Holography for Multi-Representation and Chiral Matter

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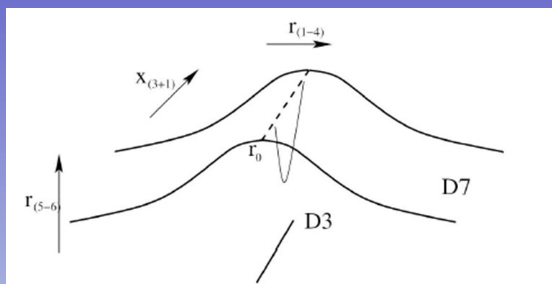
AdS/CFT Contains Non-SUSY Theories

Eg Witten black holes = finite T theories

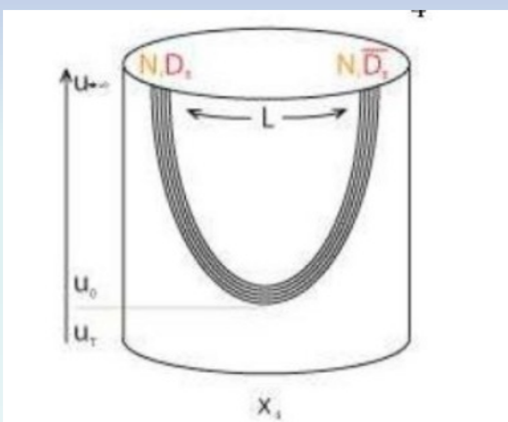
hep-ph/0501128
hep-ph/0501218

Top/down

hep-th/0306018



Sakai-Sugimoto model (D4/D8)



AdS/QCD – "fluctuation theory"

$$S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{ |DX|^2 — 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu),$$

TABLE II: Results of the model for QCD observables. Model A is a fit of the three model parameters to m_π, f_π and m_ρ (see asterisks). Model B is a fit to all seven observables.

Observable	Measured (MeV)	Model A (MeV)	Model B (MeV)
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

Condensate is a fit parameter

Probe limit DBI Action captures key elements

Crystallization to running γ

By 2011 all these systems had been shown to be "DBI + running γ"

EG Jarvinen + Kiritsis. 1112.1261 [hep-ph]

$$S = \int_{r_0}^{\infty} d^5x \sqrt{-g}\, Tr \left\{ |DX|^2 - 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

$m^2(r)$

$$m^2 = \Delta(\Delta - 4)$$

EG world-volume B field
(Johnson + Filev)
hep-th/0701001

$$\sqrt{1 + \frac{B^2}{(r^2 + X^2)^2}} = \ldots - \frac{B^2}{\sqrt{1 + \frac{B^2}{r^4}} r^6} X^2 + \ldots$$

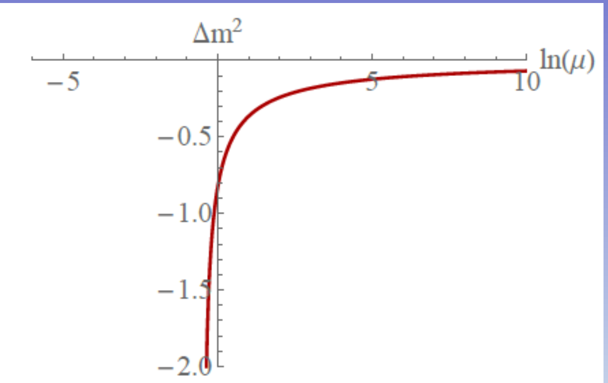
Chiral symmetry breaking is caused by a BF bound violation $m^2 < -4$ at low r ($\gamma = 1$ criteria)

Dynamic AdS/QCD

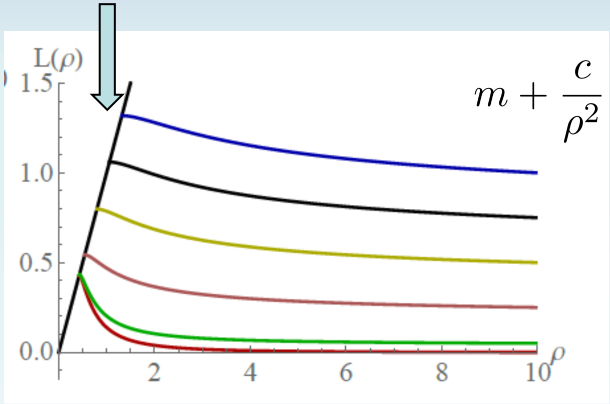
So we're unable to resist putting in the QCD running γ ! 2010.10279 [hep-ph]

$$\mu \frac{d\alpha}{d\mu} = -b_0 \alpha^2 - b_1 \alpha^3 ,$$

$$b_0 = \frac{1}{6\pi}\left(11 N_c - (N_f + \bar{N}_f)\right) ,$$
$$b_1 = \frac{1}{24\pi^2}\left(34 N_c^2 - 5 N_c (N_f + \bar{N}_f) - \frac{3}{2}\frac{N_c^2 - 1}{N_c}(N_f + \bar{N}_f)\right) ,$$
$$\gamma = \frac{3(N_c^2 - 1)}{4 N_c \pi}\alpha .$$



On mass shell IR b.c.

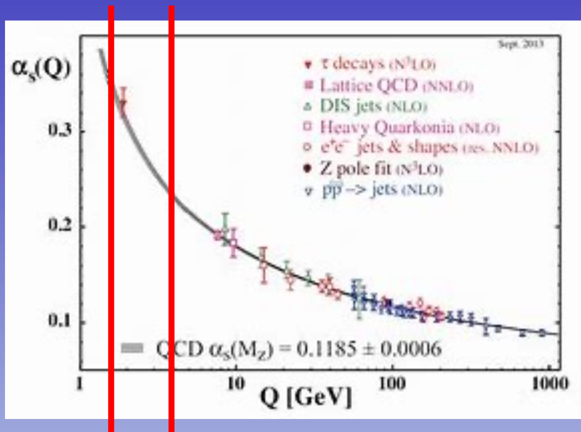


Observables (MeV)	QCD	AdS/SU(3) 2 F 2 $\bar{F}$	Deviation
M_ρ	775	775*	fitted
M_A	1230	1183	- 4%
M_S	500/990	973	+64%/-2%
M_B	938	1451	+43%
f_π	93	55.6	-50%
f_ρ	345	321	- 7%
f_A	433	368	-16%
$M_{\rho,n=1}$	1465	1678	+14%
$M_{A,n=1}$	1655	1922	+19%
$M_{S,n=1}$	990 /1200-1500	2009	+64%/+35%
$M_{B,n=1}$	1440	2406	+50%

Dynamical description of χSBing, right pattern…

Perfection

If you want to do better – treat as effective theory below scale QCD enters strong coupling. hep-th/0403279



Witten's "multi trace" prescription allows inclusion of HDOs at Λ_{UV} by reinterpreting bcs: hep-th/0112258

$$\frac{g^2}{\Lambda_{UV}^2}\bar{\psi}_L\psi_R\bar{\psi}_R\psi_L \xrightarrow{\langle\bar{\psi}_L\psi_R\rangle} m\bar{\psi}_R\psi_L$$

$$m = \frac{g^2}{\Lambda_{UV}^2}\langle\bar{q}_L q_R\rangle$$

$$c = \frac{\Lambda_{UV}^2 m}{g^2}$$

Small gap $\Lambda_{\chi SB}$ vs Λ_{UV}…..

Observables (MeV)	QCD	Dynamic AdS/QCD	HDO coupling
M_V	775	775	sets scale
M_A	1230	1230	fitted by $g_A^2 = 5.76149$
M_S	500/990	597	prediction $+20\%/-40\%$
M_B	938	938	fitted by $g_B^2 = 25.1558$
f_π	93	93	fitted by $g_S^2 = 4.58981$
f_V	345	345	fitted by $g_V^2 = 4.64807$
f_A	433	444	prediction $+2.5\%$
$M_{V,n=1}$	1465	1532	prediction $+4.5\%$
$M_{A,n=1}$	1655	1789	prediction $+8\%$
$M_{S,n=1}$	990/1200-1500	1449	prediction $+46\%/0\%$
$M_{B,n=1}$	1440	1529	prediction $+6\%$

2010.10279 [hep-ph]

$$\frac{g_S^2}{\Lambda_{UV}^2}|\bar{q}q|^2\,, \qquad \frac{g_V^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu q|^2$$

$$\frac{g_A^2}{\Lambda_{UV}^2}|\bar{q}\gamma^\mu\gamma_5 q|^2\,, \qquad \frac{g_B^2}{\Lambda_{UV}^5}|qqq|^2$$

Includes some stringy corrections on excited states?

Multi-Representation Theories

Traditionally one would argue that apart from the fundamental representation the size of the rep grows at least as N^2 and there is no probe limit – back reaction matters…

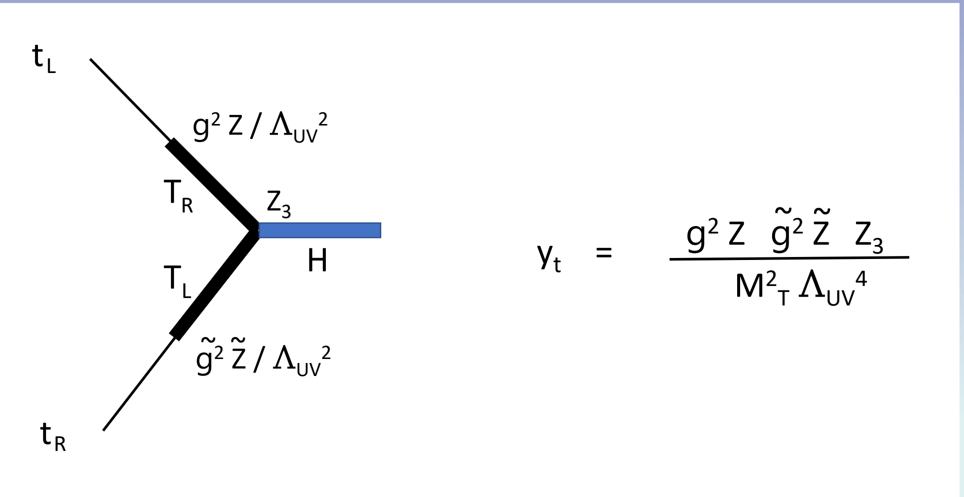
The DBI action lesson though is that a dimension 3 qq operator is described by a holographic scalar and the dynamics enters through $m^2(r)$…

Composite Higgs Models **(Reviews:2002.04914, 1506.01961)**

THE BASICS: a model must have "quark" condensates that break a global symmetry to give 4+ Goldstones that can be made the SM Higgs…

To form the top mass without FCNCs people use "partial top compositeness" (D Kaplan 1991)

Need exotic baryons made of two representations



Lattice groups have become interested in these theories…

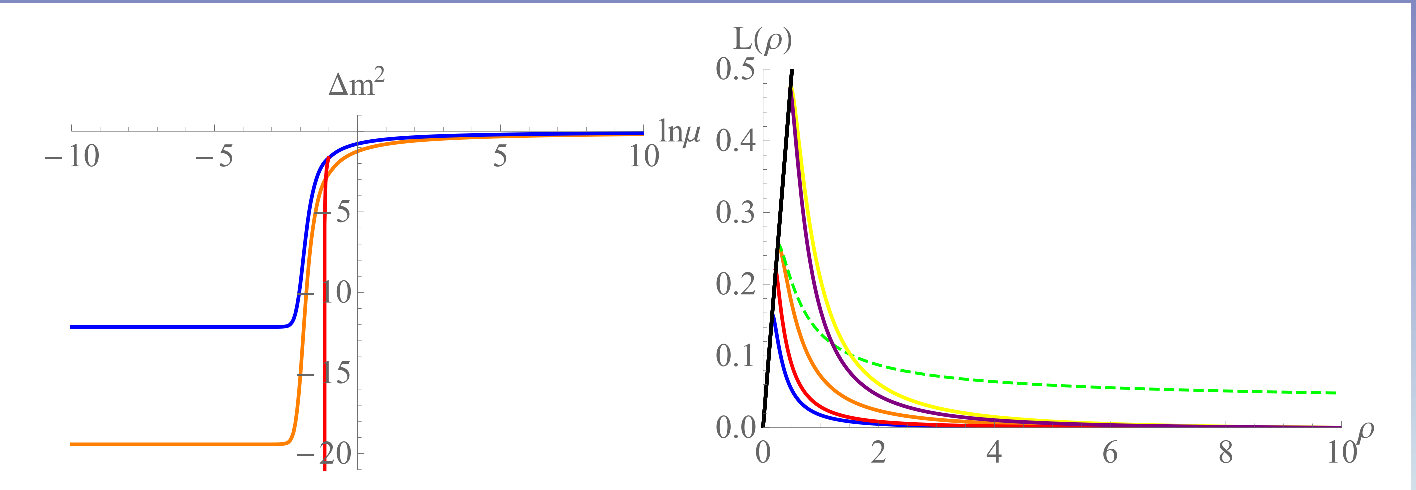
Sp(4) 4F $6A_2$

NE, Johanna Erdmenger, Kostas Rigatos and Werner Porod: 2010.10279 [hep-ph]

We run the model with two scalars – one for the F condensate and one for the A2 condensate.. We input perturbative runnings of γ in each case to fix Δm^2…

The running AdS mass

The RG mass profiles of the quarks



How you decouple the quarks is important and unknown – I'll concentrate on when they are removed below their IR mass scale. Quench = pure glue running.

The gap between F and A2 grows the less you decouple the quarks – the slower the running the more conformal the theory is around the chiral symmetry breaking point – this will lead to a lighter scalar meson…

Sp(4) 4F $6A_2$

NE, Johanna Erdmenger, Kostas Rigatos
and Werner Porod: 2010.10279 [hep-ph]

	AdS/Sp(4) A2 decouple	AdS/Sp(4) quench	lattice [79] quench
$f_{\pi A_2}$	0.120	0.103	0.1453(12)
$f_{\pi F}$	0.0701	0.0756	0.1079(52)
M_{VA_2}	1*	1*	1.000(32)
f_{VA_2}	0.517	0.518	0.508(18)
M_{VF}	0.814	0.962	0.83(19)
f_{VF}	0.364	0.428	0.411(58)
M_{AA_2}	1.35	1.28	1.75 (13)
f_{AA_2}	0.520	0.524	0.794(70)
M_{AF}	1.19	1.36	1.32(18)
f_{AF}	0.399	0.462	0.54(11)
M_{SA_2}	0.375	1.14	1.65(15) †
M_{SF}	0.902	1.25	1.52 (11)†
M_{BA_2}	1.85	1.86	
M_{BF}	1.53	1.79	

We set the scale in the A2 sector…

the pattern of mass scales is right…

F sector is lighter than the A2s

Again F sector - right pattern

KEY IMPACT: easy for us to unquench – the slower the running the lighter the sigma

[78] E. Bennett, D. K. Hong, J.-W. Lee, C.-J. D. Lin, B. Lucini, M. Mesiti, M. Piai, J. Rantaharju, and D. Vadacchino, "Sp(4) gauge theories on the lattice: quenched fundamental and antisymmetric fermions," arXiv:1912.06505 [hep-lat].

SU(4) 3 F $3\bar{F}$ 5 A_2

NE, Johanna Erdmenger, Kostas Rigatos and Werner Porod: 2010.10279 [hep-ph]

G. Ferretti, "UV Completions of Partial Compositeness: The Case for a SU(4) Gauge Group," JHEP **06** (2014) 142, arXiv:1404.7137 [hep-ph].

The lattice has simulated (unquenched) SU(4) 2 F 2 F 4 A_2

V. Ayyar, T. DeGrand, M. Golterman, D. C. Hackett, W. I. Jay, E. T. Neil, Y. Shamir, and B. Svetitsky, "Spectroscopy of SU(4) composite Higgs theory with two distinct fermion representations," Phys. Rev. D **97** no. 7, (2018) 074505, arXiv:1710.00806 [hep-lat].

	Lattice [80] $4A_2, 2F, 2\bar{F}$ unquench		AdS/$SU(4)$ $4A_2, 2F, 2\bar{F}$ decouple		AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ decouple	AdS/$SU(4)$ $5A_2, 3F, 3\bar{F}$ quench
$f_{\pi A_2}$	0.15(4)		0.0997		0.111	0.102
$f_{\pi F}$	0.11(2)		0.0953		0.109	0.892
M_{VA_2}	1.00(4)		1*		1*	1*
f_{VA_2}	0.68(5)		0.489		0.516	0.517
M_{VF}	0.93(7)		0.939		0.904	0.976
f_{VF}	0.49(7)		0.461		0.491	0.479
M_{AA_2}			1.37		1.32	1.28
f_{AA_2}			0.505		0.521	0.522
M_{AF}			1.37		1.23	1.28
f_{AF}			0.504		0.509	0.492
M_{SA_2}			0.873		0.684	1.18
M_{SF}			1.02		0.798	1.25
M_{JA_2}	3.9(3)		2.21		2.21	2.22
M_{JF}	2.0(2)		2.08		2.00	2.17
M_{BA_2}	1.4(1)		1.85		1.85	1.86
M_{BF}	1.4(1)		1.75		1.68	1.81

The pattern is right…

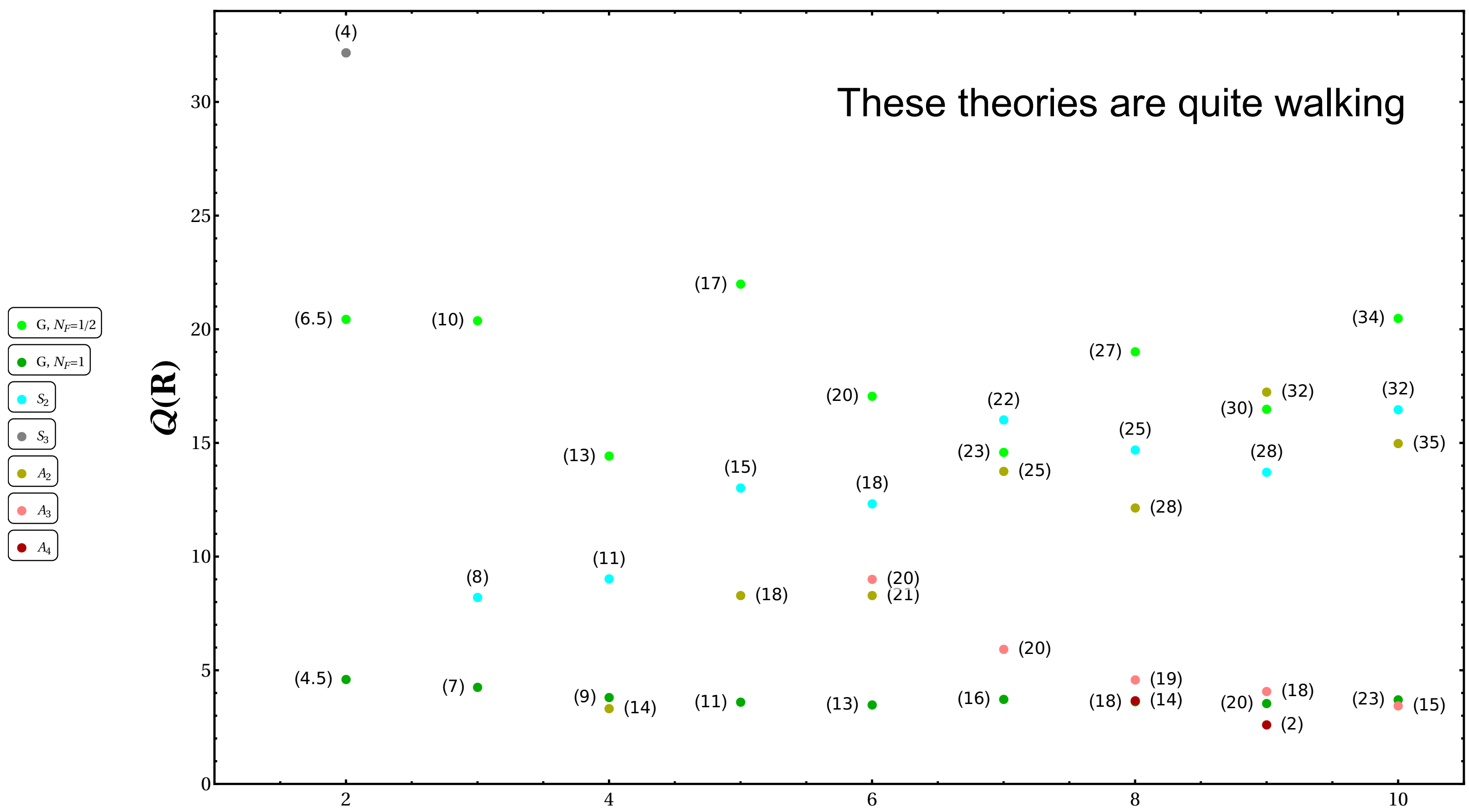
The A2-F gap is well described…

KEY POINTS:
Adding extra flavours is not a huge change…

Scalar masses get lighter as add extra flavours

It's fun that such a simple holographic model captures these more exotic theories… and leads to a number of interesting questions about the dynamcs:

Two loop results for SU(N) with fundamentals + another rep. "γ=1" criteria



Nick Evans, Kostas Rigatos. 2012.00032 [hep-ph]

$$\mathcal{Q}(R) = \frac{\Lambda_{\chi SB\ \ R}}{\Lambda_{\chi SB\ \ F}}$$

Order of magnitude gap between chiral symmetry and confinement? Would like to understand confinement as a BF bound violation for monopoles too…

Domain Wall Chiral Quarks

2106.08753

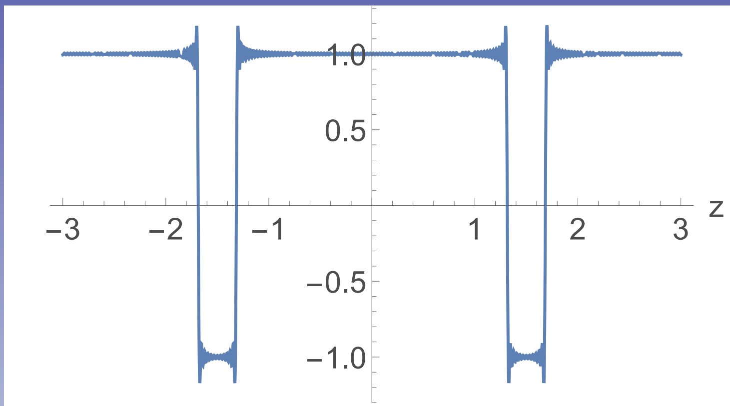
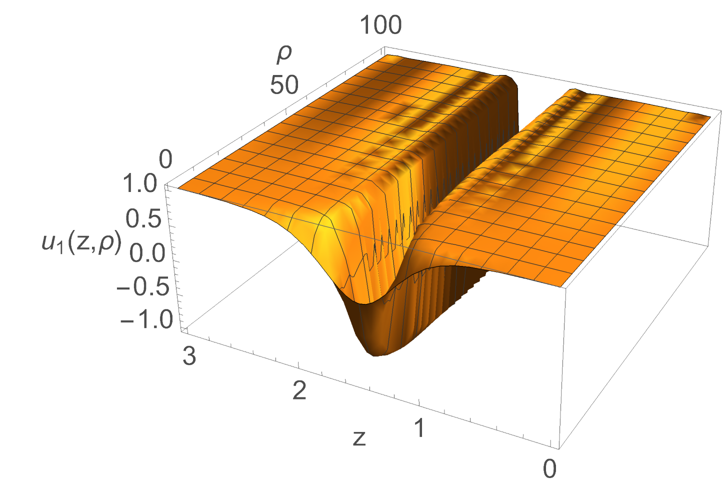
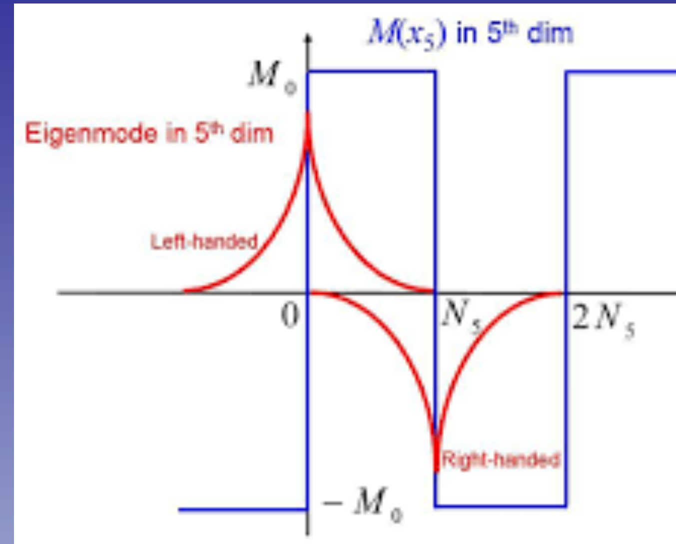


Figure 2: The Fourier representation of the even periodic mass function we use (100 Fourier terms are used) - in each period it has two domain walls separated by a width w.



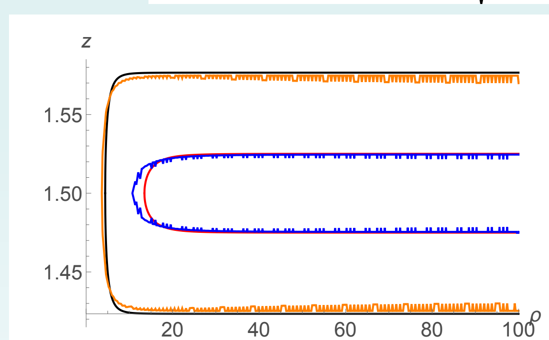


Kaplan 92

THE D3/PROBE-D7 SYSTEM & DOMAIN WALLS

	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	•	•	•	•	•	•
D7	-	-	-	-	-	-	-	-	•	•

$$S_{D7} \approx \int d^4x\; d\rho\; \rho^3 \sqrt{1 + (\partial_\rho u_i)^2 + \frac{R^4}{(\rho^2 + u_i^2)^2} (\partial_x u_i)^2}$$



Sakai-Sugimoto-esque
Us for chiral symmetry breaking…

The Large Mass Limit

u is constant except on some z(ρ) where δ_ρ u diverges

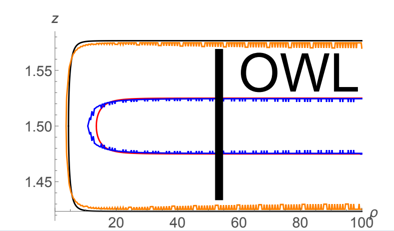
$$\partial_\rho u_i \;=\; \frac{1}{\sqrt{g_{\rho\rho}(\partial_z\rho)^2}}\,\delta(z-z_0)\Big|_{\rm locus}$$

$$S = \int d^2x\; d\rho\; \rho^2\sqrt{1+\rho^4(\partial_\rho z)^2}$$

The action is precisely that of the D3 /probe D5 anti-D5 system and the Us the same…

There remain fluctuations on the domain wall

hep-th/ 0803.3547



Non-local q_L q_R operators at the IR tip become local and mix with the 4d local qq operator – source each other?

$$\mathcal{L} \approx \rho^4(\partial_\rho z)\sqrt{1+\mathcal{A}(\partial_\rho u_i)^2+\frac{(\partial_{x_{2+1}}u_i)^2}{(\rho^2+u_i^2)^2}}$$

with

$$\mathcal{A} = 1+\frac{1}{(\partial_\rho z)^2(\rho^2+u_i^2)^2}$$

where from (5) we know

$$\partial_\rho z = \frac{\rho^4_{\rm min}}{\sqrt{\rho^{12}-\rho^8_{\rm min}\rho^4}}$$

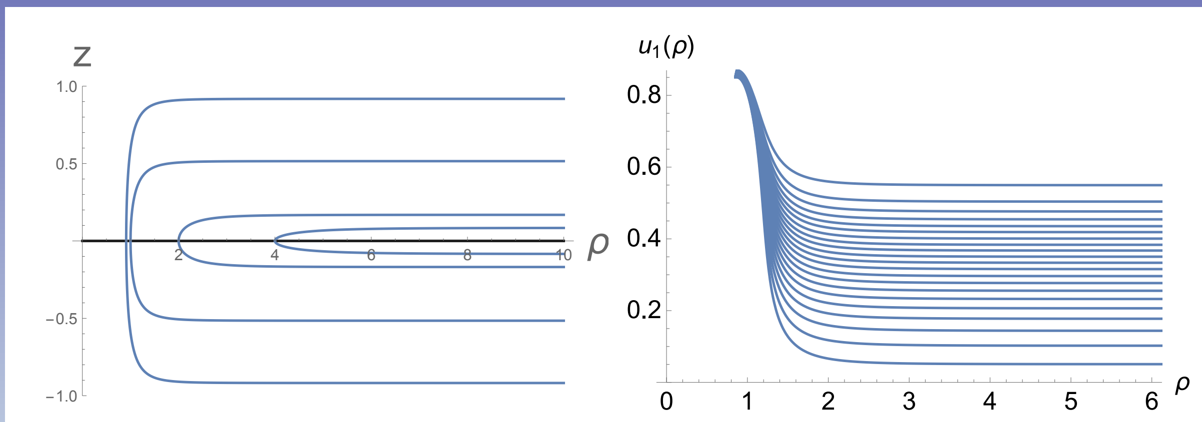
u_i = constant mass = IR gap

Suggests these theories' chiral symmetry breaking is purely a hard mass. (m_{IR} proportional 1/width)

Applied Magnetic Field/Dilaton Profile

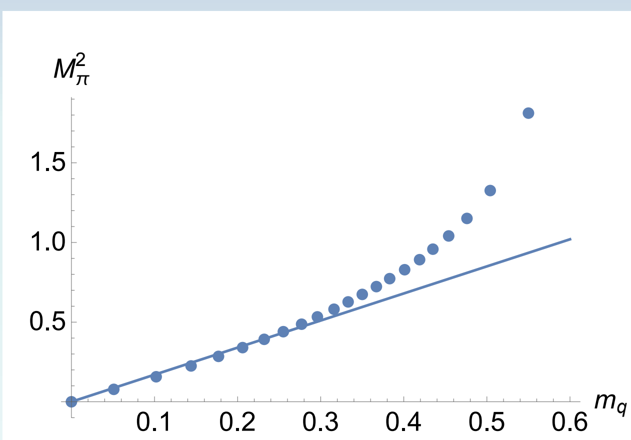
$$h^2 = 1 + \frac{1}{\rho^q}$$

Violate the BF bound by hand in the interior of the space via a dilaton profile



Witten's multi-trace prescription allows an NJL interpretation of the UV mass also…

Us pile up at IR point… surface u_i show chiral symmetry breaking and…



Goldstones show a Gell-Mann-Oakes Renner relation….

The interpretation of the set up is self consistent and the first U system we know with an explicit measure of mass and the condensate

Domain Wall AdS/QCD

2108.12152

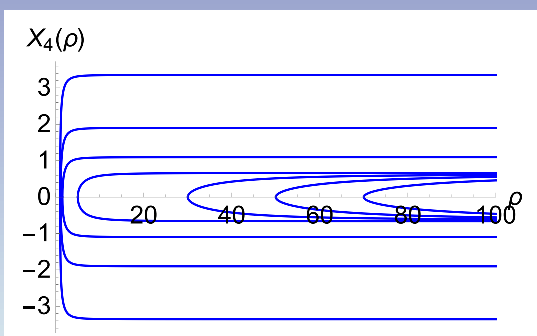
	0	1	2	3	4	5	6	7	8	9
D5	-	-	-	-	-	(-)	•	•	•	•
D7	-	-	-	-	-	•	-	-	-	•

The UV is rather odd – the fluctuations aren't normalizable… all Us asymptote to the same width irrespective of the mass gap… but IR seems OK…

hep-th/0605017

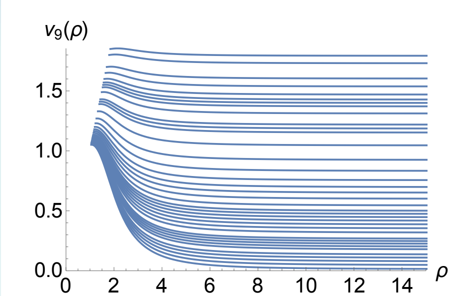
Compactifying in x_5 ➡ confinement

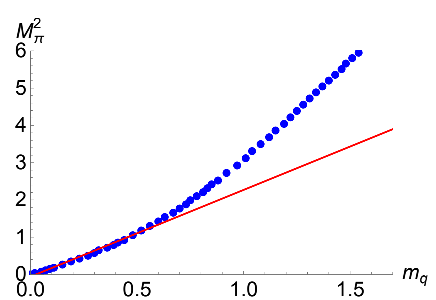
Domain wall $m(x_4)$ ➡ 3+1d chiral quarks



Us pile up… surface field shows χSBing… GMOR relation…

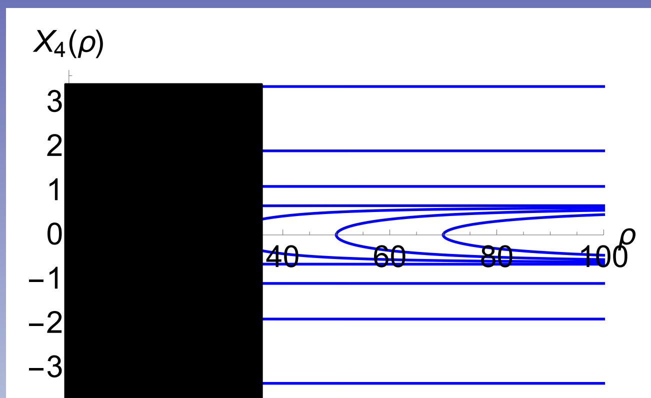
	QCD	DW AdS/QCD	Improved DW AdS/QCD
m_ρ	775 MeV	775*	$g_q = 0.247$
m_π	139 MeV	139*	$g_v = 0.656$
m_a	1230 MeV	1,955	$g_A = 1.287$
F_V	345 MeV	345*	
F_A	433 MeV	726.7	
f_π	93 MeV	135.3	128.8
$M_{v,n=1}$	1465 MeV	3284	1881.8
$M_{A,n=1}$	1655 MeV	5043	2752.5

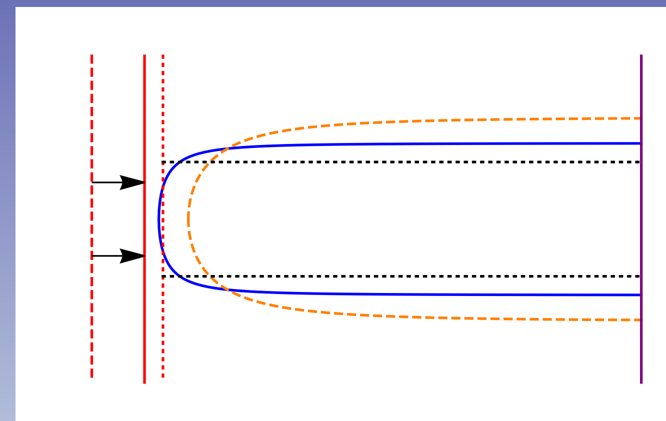




Thermal Transitions

Naively the black hole horizon eats sequential Us and there's a first order meson melting transition….





Here with Λ_{UV} and T, you must be careful not to make a one to one identification between width and UV mass… the surface fluctuation lets you precisely ID the mass… and the meson melting transition is second order…. (first top down example of that?)

Conclusions

Holographic models of QCD continue to be interesting play grounds…

Holography is a remarkably simple method to get a ball park answer for behaviour including with higher dimension reps… effective theory ideas give systematically improvement (perfect action ideas)…

Models have an interesting interplay with current lattice frontier… and highlight aspects of strong coupling we don't understand – confinement as a BF bound violation; how quarks decouple at strong coupling…

Domain wall chiral fermions are a new top down direction – refining understanding of U shaped brane configurations - alternative AdS/QCD - can we do more exotic chiral gauge theories this way?