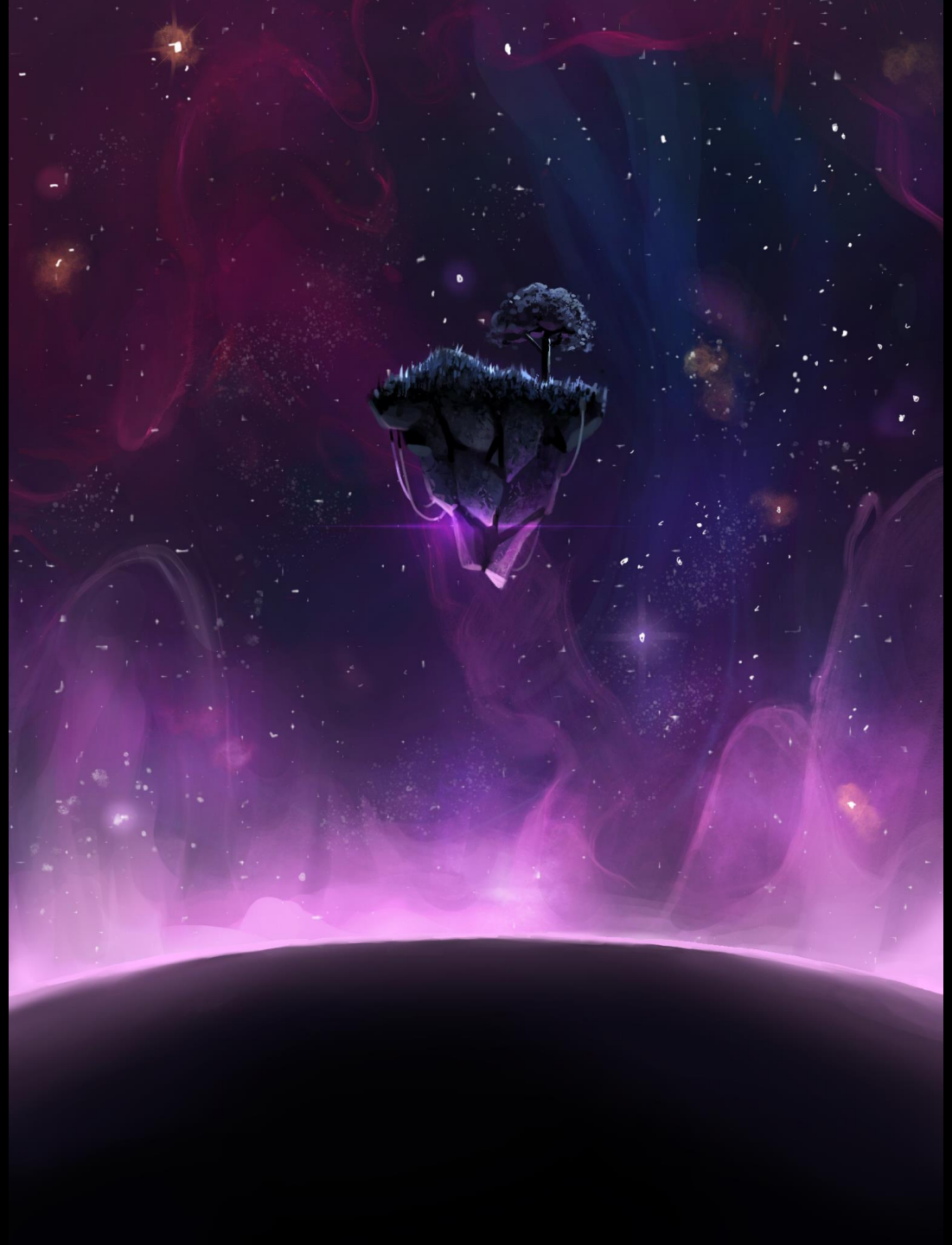


Randall-Sundrum Branes and Holography

(review talk @ Strings22)

Andreas Karch (UT Austin)

Randall-Sundrum Braneworlds

An Alternative to compactification

Lisa Randall (Princeton U. and MIT, LNS), Raman Sundrum (Boston U.)

Jun, 1999

10 pages

Published in: *Phys.Rev.Lett.* 83 (1999) 4690-4693

e-Print: hep-th/9906064 [hep-th]

DOI: 10.1103/PhysRevLett.83.4690

Report number: MIT-CTP-2874, PUPT-1867, BUHEP-99-13

View in: AMS MathSciNet, ADS Abstract Service

pdf cite 7,167 citations

(Randall, Sundrum ‘99)

OK, BOOMER

(Technically, I am Gen X)

(But everyone ignores our existence anyway…)

The basic RS problem

At the very basic level, RS is the answer to a very simple GR problem:

$$S = \frac{1}{16\,\pi\,G}\int d^{d+1}x\ \sqrt{g}\,(R - 2\Lambda) + \lambda \int d^{d}x\ \sqrt{g_I}$$

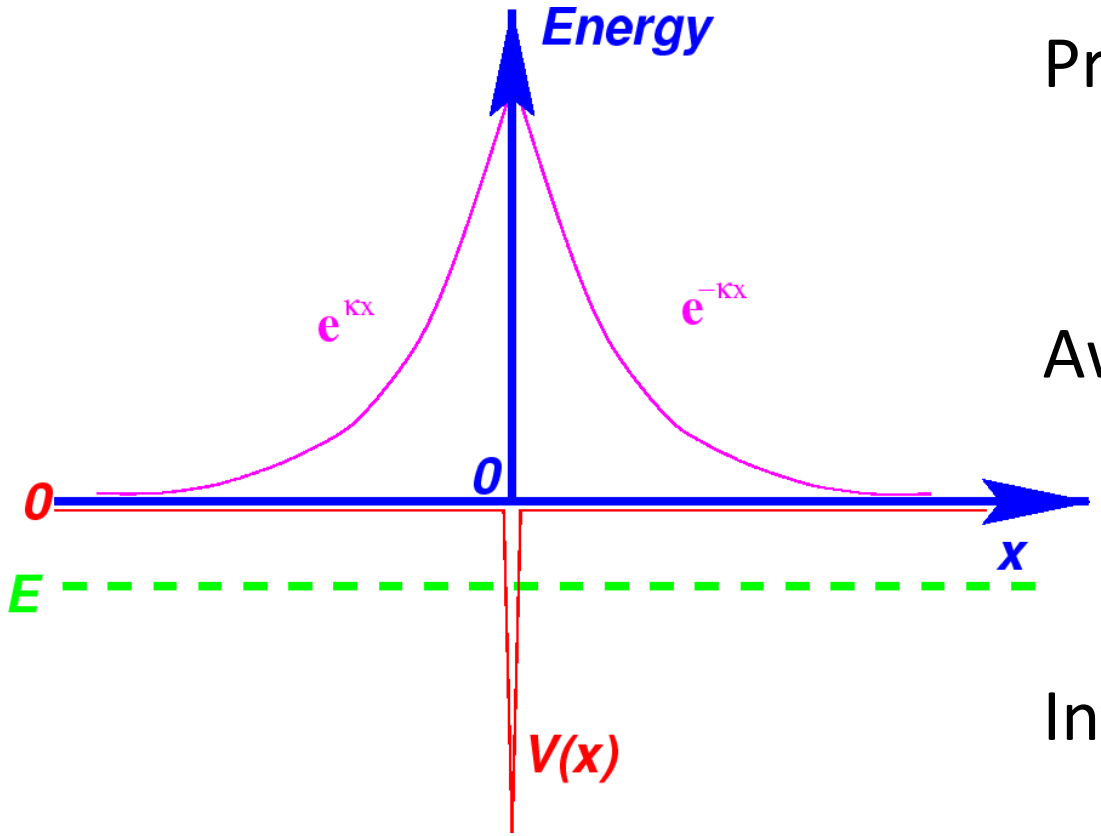
Cosmological Constant

Einstein Gravity

Thin sheet with tension

Today: d=4; 3+1 dimensional brane in AdS_5 with radius L

Quantum Mechanics Precursor:



Problem analog to δ-function potential.

$$V(x) = -V_0\delta(x)$$

Away from potential: Vacuum

$$\psi(x) = \begin{cases} Ae^{\kappa x} & x < 0 \\ Ae^{-\kappa x} & x > 0 \end{cases}$$

Integrate Schrödinger: Jump condition

$$\left.\frac{d\psi}{dx}\right|_{+\epsilon} - \left.\frac{d\psi}{dx}\right|_{-\epsilon} = \frac{2m}{\hbar^2}V_0\psi(0)$$

Vacuum Solutions:

Away from the brane: AdS_5 (or a slice thereof)

Across the brane: $$K_{\mu\nu} - h_{\mu\nu}K = 8\pi G T_{\mu\nu} = -4\pi G \lambda h_{\mu\nu}$$

(Israel '66)

RS Spacetime = 2 identical copies of (parts of) AdS_5 glued together along brane

Orbifolded RS Spacetime = part of AdS_5 terminated by brane

(A)dS sliced Randall Sundrum

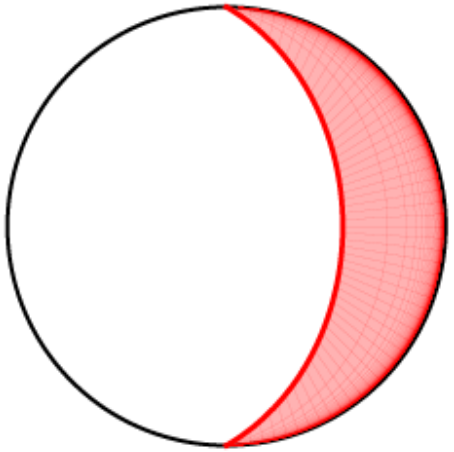
Competition between bulk cc and brane tension leaves three options:

$\lambda > \lambda_C$ $\Rightarrow$ De Sitter Brane (supercritical)

$\lambda = \lambda_C$ $\Rightarrow$ Minkowski Brane (critical)

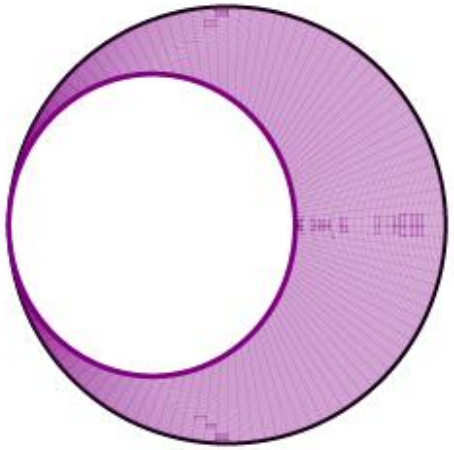
$\lambda < \lambda_C$ $\Rightarrow$ Anti-De Sitter Brane (subcritical)

Examples in (Euclidean) AdS



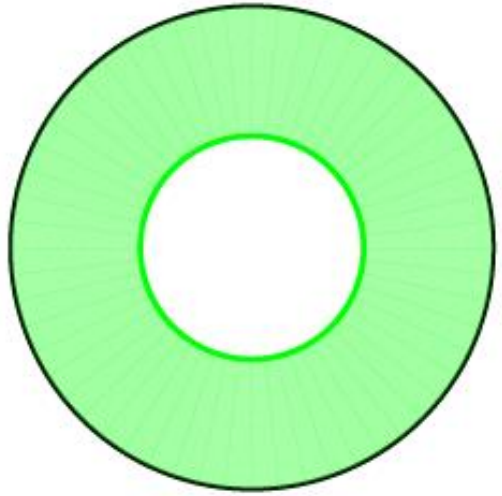
$\lambda < \lambda_c$

“AdS brane”



$\lambda = \lambda_c$

“Minkowski brane”

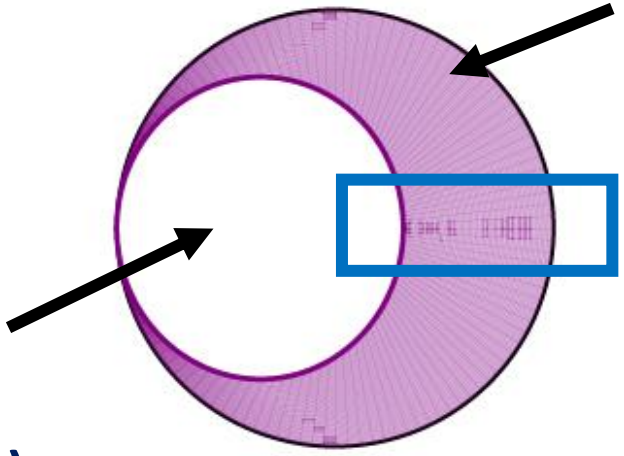


$\lambda > \lambda_c$

“dS brane”

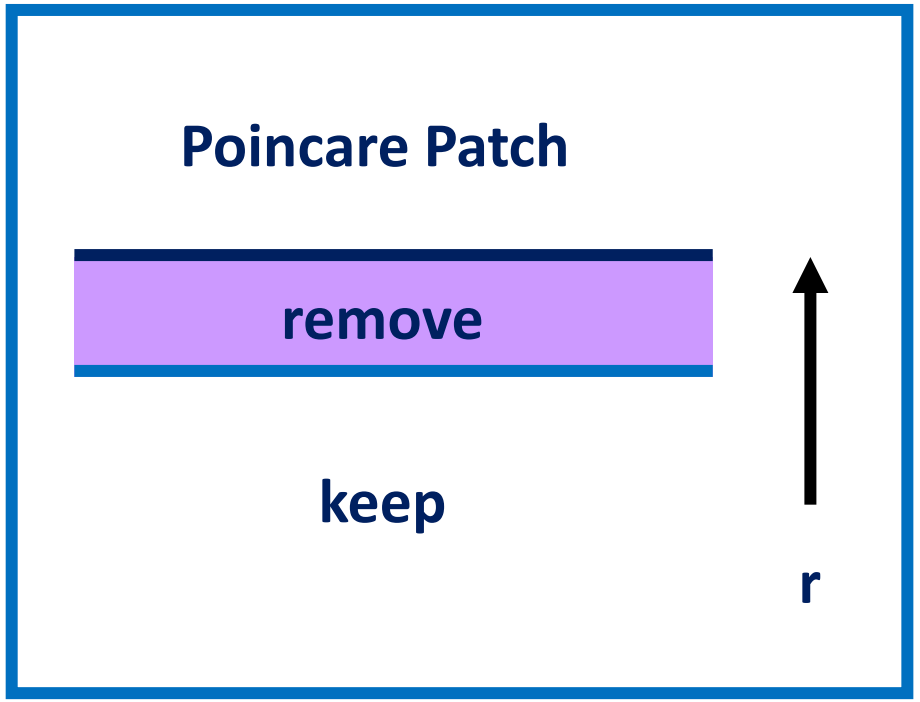
Examples in (Euclidean) AdS

for positive tension brane



orbifolded RS: keep
(spacetime ends on brane)

2-sided RS: 2 copies of white region

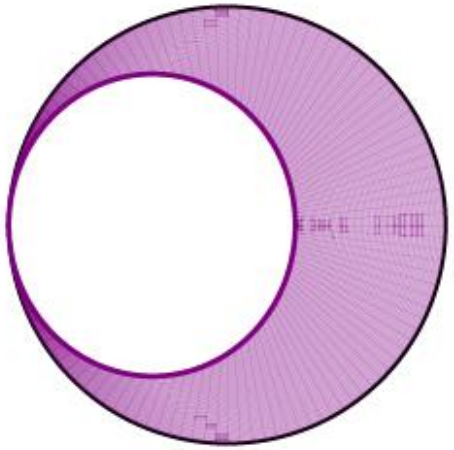


Examples in (Euclidean) AdS

Interesting applications in

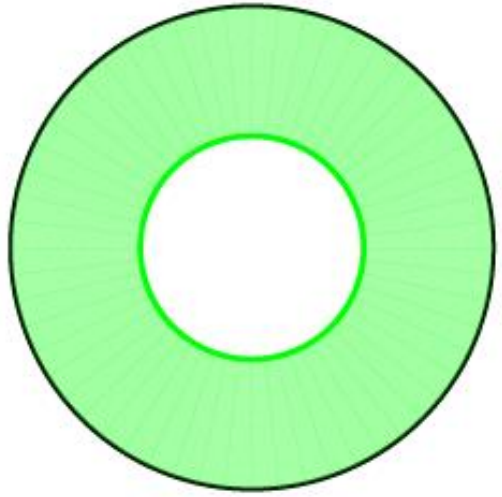
- Phenomenology
- Cosmology

Geometrizes Hierarchies



$\lambda = \lambda_c$

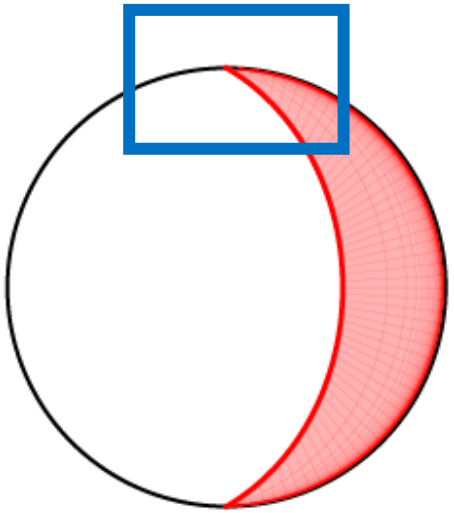
“Minkowski brane”

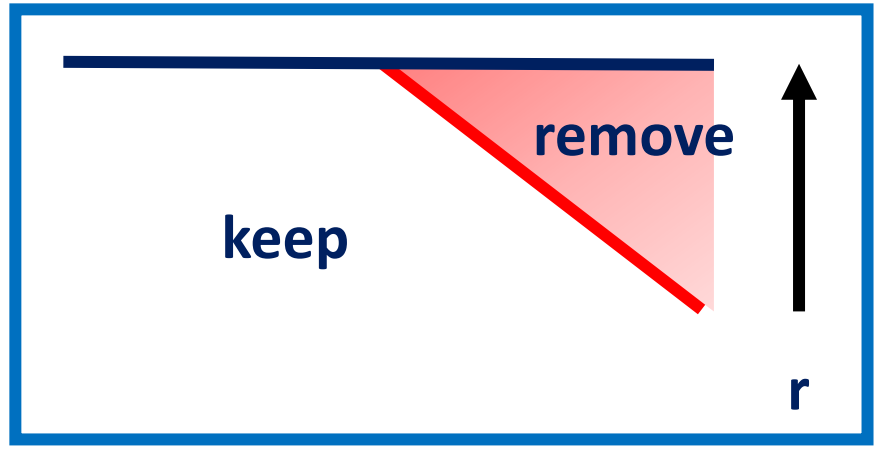


$\lambda > \lambda_c$

“dS brane”

Examples in (Euclidean) AdS





$\lambda < \lambda_c$

“AdS brane”

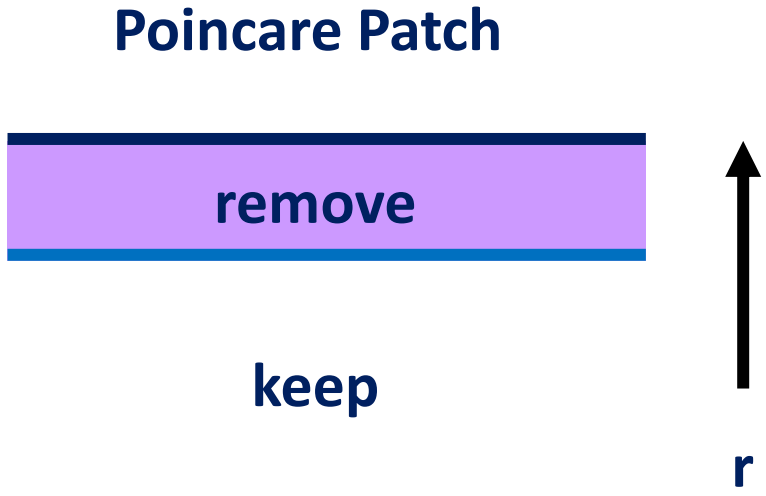
Subcritical RS brane:

- AdS/CFT with boundary
- Double Holography
- massive gravity

(AK, Randall ‘00)

Fluctuations and Holography

Essential features of critical RS:

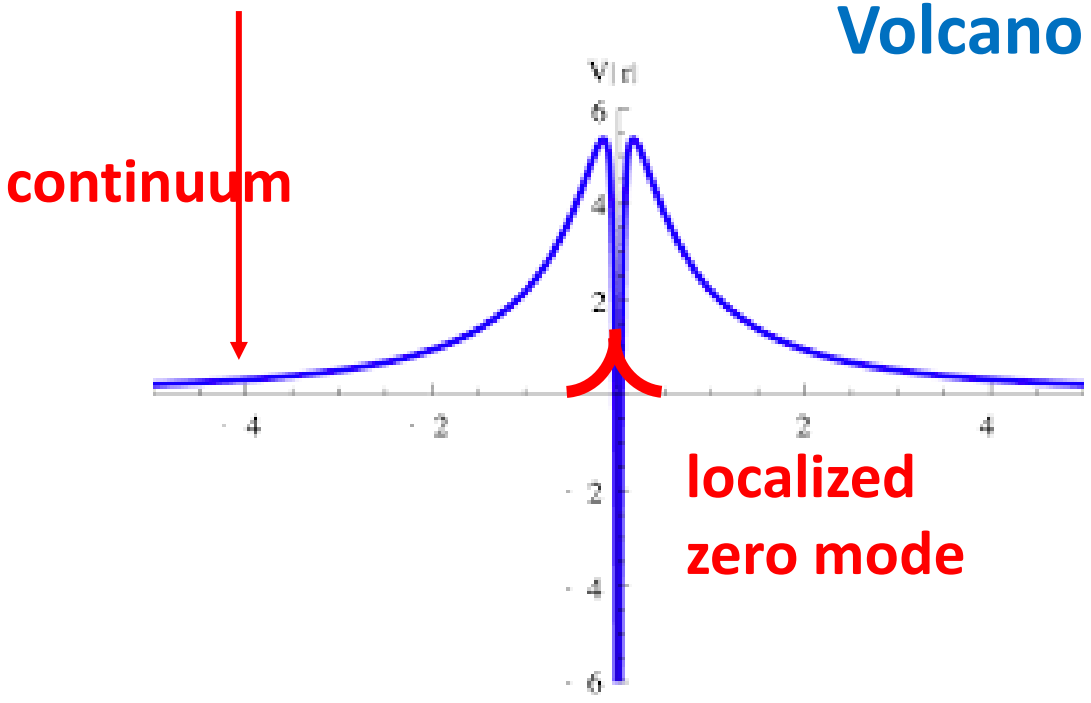


- localized gravity
- holographic interpretation

Lays groundwork for subcritical setting of interest

Randall Sundrum: graviton fluctuations

Volcano Potential



Small fluctuations of the metric around this solution give rise to a trapped, massless 3+1 - dimensional graviton on the brane

Linearized Einstein = Schrödinger with Volcano Potential

An alternative to compactification!

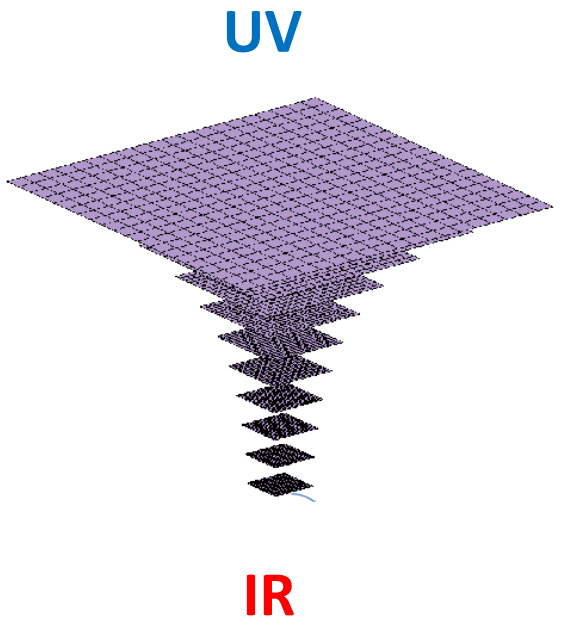
Critical RS and holography

The connection with AdS_5 suggests holographic description of critical RS:

Recall:

Gravity on AdS_5 = 3+1 CFT on Boundary

radial direction in bulk = scale in CFT



Randall-Sundrum Holography:

Two (or one) copies of the IR part of AdS_5, glued together along a 3+1 dimensional brane with massless graviton trapped on brane.

Randall-Sundrum Holography:

(Gubser; Verlinde, Witten; … '99)

Two (or one) copies of the IR part of AdS_5, glued together along a 3+1 dimensional brane with massless graviton trapped on brane.

The two IR regions

=

The RS or "UV" brane

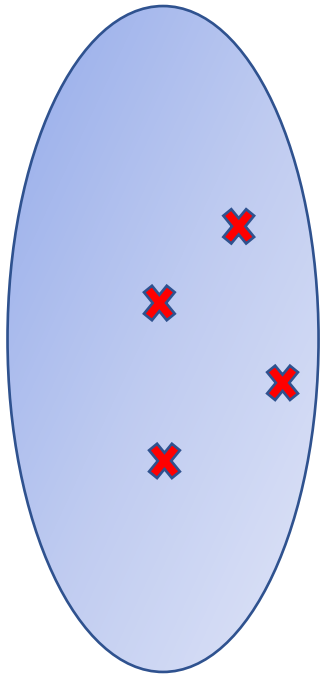
Two/one 4 dimensional CFTs with UV cutoff + dynamical gravity

The localized graviton

Gravity part of holographic dual!!!

Top/Down (critical)RS Holography

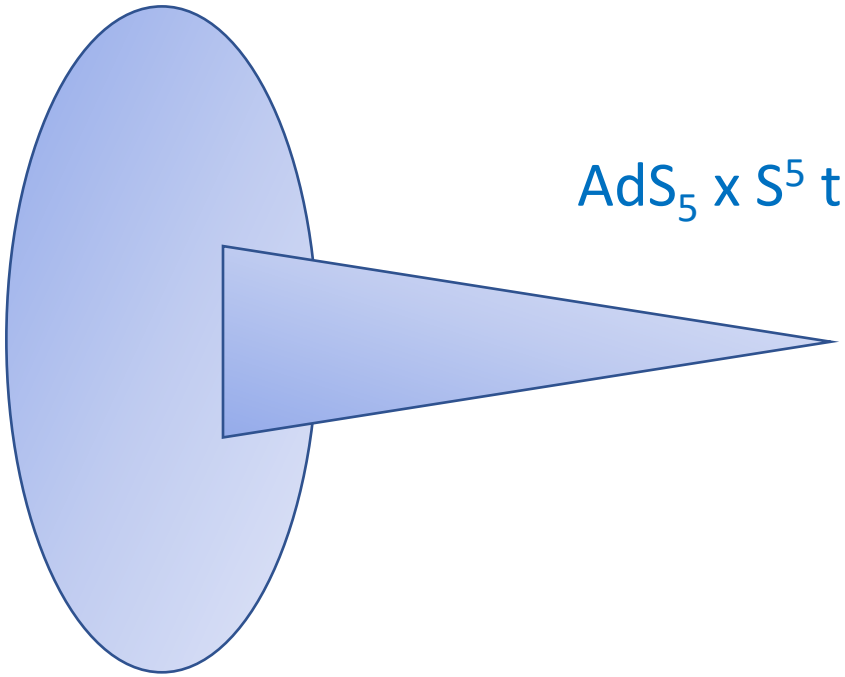
(Verlinde '99)



N>>1 D3 branes
required for
anomaly cancellation

6d base of
IIB compatification

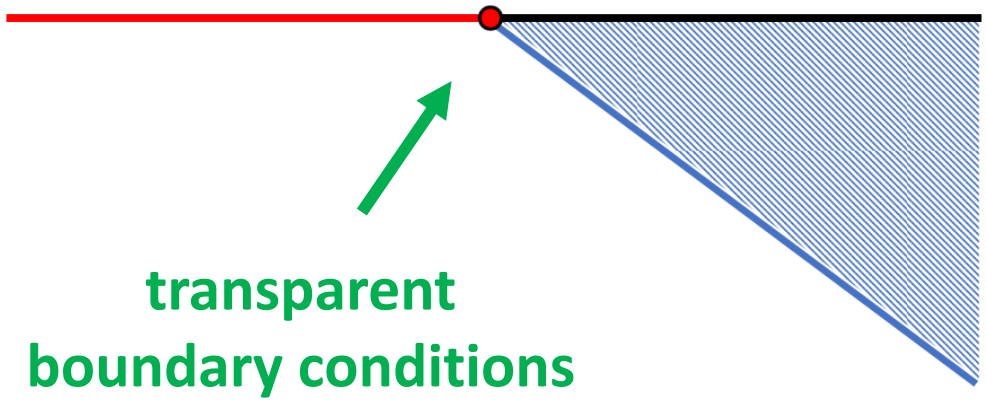
base = RS brane



D3's coincide

Subcritical Holography

(AK, Randall '00)



1. Classical Gravity of bulk + brane

↓ RS holography

2. CFT + cutoff + gravity on AdS4 communicating with CFT on half-space

dualize 3d DOFs ↑ ↓ AdS_4 holography

3. CFT with boundary (AdS/BCFT)

Suggests 3 dual description

AdS/BCFT has in fact been proposed twice

Open and closed string interpretation of SUSY CFT's on branes with boundaries #1

Andreas Karch (MIT, LNS), Lisa Randall (MIT, LNS) (Jul, 2000)

Published in: *JHEP* 06 (2001) 063 • e-Print: hep-th/0105132 [hep-th]

pdf DOI cite 458 citations

Holographic Dual of BCFT #1

Tadashi Takayanagi (Tokyo U., IPMU) (May, 2011)

Published in: *Phys.Rev.Lett.* 107 (2011) 101602 • e-Print: 1105.5165 [hep-th]

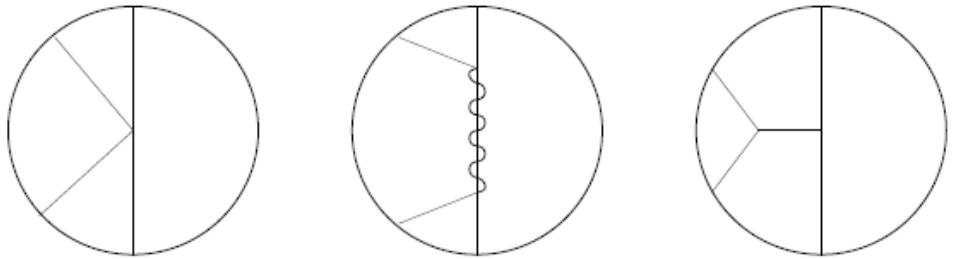
pdf DOI cite 306 citations

different philosophy, technically identical

2pt Correlators agree with BCFT structure

(Aharony, Dewolfe, Freedman, AK ’03)

(Rastelli, Zhou ’17)



- Decomposition of 5d fields in 4d modes = BOPE

(Aharony, Dewolfe, Freedman, AK ’03)

- 2-pt function obeys block decomposition

(Rastelli, Zhou ’17)

Subcritical Holography

(AK, Randall '00)

Solvable → 1. Classical Gravity of bulk + brane

Gravity + Bath → 2. CFT + cutoff + gravity on AdS4 communicating with CFT on half-space

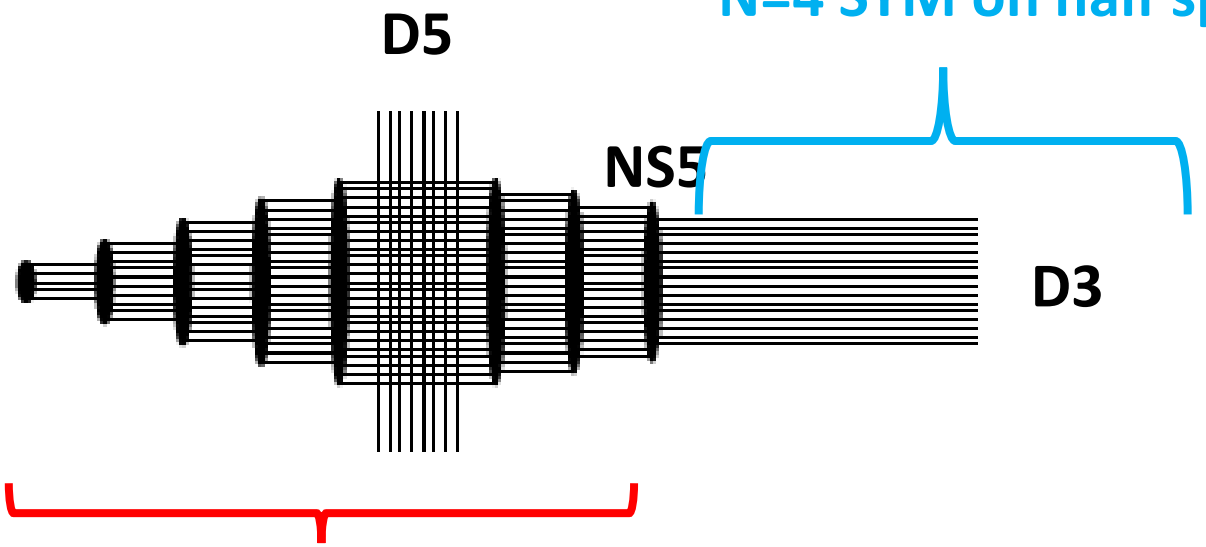
Well defined field theory → 3. CFT with boundary (AdS/BCFT)

"double holography"

Top/down AdS/BCFT

(UCLA-solution: D'Hoker, Estes, Gutperle '07, ….)

The BCFT:



K/N_5 allows to dial ratio of 4d/3d DOFs

$$U(R) - U(2R) - \ldots - U(R^2) - U(R^2 - S) - \ldots - U(2N_5K + S) - \widehat{U(2N_5K)}$$
$$|$$
$$[N_5]$$

$$R = \frac{N_5}{2} + K \qquad S = \frac{N_5}{2} - K \ .$$

(Hanany, Witten '96)

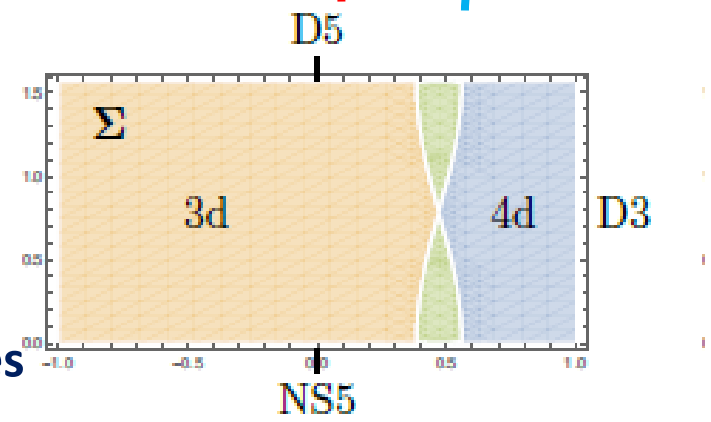
(Gaiotto, Witten '09)

Top/down AdS/BCFT

(UCLA-solution: D'Hoker, Estes, Gutperle '07, ….)

The Bulk:

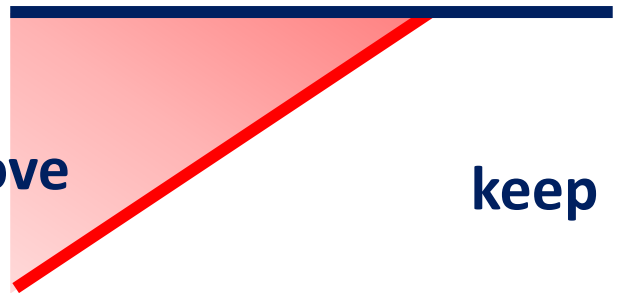
boundary, 3d CFT **N=4 SYM on half space**



S²
collapses

full top down

naïve bottom up



remove

keep

SU(2) x SU(2) R-symmetry:

$$AdS_4 \times S^2 \times S^2 \times \Sigma$$

metric functions/fields depend on 2d Riemann surface

Fluctuations: the massive graviton

Linearized fluctuations: (AK, Randall ’00)

While the subcritical RS brane still localizes a graviton, this time it has a non-zero mass!

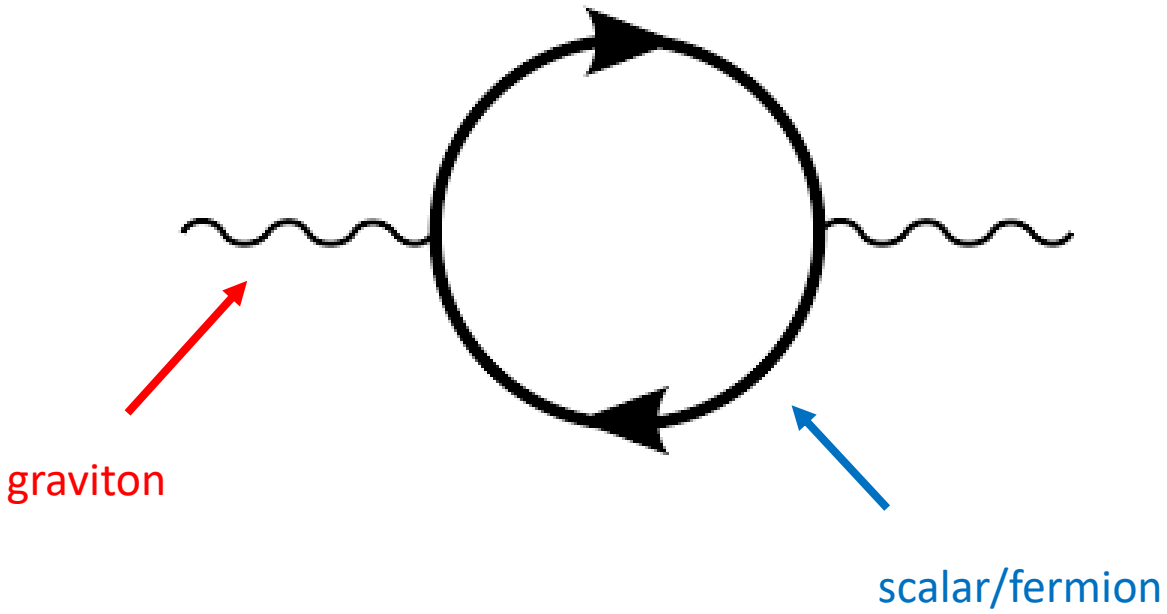
Top-down: (Bachas, Lavdas‘18)

Why? So What?

Intermediate Picture Mass

(Porrati '02)

CFT + Gravity coupled to Bath:



- Answer depends on bc for matter
- Standard (reflecting) bc: no mass
- Any other bc (non-zero Transmission): graviton gets a mass!
- Higgs Mechanism for gravity; scalar gets eaten

Mass direct consequence of coupling to bath.

The Photon analog

Consider a CFT with U(1) global symmetry, charged operator W

$$S = S_{CFT} + h \int d^d x \, W$$ explicit breaking!

Higgsing at tree level!

In the dual bulk, charged scalar profile from boundary value h

For gravity this also works: holographic lattice → Gravity Higgsed

(Blake, Tong, Vegh '13)

But what if U(1) is broken by multi-trace?

$$S = S_{CFT} + h \int d^d x \, W^2$$

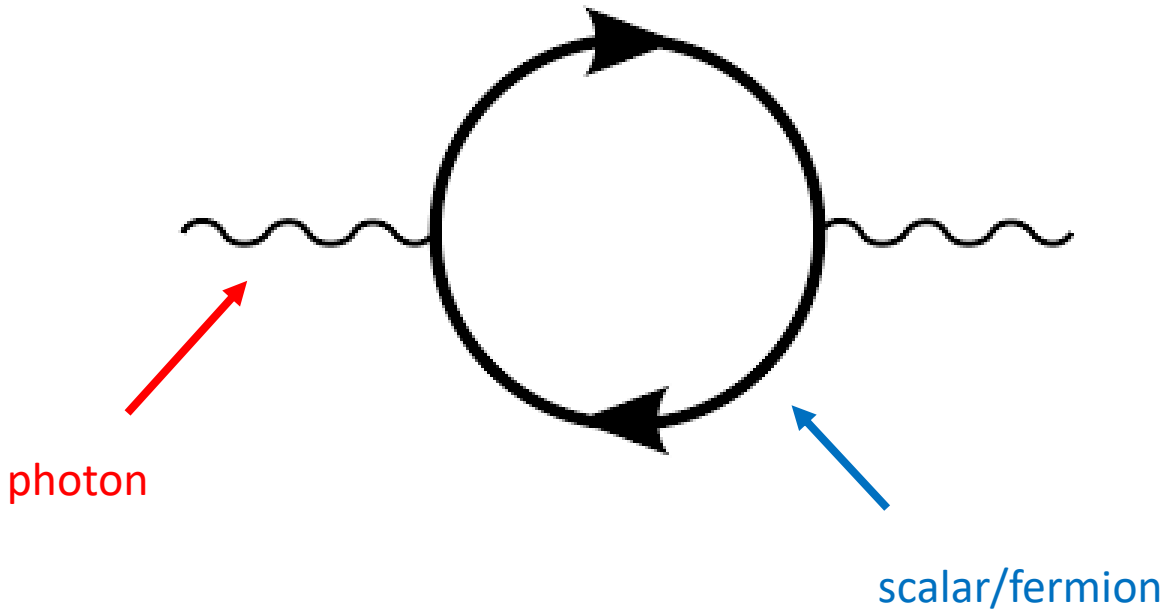
(double trace breaking)

$$S = S_{CFT1} + S_{CFT2} + h \int d^d x \, W_1 \, W_2$$

(double trace coupling to bath)

The photon analog

(Rattazi, Redi ’09)



- Matter loop = bi-linear condensate
- Standard Higgs mechanism
- Corresponds to deforming CFT by charged double trace (either within CFT our coupled to bath)

Comments

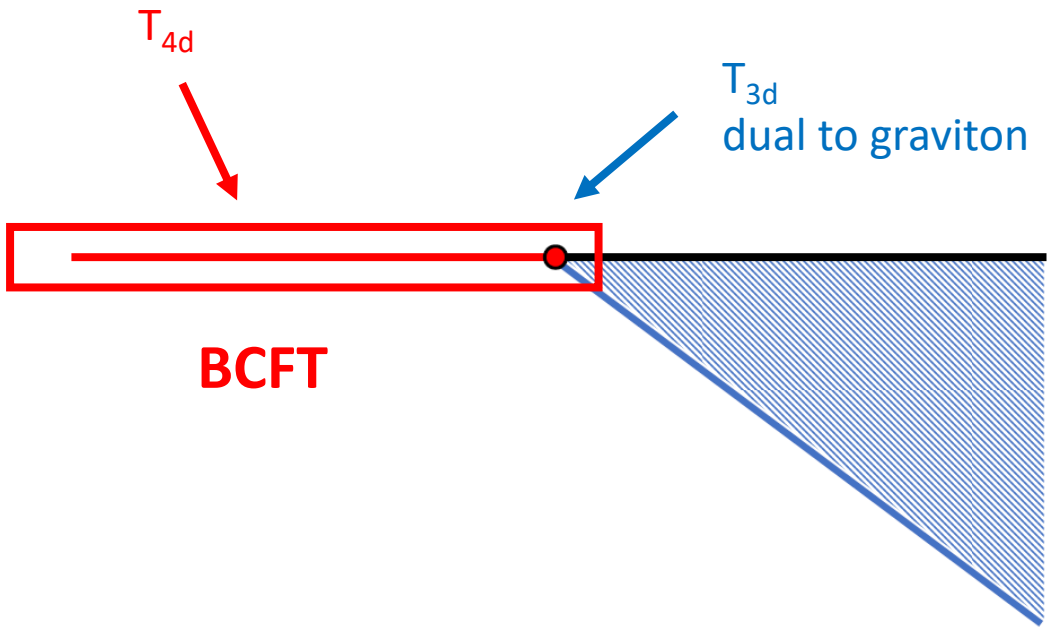
- For a single scalar this mass is of order $G \sim N^{-2}$
- RS branes have a CFT with N^2 scalars, so mass of order 1
- While matter propagator only modified due to bc, the graviton/photon equation of motion changed everywhere
- effective action for graviton has explicit Pauli Fierz mass term

Coupling to bath drives gravity/E&M into a Higgs phase.

BCFT Mass:

(Aharony, DeWolfe, Freedman, AK '03)

CFT with boundary (AdS/BCFT)

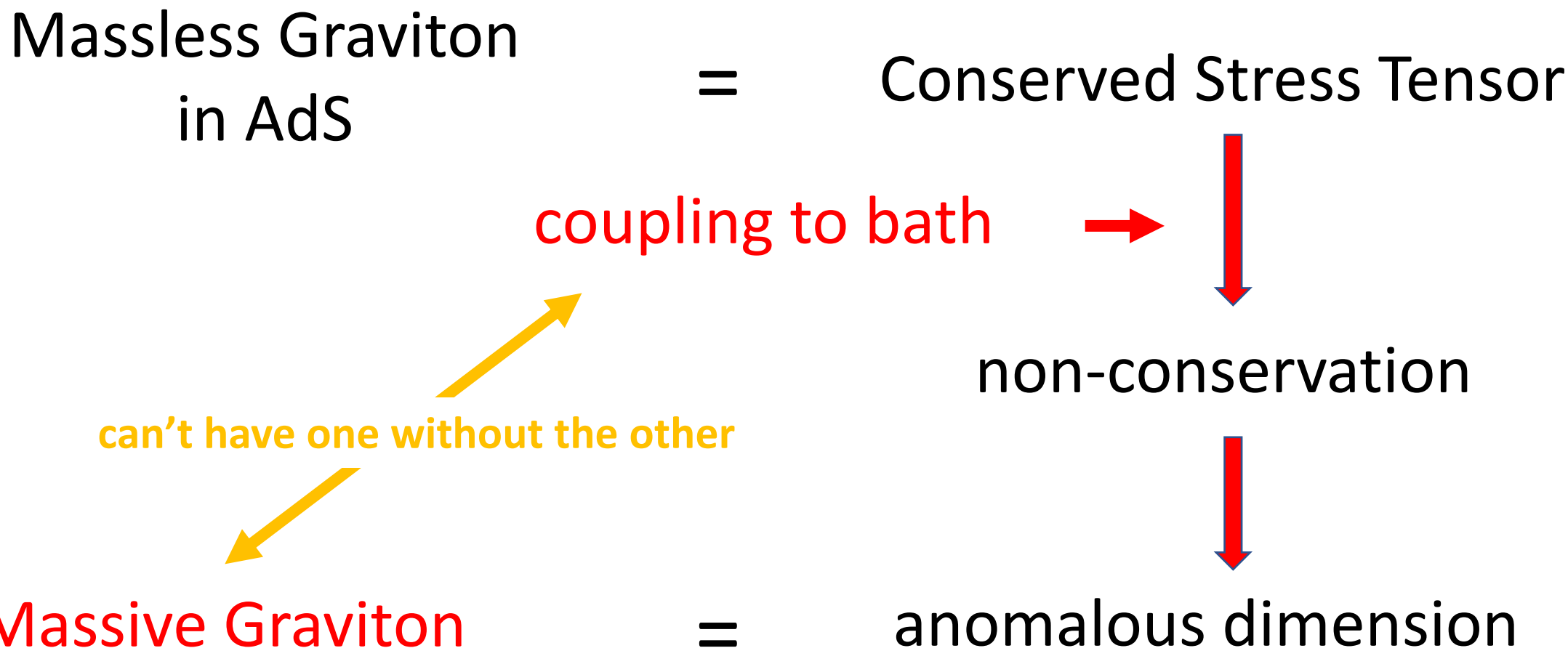


- **$T_{4d} + T_{3d}$ conserved**
- **T_{3d} alone not conserved**
- **anomalous dimension for T_{3d} = graviton mass**

verified by explicit calculations in string theoretic D3/D5 model

Mass/Bath beyond AdS/BCFT

(Aharony, Clark, AK '06; Kiritsis '06)



Coupling to bath drives gravity/E&M into a Higgs phase.

Mass/Bath made quantitative

(Aharony, Clark, AK '06)

$$S = S_{CFT1} + S_{CFT2} + h \int d^d x \, O_1 \, O_2$$

1-loop graviton mass

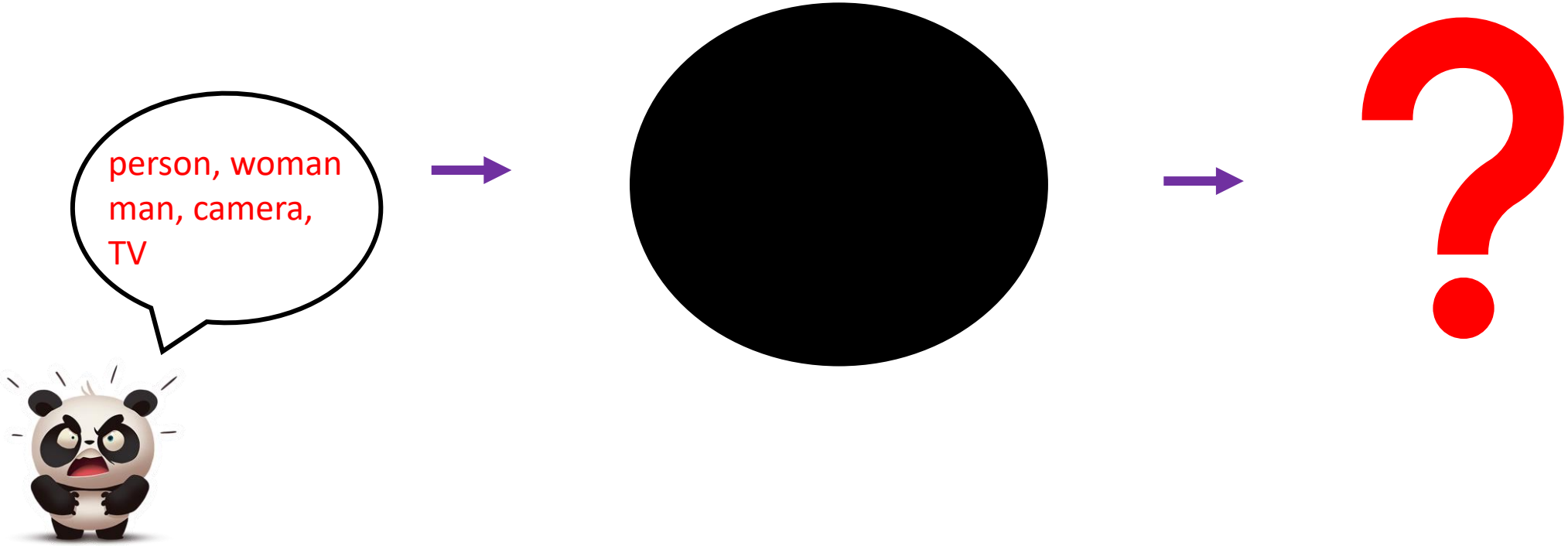
=

conformal perturbation theory for anomalous dimension of T to leading order in h

$$M^2_{grav} = d\,(\Delta_T - d) = h^2 \left(\frac{1}{c_1} + \frac{1}{c_2}\right) \frac{\Delta_1 \Delta_2 d}{(d+2)(d-1)}$$

Applications to BH Information

Do black holes lose information?



Recent Black Hole Revolution

Unitary evaporation of black holes (Page curve) from semi-classics:

yes, info
does come back!

Ingredients: QES, Islands, Wormholes

Basic construction: (Pennington '19)

Concrete implementation:

d=2: Almheiri, Engelhardt, Marolf, Maxfield '19, ….) → 2d gravity + bath

d>2: (Almheiri, Mahajan, Santos '19) bottom-up

(Uhlemann '19)
(Demulder, Gnecchi, Lavdas, Lust '21) top-down

} sub-critical RS branes!

Recent Black Hole Revolution

Unitary evaporation of black holes (Page curve) from semi-classics:

yes, info
does come back!

Ingredients: QES, Islands, Wormholes

All Page curves calculated to date were obtained in a theory in which gravity is in **Higgs phase!**

Basic construction: (Pennington '19)

Concrete implementation:

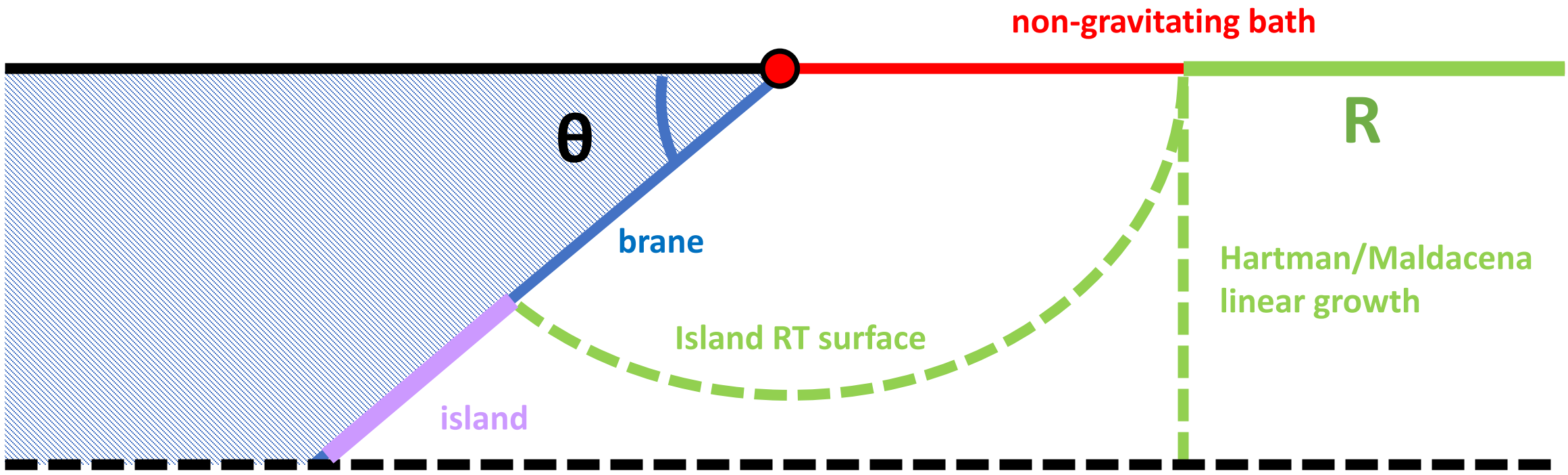
d=2: Almheiri, Engelhardt, Marolf, Maxfield '19, ….)

d>2: (Almheiri, Mahajan, Santos '19)

(Uhlemann '19)
(Demulder, Gnecchi, Lavdas, Lust '21)

gravity + bath

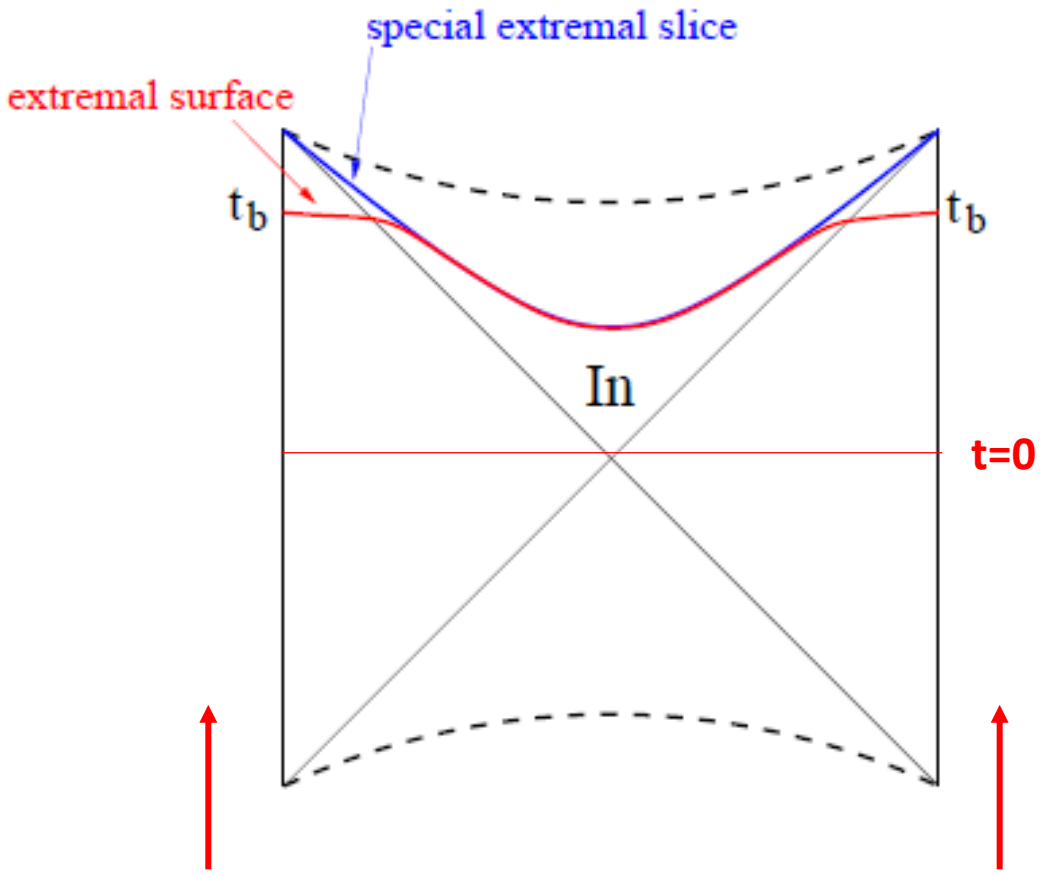
Double Holographic Islands



(Almheiri, Mahajan, Santos ’19, Almheir, Mahajan, Maldacena ‘19)

(Chen, Myers, Neuenfeld, Reyes, Sandor ’20, Geng, AK, Perez, Raju, Randall, Riojas, Shashi ‘20)

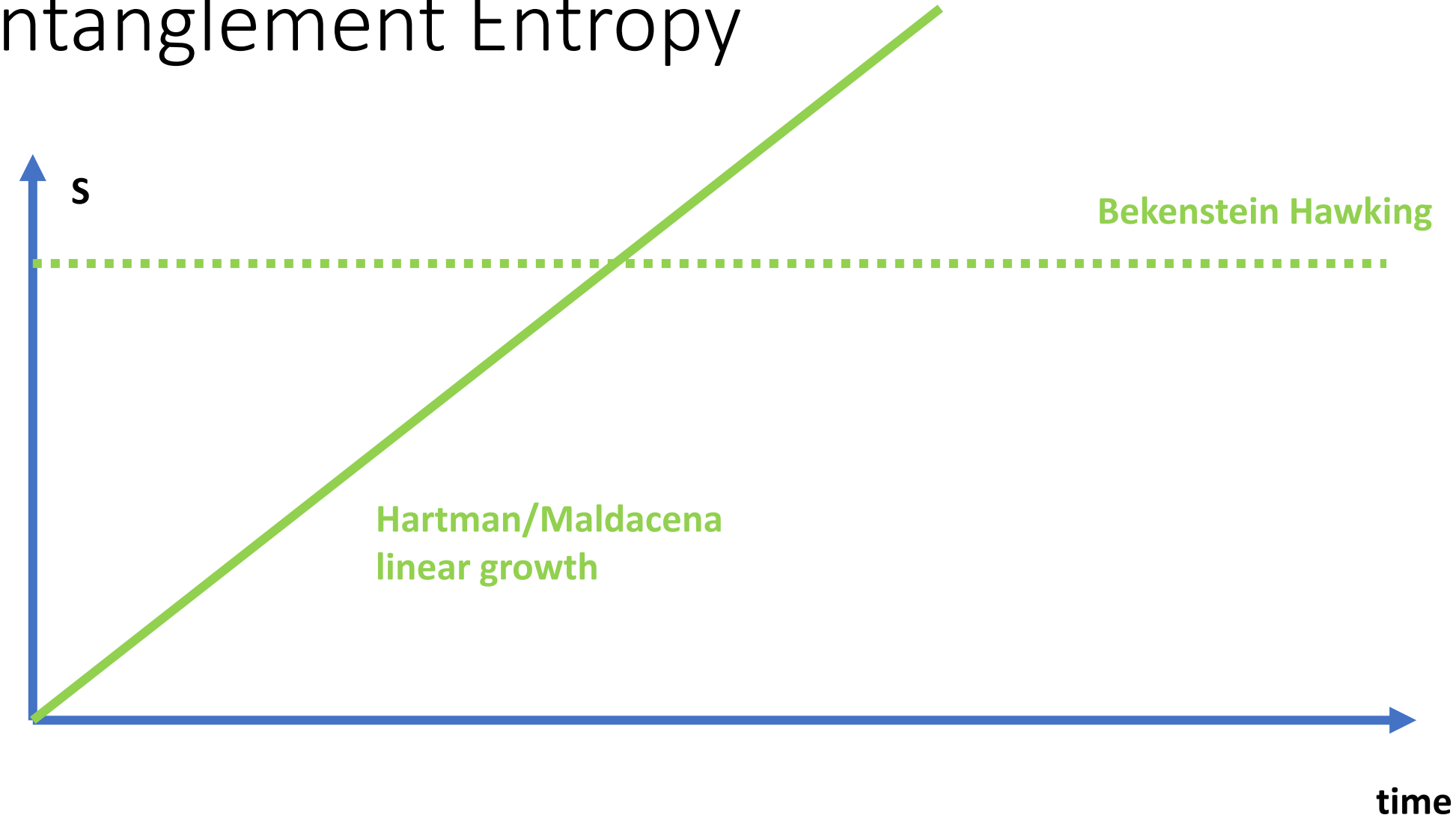
Paradox of the eternal black hole (Hartman, Maldacena '13)



Entanglement grows
linear with time forever

Puzzle: should saturate as S_{BH}

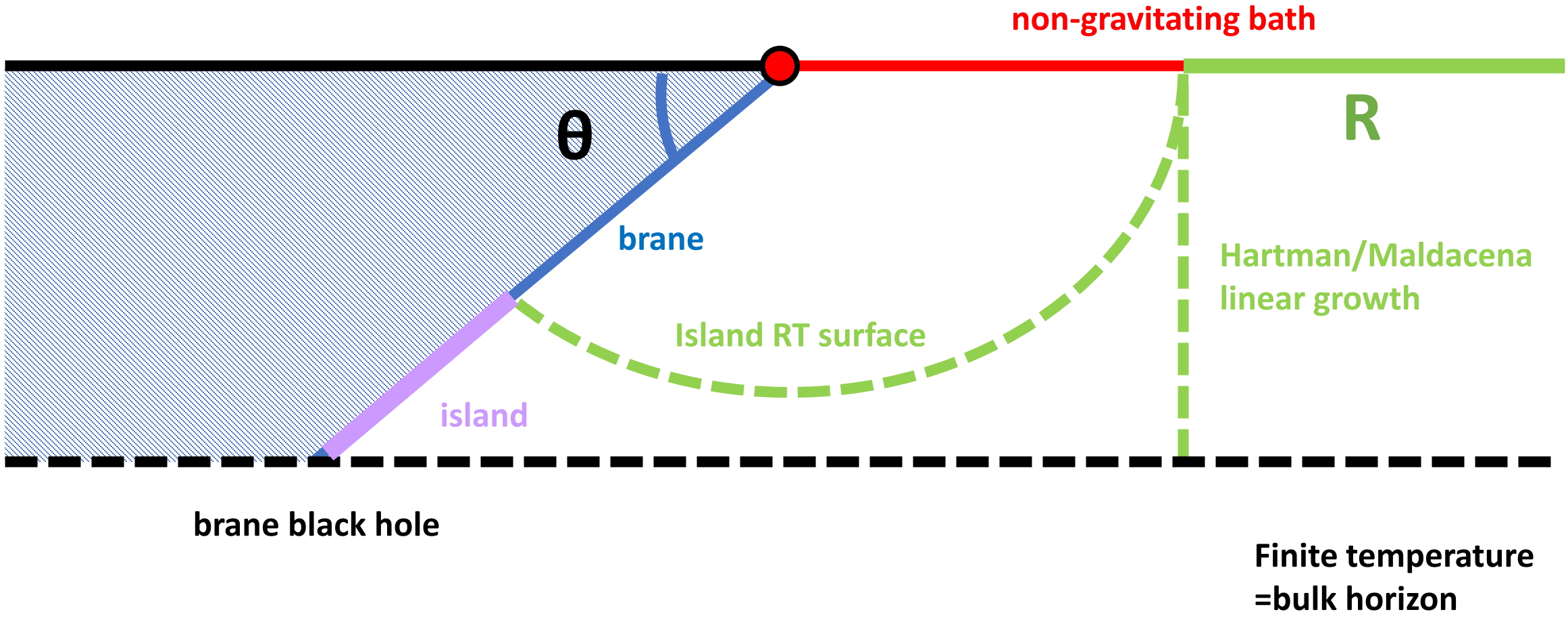
Entanglement Entropy



Double Holographic Islands

(Almheiri, Mahajan, Santos '19)

(Almheiri, Mahajan, Maldacena '19)



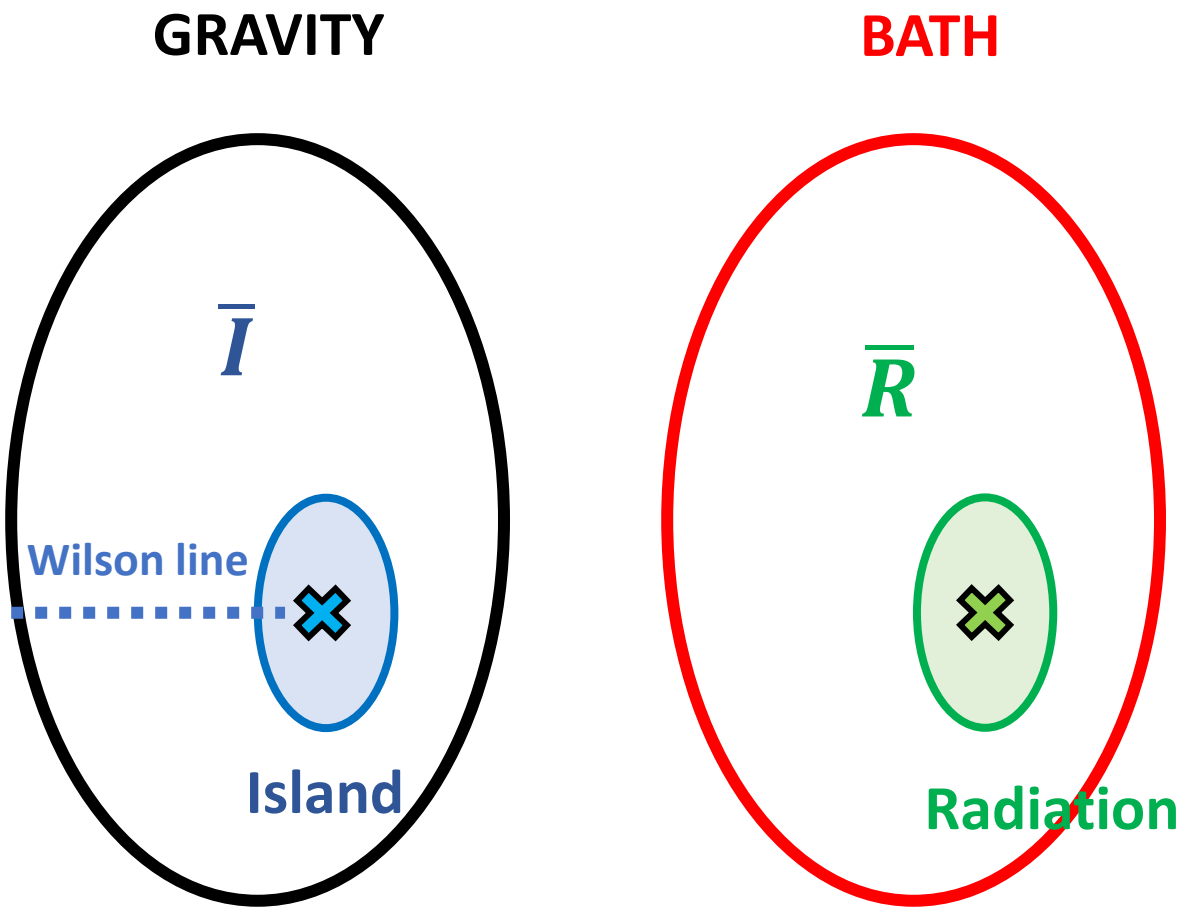
Entanglement Entropy

Success: Page curve of gravity in its Higgs phase!

can also be seen in BCFT (Sully, Van Raamsdonk, Wakeham ‘20)

Page curves without mass?

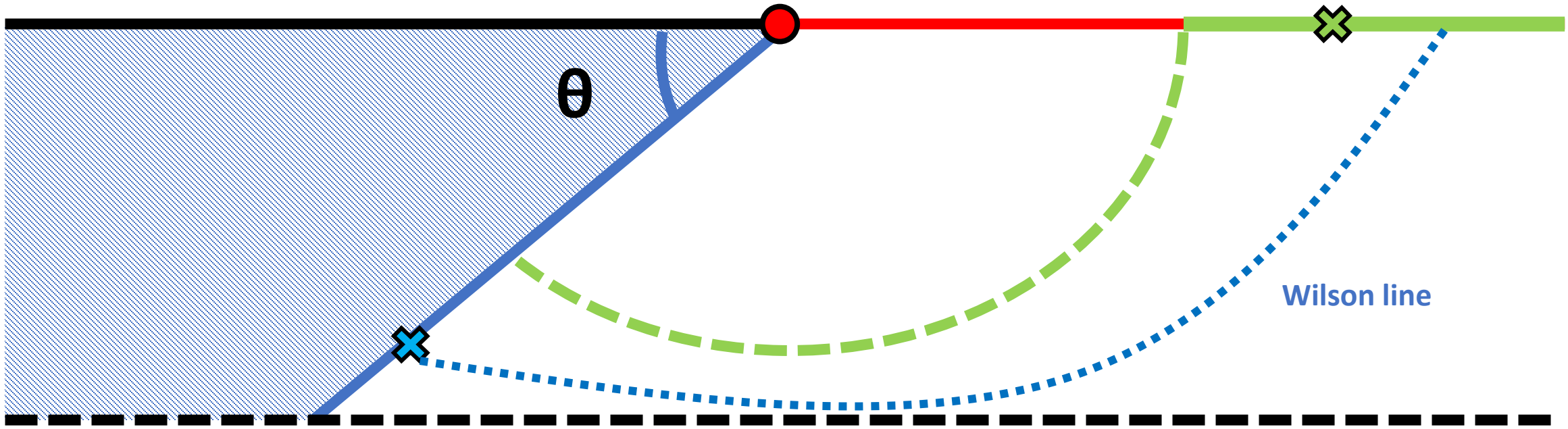
(Geng, AK, Perez, Raju, Randall, Riojas, Shashi ‘21)



The fact that operators in the island need to be gravitationally dressed clashes with standard notions of Wedge reconstruction

Claim: no islands in massless gravity

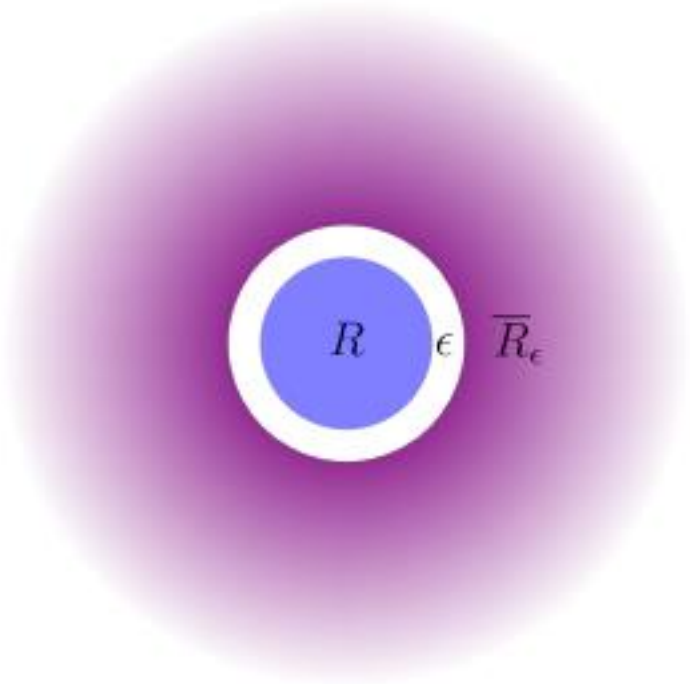
But how come islands exist in RS?



Excitation in I can be dressed to R via higher dimensional bulk!

In the intermediate picture this is represented as the screening by the graviton mass.

Gravity fails split property (Raju'21)



- Split property of QFT allows us to define EE in regions
- Massless gravity fails to satisfy split property
- Assuming it was the error in Hawking's original calculation
- Instead: in massless gravity all information accessible at infinity all the time

Massless Gravity = Flat Page Curve

Quantum Information in Massless Gravity

- The problematic assumption is that one can meaningfully define the EE of a local region in a theory of gravity.
- Quantum effects due to dressing ambiguities go as $G\,E^{d-2}$. Precision required to resolve Page curve goes as e^{-S} . For black holes (unlike ordinary matter) these can not be separated.
- Coupling to bath = Higgsing Gravity resolves this tension.

Open problems

Massless Page curves for other observables?

- Gauge invariant operators can be defined relationally. Local enough to see Page curve? (surely not in eternal black hole)

$$O = \int e^{-aR(x)^2}\, \psi(x)$$

- Define coarse grained entropy or gauge fixed observables directly in bulk instead of fine-grained EE of R. Clearly Page curve can go up. But can we find observable that sees it go down without getting a flat Page curve?

Intermediate Dictionary?

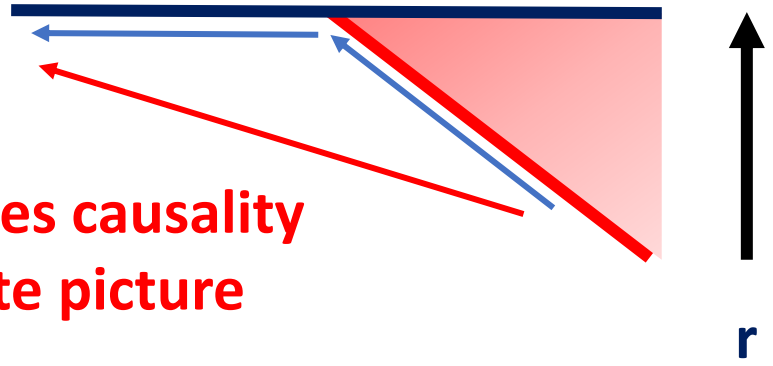
Bulk and BCFT are sharply defined. But what about intermediate picture?

Bottom-up, 3d Dof >> 4d Dof: Generalized Differentiate Dictionary

(Neuenfeld ‘21)

Potential problem: Shortcuts!

(Omiya, Wei ‘21)



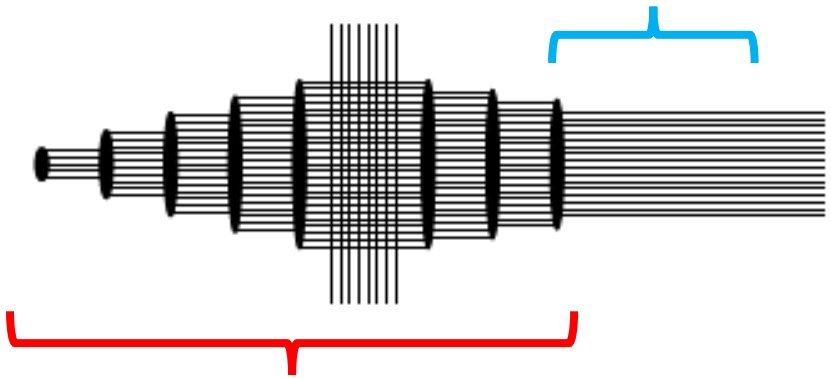
bulk geodesic violates causality
in naïve intermediate picture

Intermediate Dictionary?

(AK, Uhlemann, Sun '22)

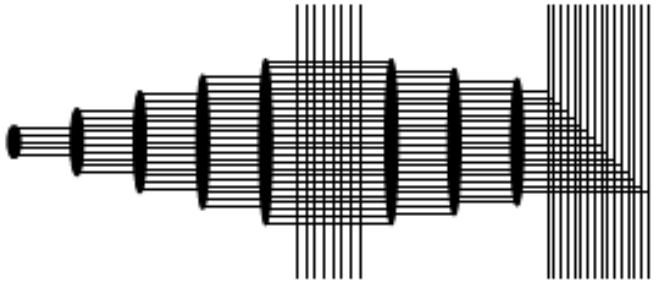
Top-down gives an answer: dualize only 3d CFT!

N=4 SYM on half space



boundary, 3d CFT

genuine 3d CFT with global symmetry

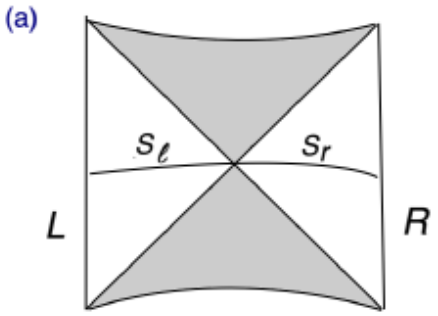


has UCLA-type AdS_4 x internal dual;
can be coupled to N=4 on half-space

Geometry genuinely different! Is intermediate picture "on brane" possible?

Algebraic Approach

(Leutheusser, Liu '21, Witten '22)



U=H_R/N

At leading order in large N algebra of observables:

$$\mathcal{A}_R = \mathcal{A}_{R,0} \otimes \mathcal{A}_U \qquad \mathcal{A}_L = \mathcal{A}_{L,0} \otimes \mathcal{A}_U$$

"type 3": local fields in the bulk

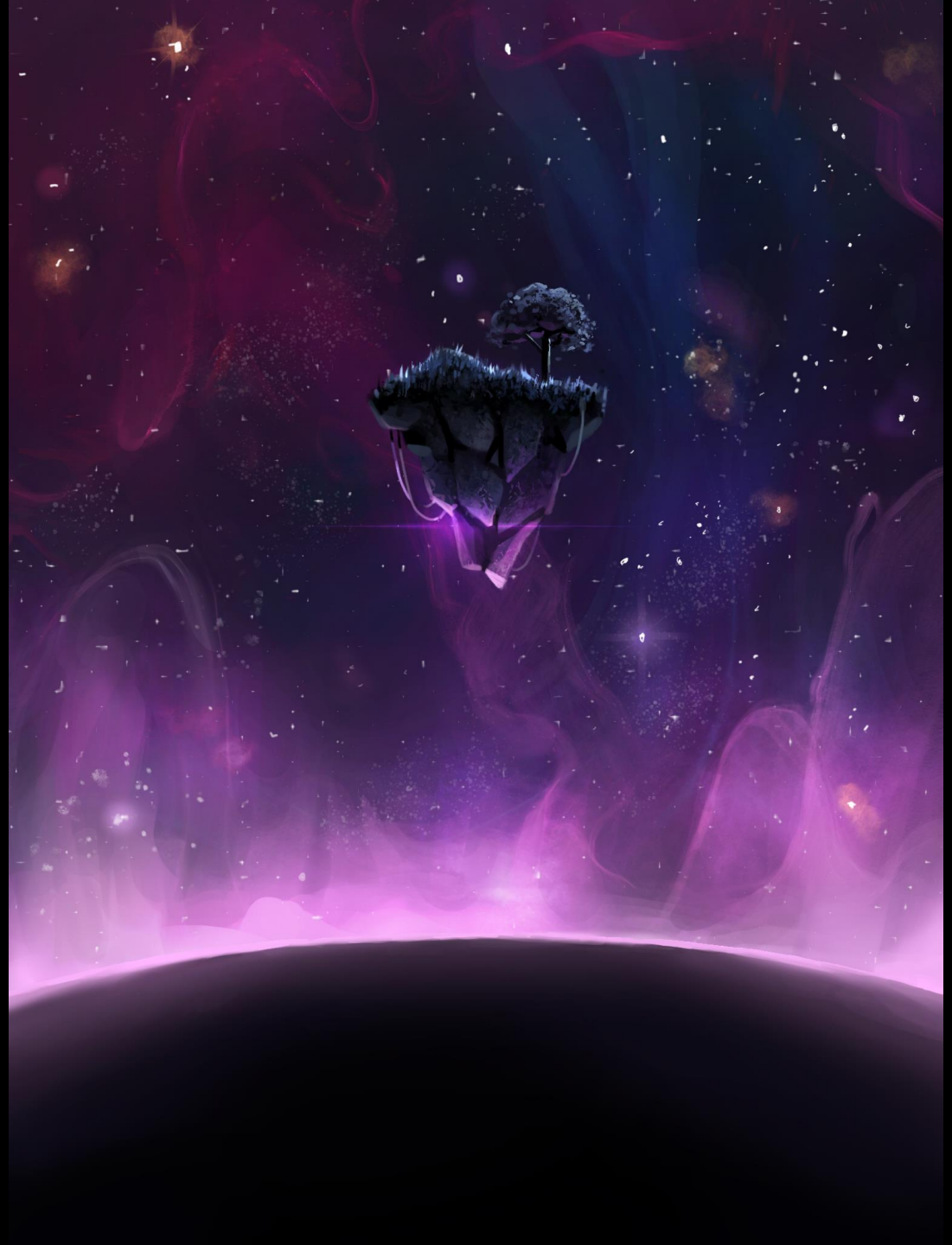
Coupling to Bath?

$$S = S_{CFT1} + S_{CFT2} + h \int d^d x \, O_1 \, O_2$$

U fails to commute with *A* even at leading order!

(Fan, AK, ongoing)

Higgsing? Islands?



Thank you