Null energy+Raychaudhuri's equation – congruence of null geodesics leaving the AdS boundary cannot converge and then diverge.
- Loopholes!
  - 2d geometries – no sphere! (Centaur: dS_2 in AdS_2)
    [Anninos, Hofman; Anninos, Galante, Hofman]
  - Embedding dS_4 in $AdS_2 \times S_2$
    [Anninos, Galante, Mühlmann]



Centaur (Flow) Geometries

(from now on $\ell = 1$)

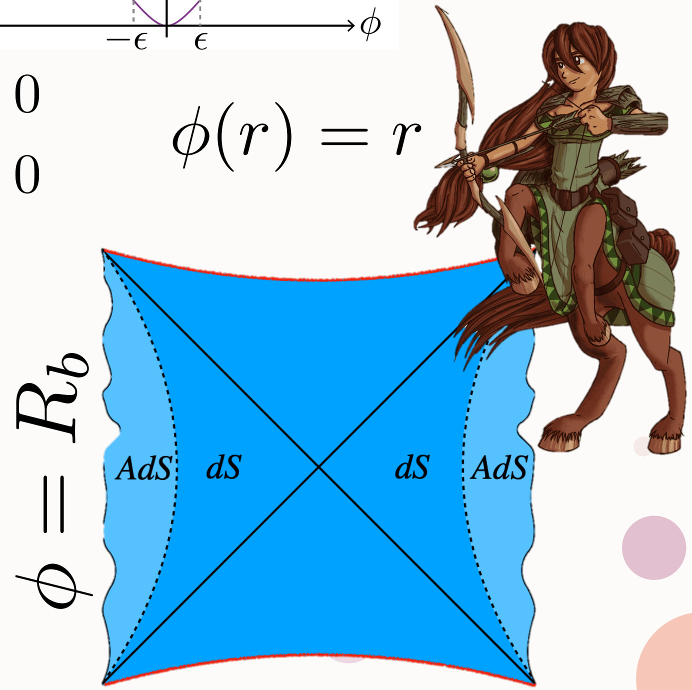
- Solutions to 2d dilaton-gravity $S = \frac{1}{16\pi G}\int d^2x\sqrt{-g}\left(\phi R + \ell^{-2}U(\phi)\right) + \begin{matrix}\text{topological/boundary}\\ \text{terms}\end{matrix}.$
- With a piecewise potential $U(\phi) = \begin{cases} 2\phi & \phi \gg \epsilon \\ -2\phi & \phi \ll -\epsilon \end{cases}$



- Metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} \qquad f(r) = \begin{cases} 1 - r^2 & r < 0 \\ 1 + r^2 & r > 0 \end{cases} \qquad \phi(r) = r$$

- Interpolates from a UV AdS_2 boundary to a dS_2 spacetime in the IR.
- AdS_2 Caps off at $\phi = R_b$.
- For our calculation use a sharp transition.

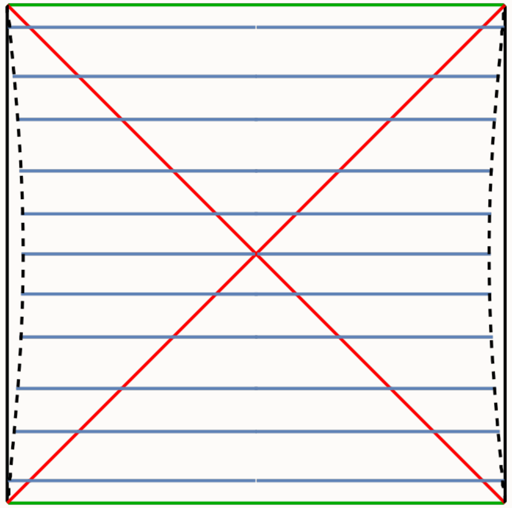


[Anninos, Hofman]

Complexity in Centaur Geometries

- Complexity – minimal number of operations to construct a state.
- Here we use the complexity=volume conjecture

AdS_2 BH



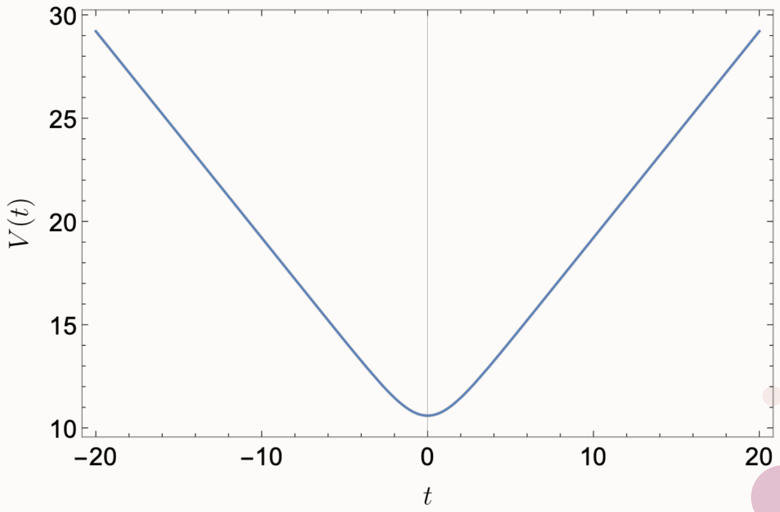
Inverse temperature
$\beta = \frac{1}{T} = 2\pi\ell$

$$L(t) \cong 2\log\left(2R_b\cosh\frac{t}{2\beta}\right)$$

valid for:
$R_b \gg \ell = 1$

at late times:

$$L(t) \sim 2\log R_b + |t|/\beta + \cdots$$

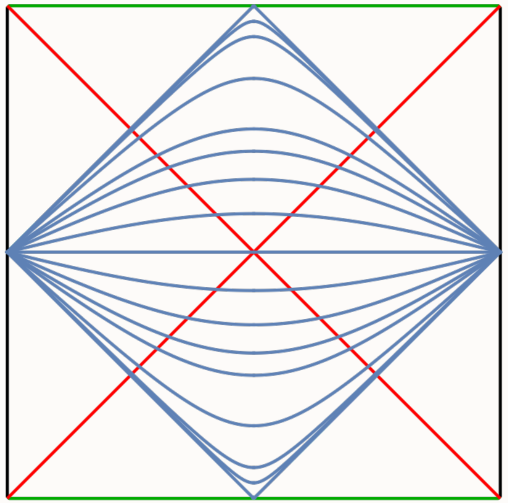


Late time growth – chaos?

[SC, Galante, Kramer JHEP 02 (2022) 198]

Complexity in Centaur Geometries

dS_2 volume/length
(no complexity interpretation)

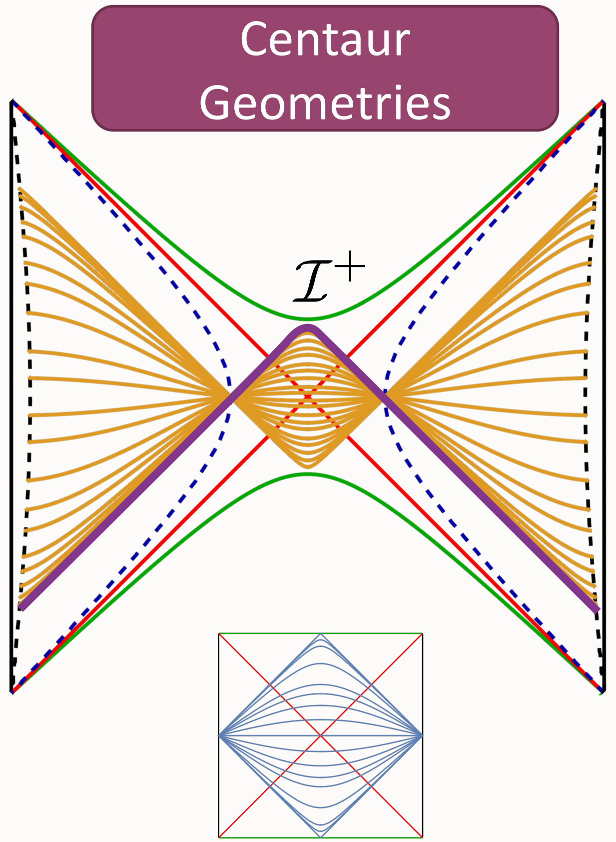


$$L(t) = \pi$$

[SC, Galante, Kramer JHEP 02 (2022) 198]

Complexity in Centaur Geometries

- Complexity=volume conjecture

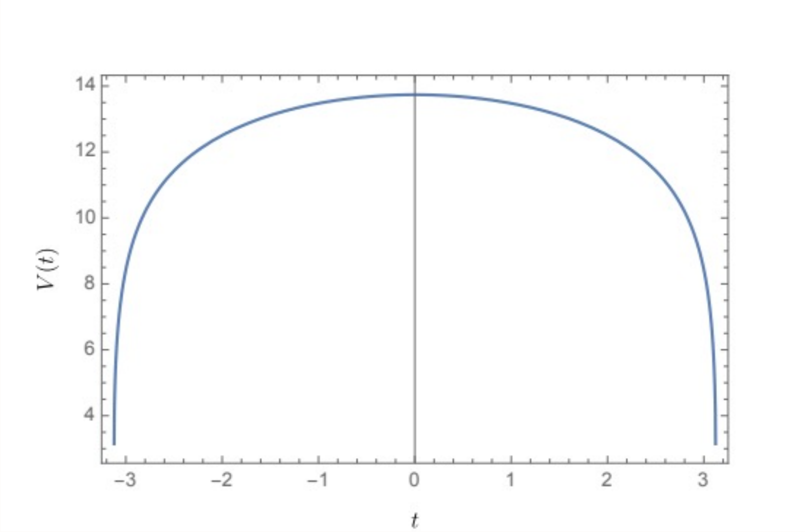


$$L(t) \cong \pi + 2\log\left(2R_b\cos\frac{t}{2\beta}\right)$$

Inverse temperature
$\beta = \frac{1}{T} = 2\pi\ell$

valid for:

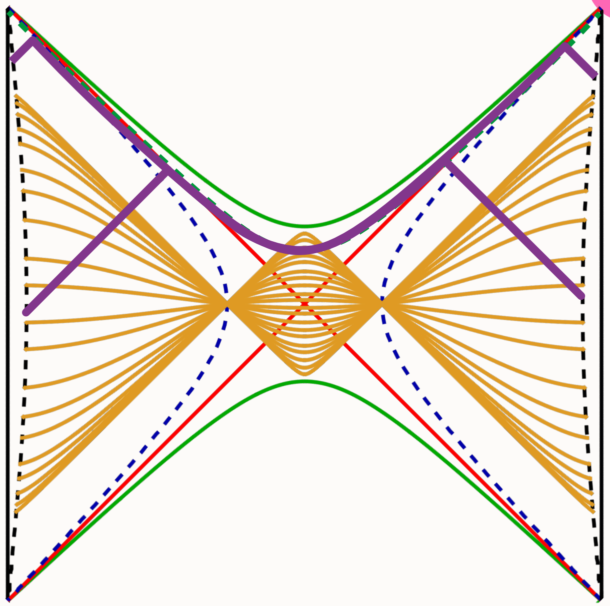
$$R_b \gg \tan\frac{|t|}{2}$$



1. Geodesics do not grow linearly with time. In fact, they decrease in time!
2. Geodesics only exist for $|t/\beta| \leq \pi$.

[SC, Galante, Kramer JHEP 02 (2022) 198]

Questions and Speculations



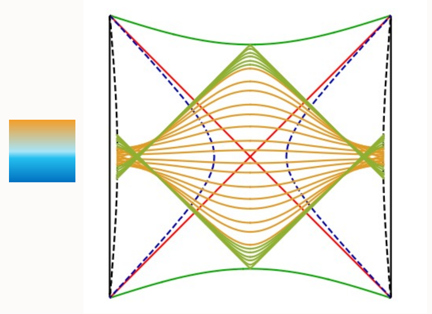
- What happens aftertime $t_c = \pi\beta$?
- One option – place a cutoff at $\mathcal{J}^+$ and consider also piecewise geodesics along this cutoff [Jørstad, Myers, Ruan]
- Complexity will grow linearly with time at a rate dictated by the cutoff - hyperfast scrambling ideas of [Susskind et al.]. What is this cutoff at $\mathcal{J}^+$ - cutoff for Euclidean CFT?
- Upon analytic continuation, the length becomes complex. In the context of complexity, not clear what it means.
  - A proposal that the Centaur geometries consistent with SYK with complex deformation appears in [Anninos, Galante]
- Another option – take it seriously?
  The decrease in complexity seems to indicate that the dual theory is not Chaotic.
  Additional indications:
  - OTOC does not display the exponential Lyapunov behavior but rather oscillates. [Anninos, Galante, Hofman]
  - Quasi normal modes have a large real part – less efficient dissipation. [Anninos, Hofman]
  - ….

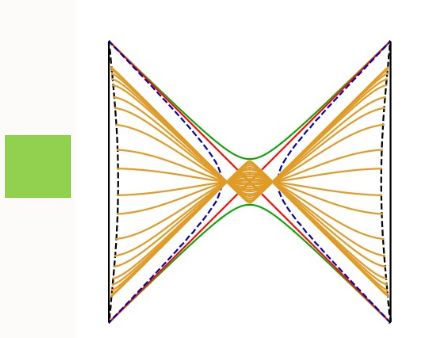
Further Tests

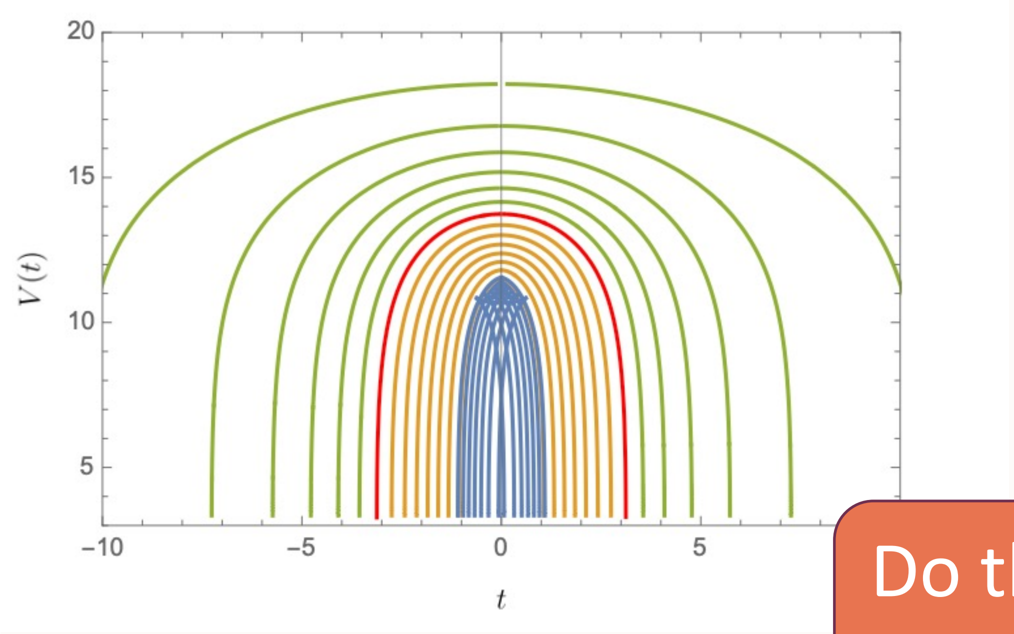
- Is this due to the sharp gluing? Not really.

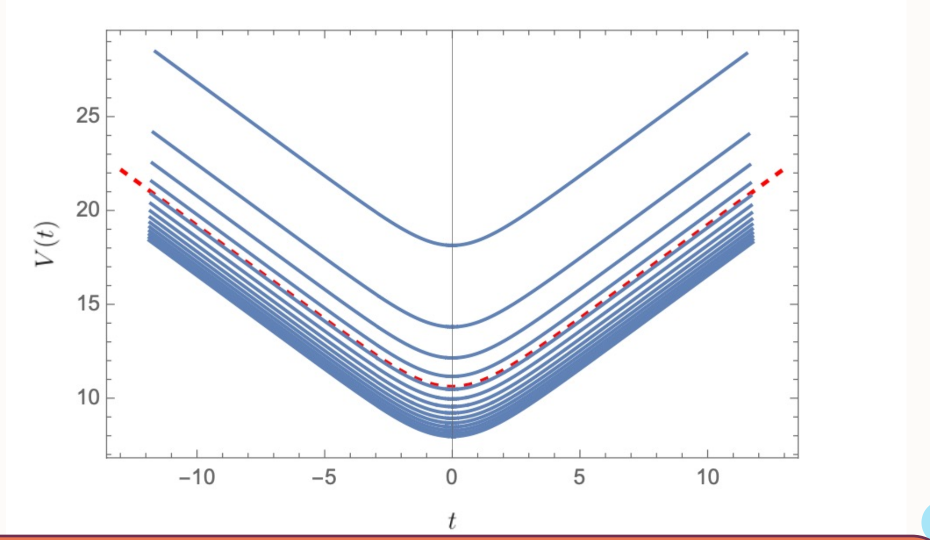
Gluing to larger/smaller portion of dS_2

AdS_2/AdS_2 transitions





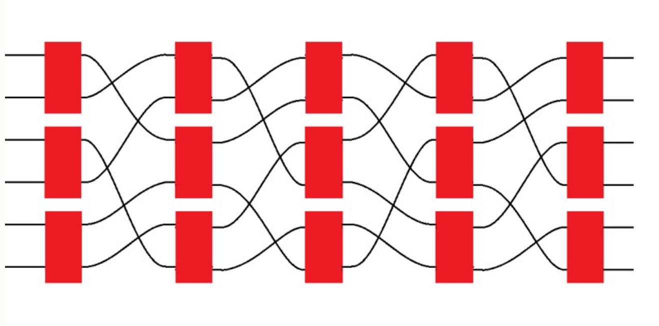


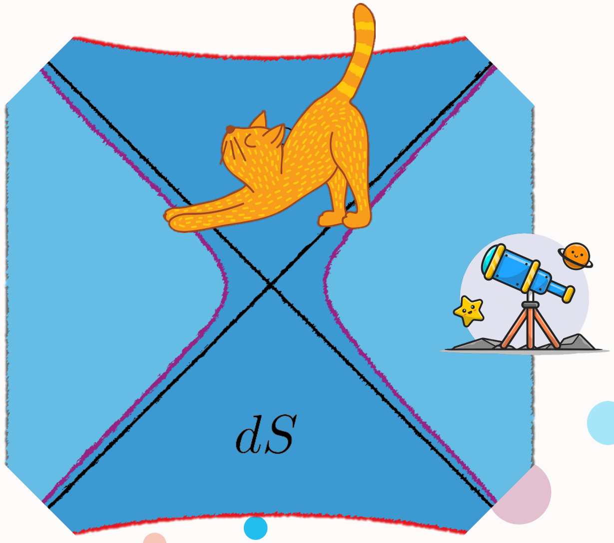


Do those results persist for dS_4 flow geometries?

Second Approach

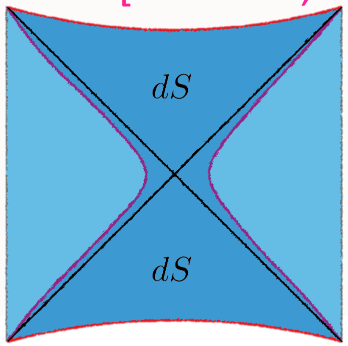
Holographic Complexity on the Stretched Horizon



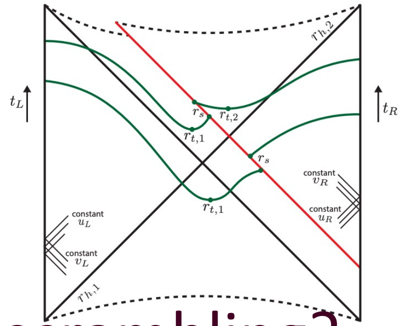


Holography on the Stretched Horizon

- Holographic theory with finite number of degrees of freedom (perhaps double scaled SYK) can be associated to a holographic screen at the stretched horizon. This is the surface of maximal area within the region we have access to. [Susskind, Shaghoulian, Lin; …]



- Gravity does not decouple - maybe SYK coupled to gravity.
- Suggested that the theory at the stretched horizon is hyperfast scrambling (SYK with hamiltonian terms that act on many fermions at a time $q \sim N^{\alpha}, 0 < \alpha < 1$).
  – complexity diverges at finite critical time and then regularized by a cutoff at $\mathcal{J}^{+}$.
  [Susskind et al.; Jørstada, Myers, Ruan;…]



- How can we test if the theory at the stretched horizon is hyperfast scrambling?
- In the context of holographic complexity - reaction to perturbations, e.g., shockwaves in black holes [SC, Marrochio, Myers]

Complexity in Fast Scrabling Systems

$$|TFD(t_L, t_R)\rangle_{pert} = U_R(t_R + t_w)\, \mathcal{O}_R\, U_R(t_L - t_w)\, |TFD\rangle\ .$$

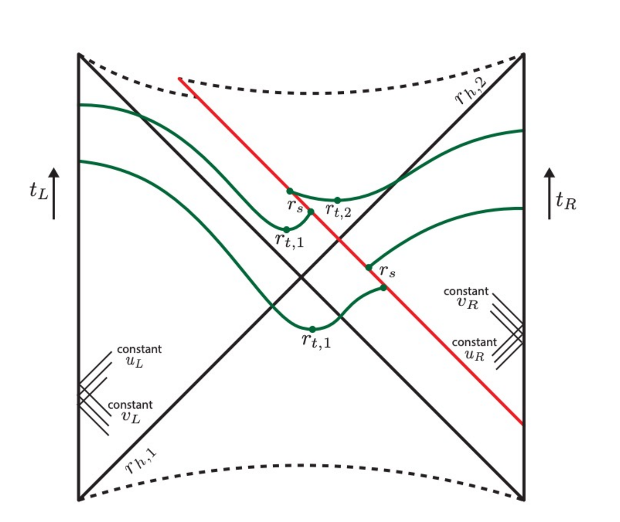
$t_L = t_R = t/2$

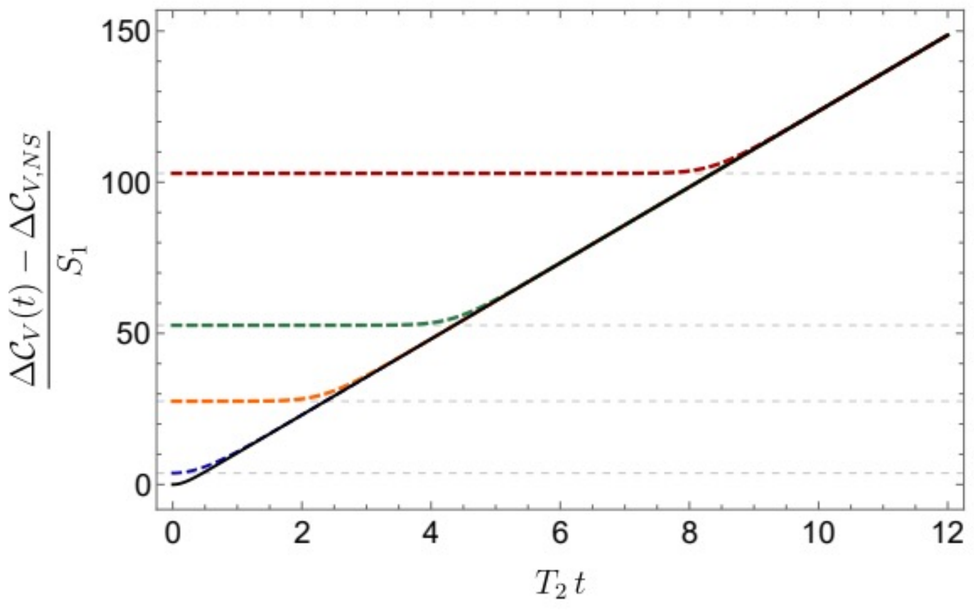
- The complexity starts growing linearly, but only after a long plateau of size $\Delta t = 4(t_w - t^*_{scr})$ where $t^*_{scr} = \frac{1}{2\pi T}\log(1/\epsilon)$

T – temperature

$\epsilon \ll 1$- relative size of the perturbation (energy of the shock)

$-t_w$ - time of perturbation.





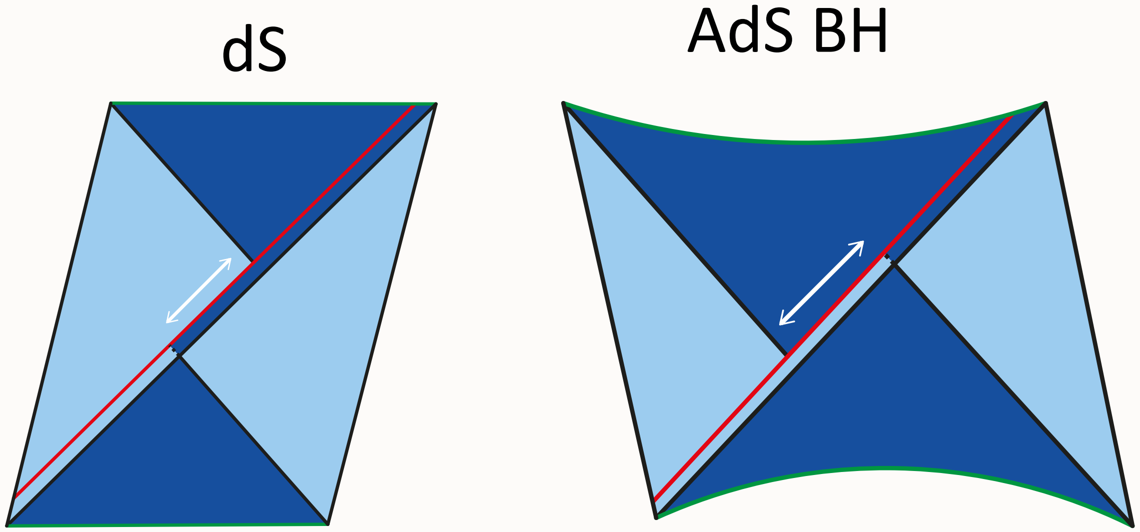
[SC, Marrochio, Myers]

Sockwaves in dS Geometries

- Positive Energy Shockwaves push the horizon away and make more space available to the observer. [Gao, Wald, 2000]
- This is very different from what happens for AdS black holes where this is only possible for negative energy shockwaves.

$$ds^2 = -F(r,u)du^2 - 2dr\,du + r^2 d\varphi^2$$

$$F(r,u) = 1 - \frac{r^2}{L^2} - 8G_N\mathcal{E}(1-\Theta(u-u_s))$$

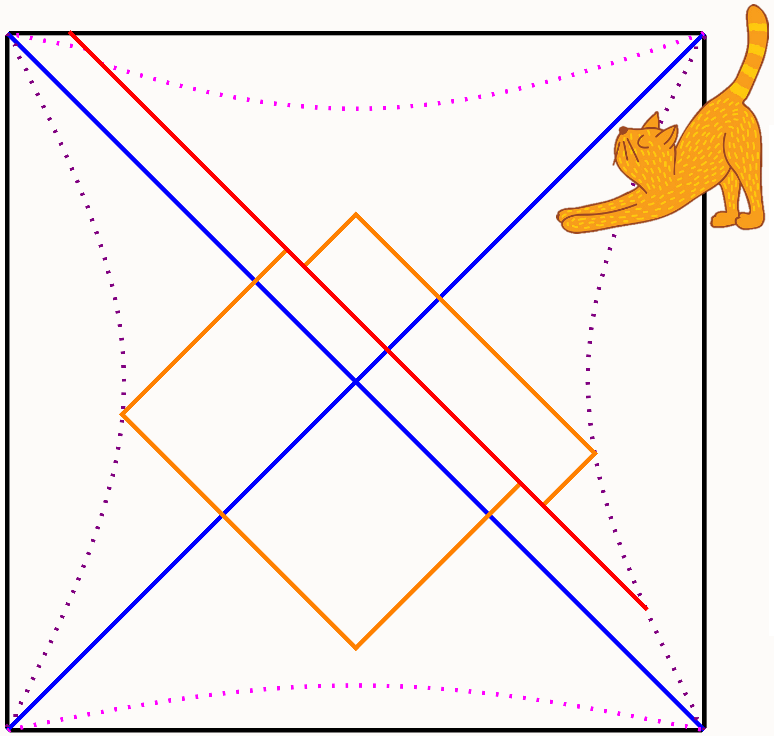


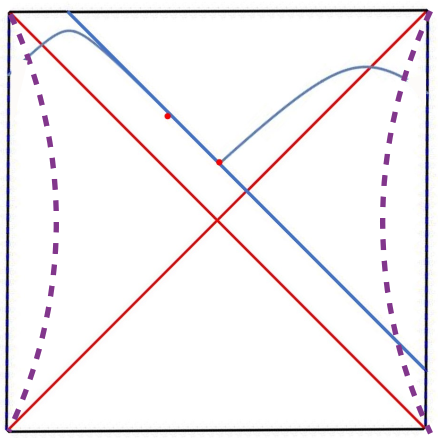
[Hotta Tanaka; Sfetsos; …]

Complexity for dS shockwaves

- How does complexity of the theory at the stretched horizon reacts to the shockwave?
- Anchor complexity quantities there. [Jørstada, Myers, Ruan]
- Use a cutoff at $\mathcal{J}^+$.
- Complexity=spacetime volume of the Wheeler-DeWitt patch.
- Preliminary results in 3d. Similar behavior in higher dimensions too.

(Here the diagrams discontinuous in the radial direction)





[work in progress Baiguera, Berman, SC]

Results so Far

- Similarly to black holes: the complexity grows linearly after an initial plateau!
- Linear growth governed by the cutoff at $\mathcal{J}^+$

$$\frac{dC}{dt} = \frac{2\pi}{G_N L^2 d}(r_{max})^{d-1}$$

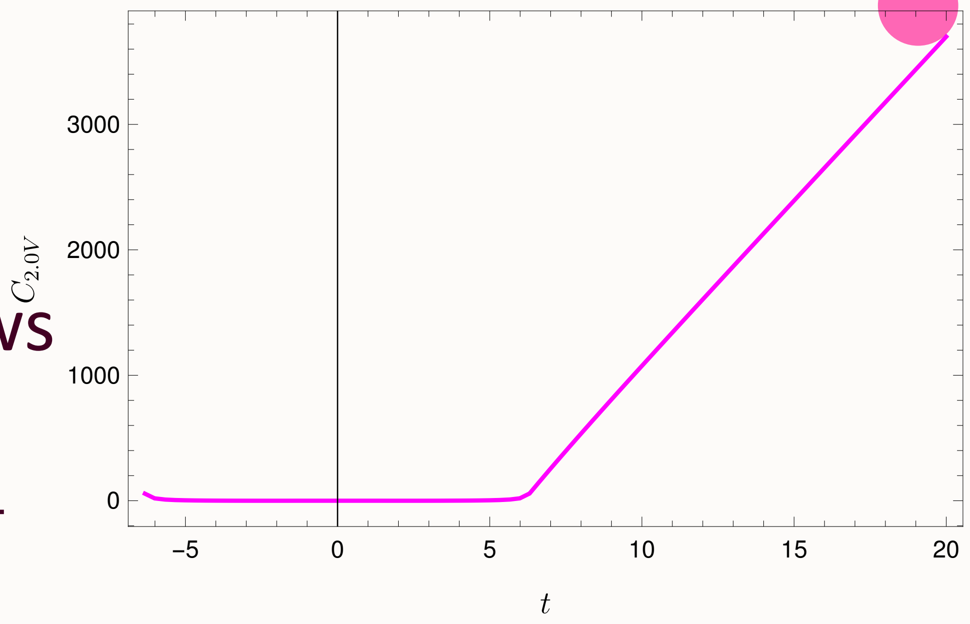
- The size of the plateau depends on the size of the perturbation

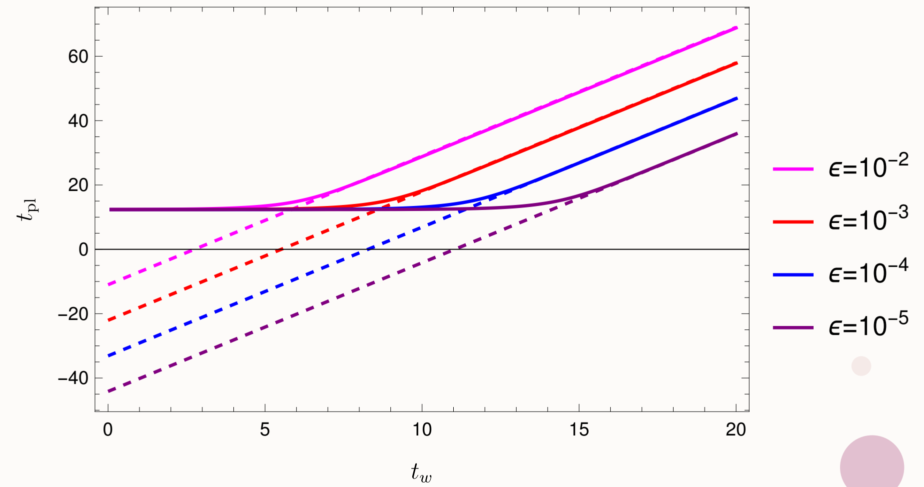
Scrambling time

$$\Delta t = 4(t_w - \log(1/\epsilon) + \mathcal{G}(\rho))$$

ϵ is the relative size of the perturbation.

ρL is the location of the stretched horizon





[work in progress Baiguera, Berman, SC; related results: Aalsma, Shiu]

Exploring the Plateau

- Usual scrambling time for fast scrambling systems.
- In higher dimensions and with other types of shocks (SdS to SdS) – similar results.
- Universal geometric property of horizons? Can we prove this?
- Word of caution: very dependent on notion of time. Here, we used metric time in the static patch. Using proper time ($\tau_p = \sqrt{1-\rho^2} L\, t$) would perhaps add some blue shift factors which could reduce the scrambling time.

[work in progress Baiguera, Berman, SC]

Summary

- Revival of dS holography - different approaches on the market.
- Interesting to explore with quantum information lens.
- Centaur geometries - access to a timelike boundary!
- Complexity (=volume) intially decreased and stopped existing after a certain time. Interpretations?
  - Theory is not chaotic?
  - Complex geodesics?
  - UV cutoff?
- Complexity (=spacetime volume) at the stretched horizon with shockwaves.
  - Scrambling time indicates chaotic behavior.

Outlook and Open Questions

- Flow geometries.
  - What can we say in higher dimensions?
  - What is the QFT interpretation (complex couplings?)
- Stretched horizon approach:
  - What is the QFT interpretation of the stretched horizon (SYK/matrix model coupled to gravity?)
- Is the theory dual to de-sitter space chaotic?
- Can we use these holographic setups to answer basic questions about quantum gravity in dS?
  - What are good observables?
  - What is the origin of the dS entropy?