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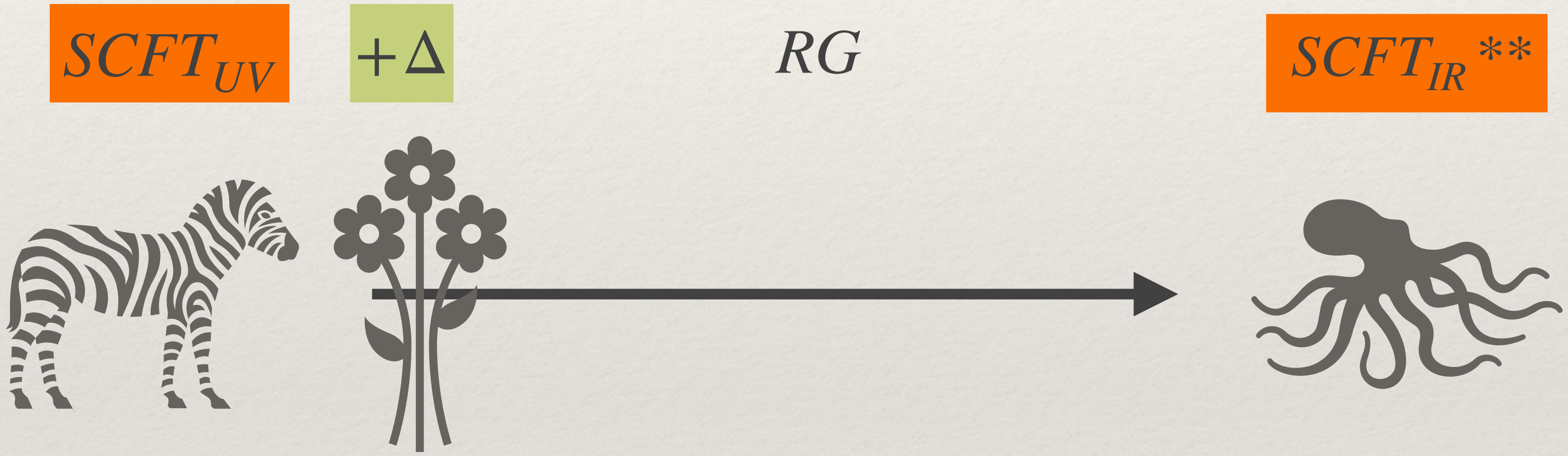
News from the higher $\mathscr{D}$ SQFT frontier

19/7/2022
Strings 2022
Vienna

$\mathscr{D}$ and SQFT

- **SQFT for us** — Unitary, Poincare, (at least) Minimal Supersymmetry
- **SCFTs** — Superconformal symmetry; Fixed points of RG flows*



$SCFT_{UV}$ $+\Delta$ $\xrightarrow{RG}$ $SCFT_{IR}$**

- Typically discussion starts by choosing a space-time dimension

$$\mathscr{D}$$

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* Unless explicitly stated all constructions will be assumed to be at least minimally supersymmetric

** Might be interacting SCFT, gapped, or free theory

Higher $\mathscr{D}$ SQFTs

- Classification of superconformal algebras in $\mathscr{D} \leq 6$ *(Nahm 77; Kac 77; Minwalla 97)*

Interacting SCFTs

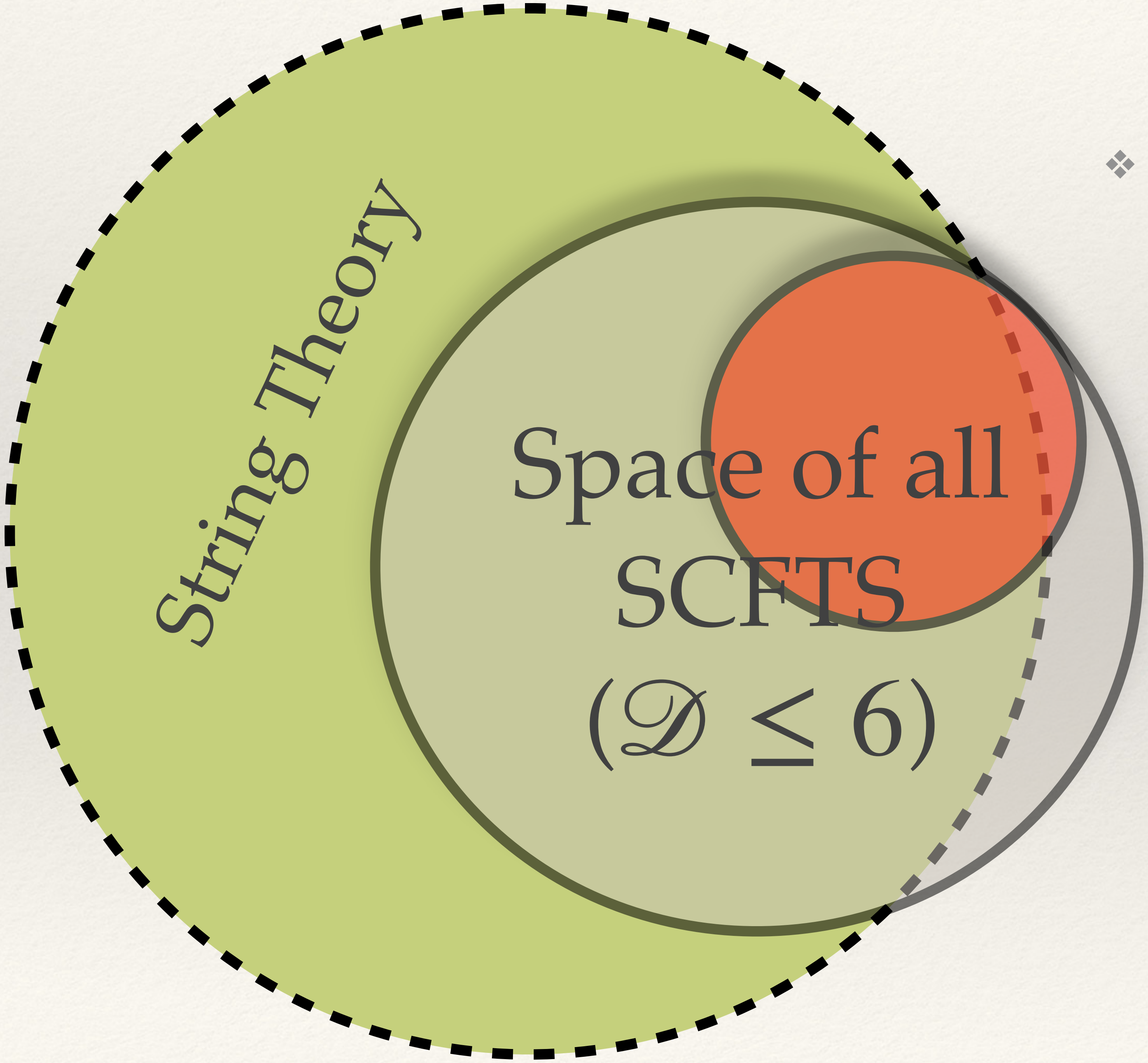
- in $\mathscr{D} > 4$ are not deformations of a free theory in $\mathscr{D}$
- in $\mathscr{D} = 4$ can have Lagrangian descriptions in $\mathscr{D}$
- in $\mathscr{D} < 4$ have a plethora of such descriptions

- Three known roads to constructing interacting SCFTs
  - **Field theoretic:** Lagrangians in $\mathscr{D} \leq 4$
  - **Stringy:** String/M-theory constructions in all $\mathscr{D}$

    (Singular) geometry; branes; holography
  - $\oplus$ **Hybrid:** Stringy construction → QFT deformations

- Numerous relations between the three

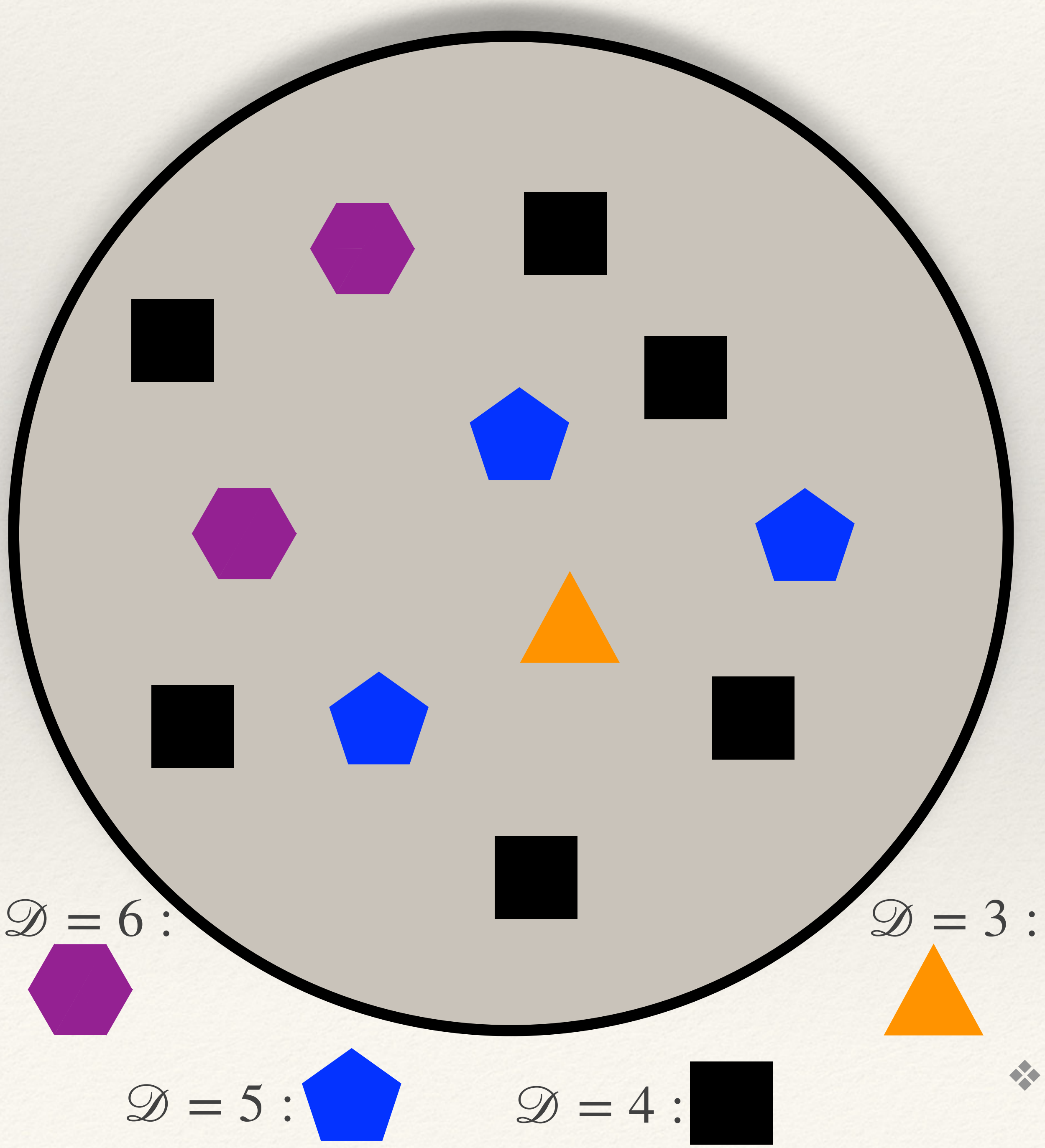
Space of ALL SCFTs



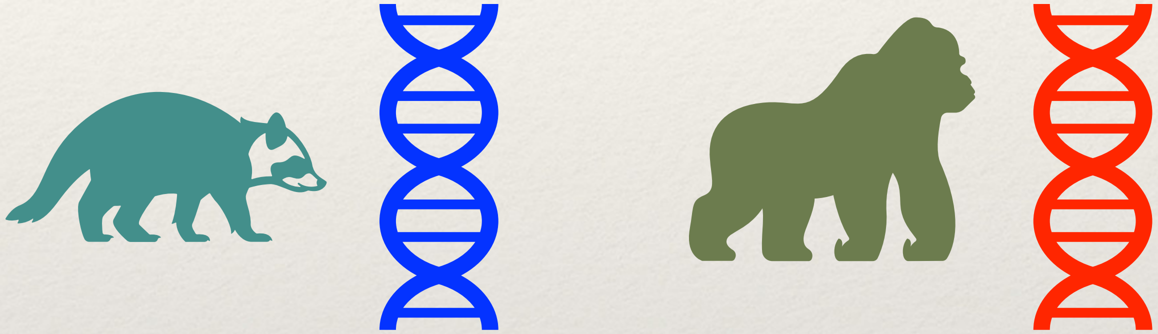
- This space includes:
  - Free fixed points
    - *Deformation of these*
  - Limits of String Theory
    - *Deformations of these*
  - Anything else (??)

Charting the Space of ALL SCFTs

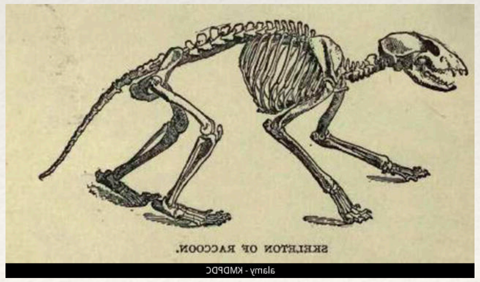
- Every known SCFT has a name: SQCD, T_N, AD, E_8 MN, E-string, conformal matter…



- The animal={Operators, all correlation functions}



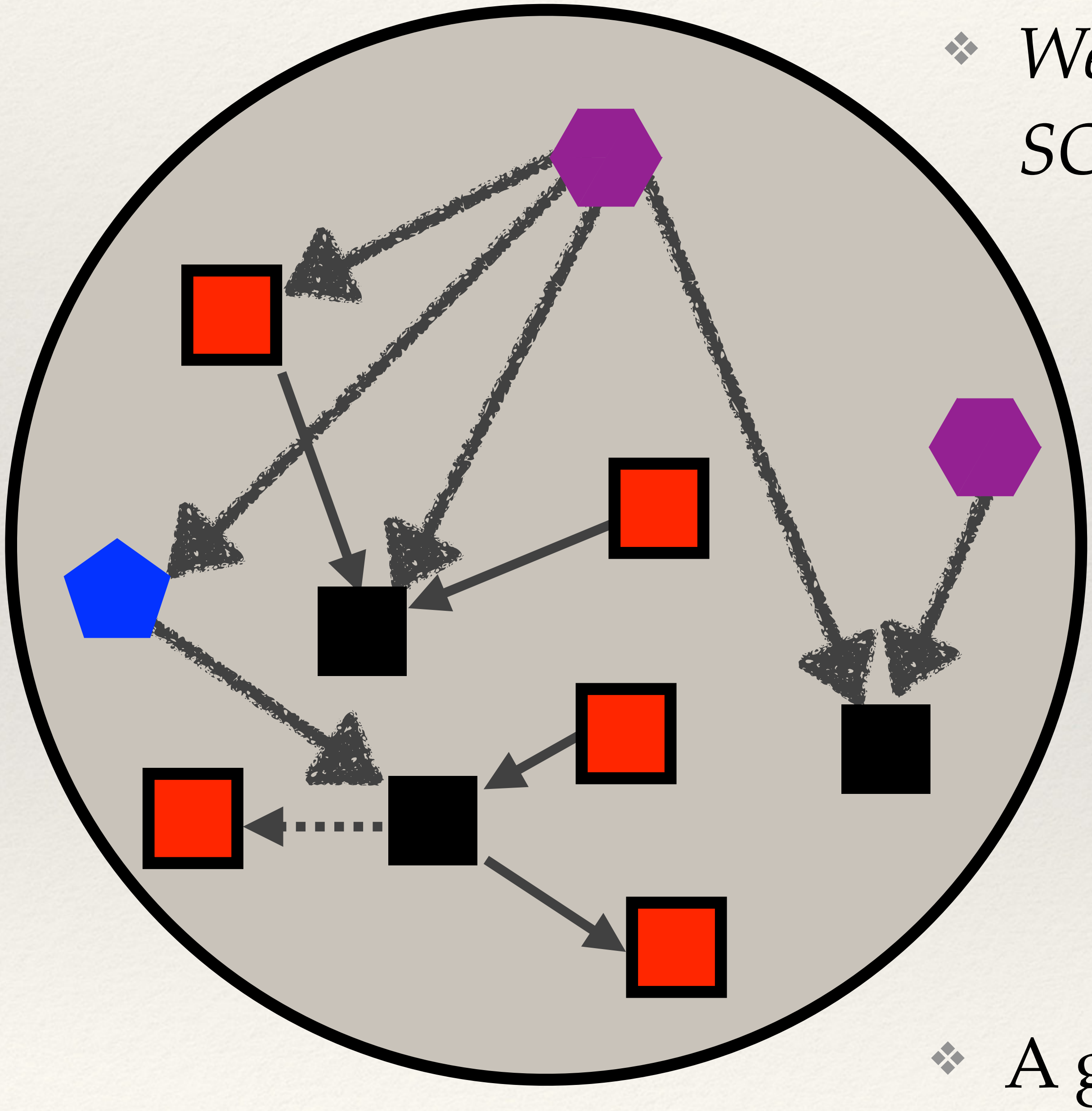
- *The skeleton={Symmetries, Anomalies, BPS spectra, Moduli spaces, Conformal manifolds, (S)Partition functions}*



- Construction: Lagrangian or Stringy

Structure of the Space of ALL SCFTs

- The names are not unique



- *We might be able to construct the same SCFT using various construction*
  - A stringy construction
  - A deformation of a free fixed point
    (red ≙ Lagrangian)
  - A deformation of a strongly coupled SCFT (superpotential, gauging)
- A geometric deformation of an SCFT (place on a compact geometry)

Progress on the following questions:

- *Goal: Understand the structure of the space of all SCFTs*

  - Enumerate SCFTs

    (Different constructions, string theoretic and field theoretic)

    See Strings 2015 Heckman, Strings 2017 Kim

  - Find the skeletons

    (Moduli spaces, (generalized) symmetries and anomalies, BPS states)

    See Strings 2021 Shao

  - Derive the structure of relations

    (Various types of dualities and emergence of symmetry)

- Much of the progress follows exploring relations between different SQFTs in a given $\mathscr{D}$, across different $\mathscr{D}$, and SQFTs and string theory

  See Strings 2014 Tachikawa; Strings 2018 Cordova

Plan:

- SQFTs from String Theory
- SQFTs from SQFTs
  - SQFTs from SQFTs in given $\mathscr{D}$
    - *Moduli spaces*
    - *In-$\mathscr{D}$ dualities*
    - *Lagrangians*
  - SQFTs from SQFTs across $\mathscr{D}$ ↓
    - *Across-$\mathscr{D}$ dualities*
    - $\mathscr{D} = 5$ *from* $\mathscr{D} = 6$
    - $\mathscr{D} = 4$ *from* $\mathscr{D} = 6$
    - *In-$\mathscr{D}$ dualities from Across-$\mathscr{D}$ dualities*
    - $\mathscr{D} = 6$ *Dualities*
  - SQFTs from SQFTs across $\mathscr{D}$ ↑
    - $\mathscr{D} = 6$ *SCFTs from* $\mathscr{D} < 6$ *SCFTs*

SCFTs from string theory: general $\mathscr{D}$

- Construct SCFTs from String Theory by decoupling gravity
  - Brane constructions
  - Holography
  - String theory on non compact spaces with singularities

- Conjectured classifications of $\mathscr{D} = 6$ SCFTs

((1,0) *or* (2,0) *supersymmetry: at least 8 supercharges)*

(Branes; Branes probing singularities; F-theory)

Eg Heckman, Morrison, Vafa 13; Heckman, Morrison, Rudelius, Vafa 15; Bhardwaj, Morrison, Tachikawa, Tomasiello 18; Heckman, Rudelius 18

See Strings 2015 Heckman

- A large variety of $\mathscr{D} = 5$ SCFTs

($\mathscr{N} = 1$ supersymmetry: 8 supercharges)

Starting with Seiberg 96; Morrison, Seiberg 96; Katz, Klemm, Vafa 96; Intriligator, Morrison, Seiberg 97 Aharony, Hanany 97; Brandhuber, Oz 99; many many others

- *A variety of $\mathscr{D} = 4$ SQFTs*

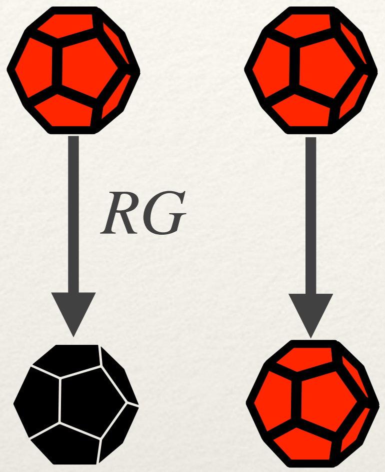
($\mathscr{N} \geq 2$ supersymmetry: at least 8 supercharges; $\mathscr{N} = 1$: 4 supercharges)

Eg Giveon, Kutasov 98; Shapere, Vafa 99; Acharya, Witten 01; many many others

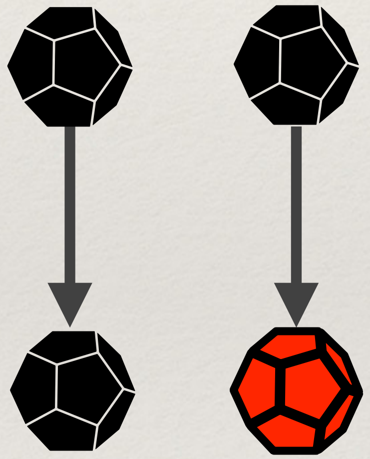
Plan:

- SQFTs from String Theory
- SQFTs from SQFTs
  - SQFTs from SQFTs in given $\mathscr{D}$
    - *Moduli spaces*
    - *In $\mathscr{D}$-dualities*
    - *Lagrangians*
  - SQFTs from SQFTs across $\mathscr{D}$
    - *Across $\mathscr{D}$ dualities*
    - $\mathscr{D} = 5$ *from* $\mathscr{D} = 6$
    - $\mathscr{D} = 4$ *from* $\mathscr{D} = 6$
    - *In-$\mathscr{D}$ dualities from Across-$\mathscr{D}$ dualities*
    - $\mathscr{D} = 6$ *Dualities*
  - SQFTs from SQFTs across $\mathscr{D}$
    - $\mathscr{D} = 6$ *SCFTs from* $\mathscr{D} < 6$ *SCFTs*

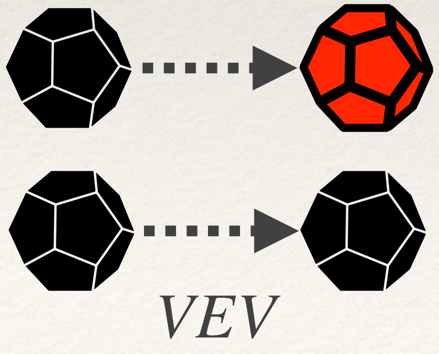
Relations I: SQFTs from SCFTs in Fixed $\mathscr{D}$



- For $\mathscr{D} \leq 4$ start from free fields and deform by relevant deformations (superpotential, gauging global symmetries)
- *Resulting theory might be a gapped, free, or interacting SQFT*



- For $\mathscr{D} \leq 5$ deforming an SCFT by relevant deformations a given SCFT can flow to an interacting, gapped or free theory
- *In $\mathscr{D} = 5$ real mass deformations can lead to IR free gauge theory, deformations of which are again IR free*



- For $\mathscr{D} \leq 6$ can explore moduli spaces of vacua (VEVs) to construct new SQFTs: might be free or interacting
- Such VEV deformations (tensor branch) of SCFTs are described by anomaly free Lagrangians in $\mathscr{D} = 6$

Cordova, Dumitrescu, Intriligator 16

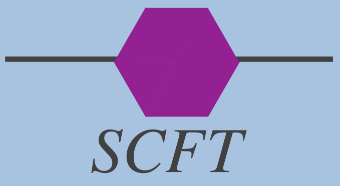
Skeletons I: Moduli Spaces of Vacua

- *Theories with 8 supercharges* ($\mathscr{D} > 4$ *minimal supersymmetry,* $\mathscr{D} = 4$ $\mathcal{N} > 1$): Invariant definition of branches of Moduli Spaces of Vacua: Higgs branch and Tensor/Coulomb branch

- *Tensor* ($\mathscr{D} = 6$)/*Coulomb* ($\mathscr{D} < 6$) *branch:* Typically described on general locus by a simple gauge theory in $\mathscr{D} > 3$

Classification of $\mathscr{D} = 6$ *SCFTs via the tensor branch; Eg single gauge group no matter:*

Bhardwaj 15



$G_{gauge} \in \{SU(3)\,,\ SO(8)\,,\ F_4\,,\ E_6\,,\ E_7\,,\ E_8\}$

Seiberg 96; Bershadski, Vafa 96

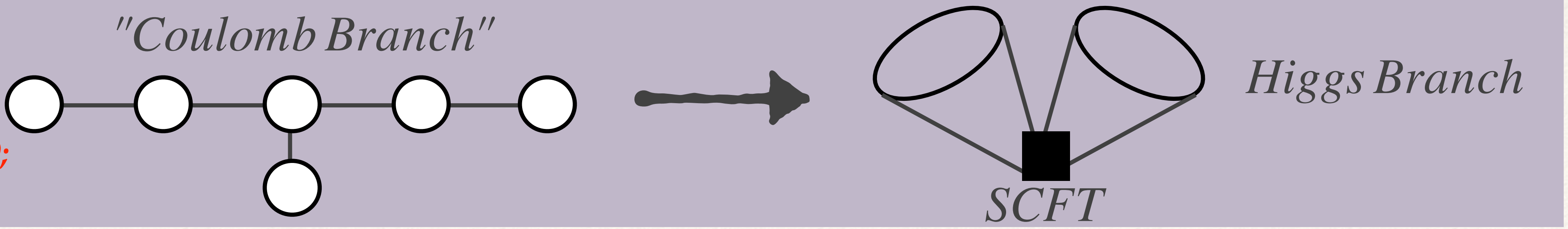
Classification of $\mathscr{D} = 4$ *SCFTs via the Coulomb branch geometries (Seiberg-Witten curves)*

Starting with* Seiberg, Witten 94; *Argyres, Martone 20

- Higgs branches more complicated: A prescription (motivated by brane constructions) to conjecture the structure of the Higgs branches: an auxiliary object called the ``*magnetic quiver*"

Cabrera, Hanany, Yagi 18; many more

Bourget, Cabrera, Grimminger, Hanany, Sperling, Zayac, Zhong 19; many more



Skeletons I: Moduli Spaces of Vacua

- * In $\mathscr{D} = 3$, as the $\mathscr{N} = 4$ R-symmetry is $SU(2)_H \times SU(2)_C$, the Higgs and the Coulomb branches are similar *(leading to the phenomenon of Mirror symmetry)*: two branches are distinct but no invariant way to say which is Coulomb and which is Higgs
- * In $\mathscr{D} = 4$ with $\mathscr{N} = 4$ the known theories have moduli spaces related to real crystallographic groups while for $\mathscr{N} = 3$ the moduli spaces are conjectured to be given by complex crystallographic groups Γ

$$\mathscr{M} = \mathbb{C}^{3\,rank}/\Gamma$$

Eg Argyres, Bourget, Martone 19; Tachikawa, Zafrir 19; Kaidi, Martone, Zafrir 22

- The two branches are distinct in general but there are hints of interesting interplay.

- * Intriguing connection between Coulomb branches of $\mathscr{N} = 2$ $\mathscr{D} = 4$ theories and their Higgs (Schur) branches through VOA algebra computations. *Cordova, Gaiotto, Shao 16*

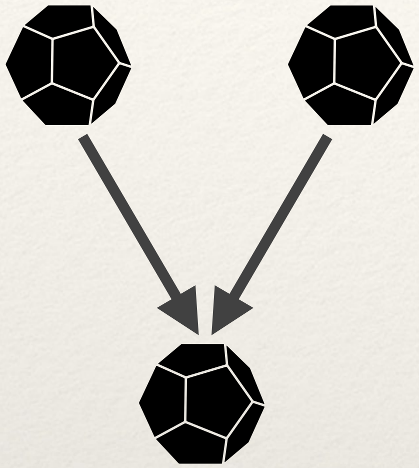
Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees 13

- * Intriguing connection between mirror symmetry of $\mathscr{N} = 4$ $\mathscr{D} = 3$ theories and phenomena of self-duality / emergence of symmetry of $\mathscr{N} = 1$ $\mathscr{D} = 4$ theories

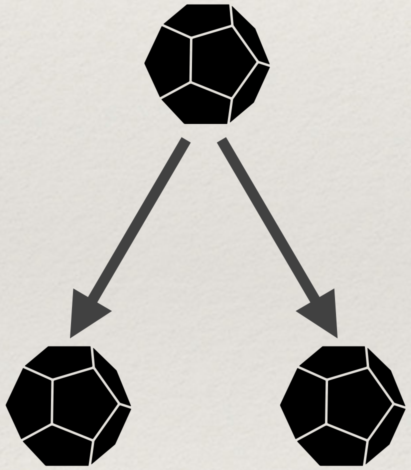
Hwang, Pasquetti, Sacchi 20/21; Pasquetti, SR, Sacchi, Zafrir 19 *See talk by Pasquetti*

- * There is no universal understanding of branches of vacua with less than eight supercharges

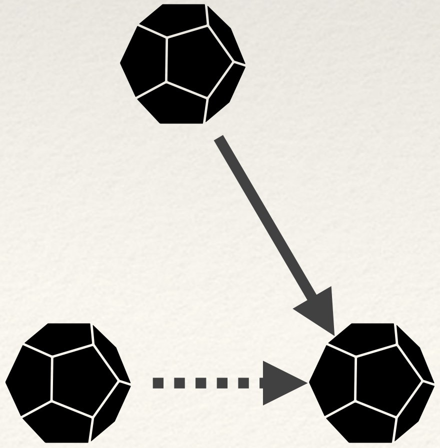
Relations II: in-$\mathscr{D}$ Dualities



- The same SCFT can be obtained by deforming different SCFTs: Duality (*two or more equivalent descriptions)*
- *This can be an IR duality if RG flow is involved, or conformal duality if no flow is involved*



- An SCFT can be deformed in different ways to obtain different SQFTs: UV Duality
- *For $\mathscr{D} = 5$ this becomes interesting as we can learn about the strongly coupled UV SCFT through the different IR SQFTs*

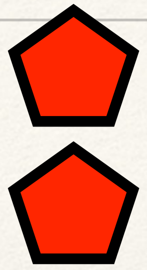


- One might be able to obtain a given SCFT by exploring moduli space of one SCFT and deforming another
- *An example is AD theories: moduli spaces of $\mathcal{N} = 2$ SCFTs and RG fixed point of deformed free fixed points*

Maruyoshi, Song 16;

Duality examples: Fixed $\mathscr{D}$

- $\mathscr{D} = 5$ $\mathscr{N} = 1$ UV dualities; Eg:



$SU(2) \times SU(2)$ *with hyper in* $(\mathbf{2}, \mathbf{2})$

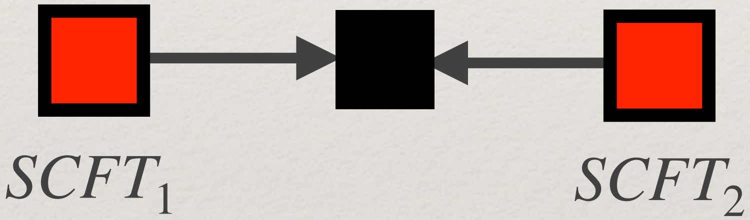
$SU(3)$ *with* $2 \times \mathbf{3}$ *hyper*

See eg Aharony, Hanany 98; Bergman, Rodriguez-Gomez, Zafrir 14; Bhardwaj 19; Apruzzi, Schafer-Nameki, Wang 19

- $\mathscr{D} = 4$ $\mathscr{N} = 1$ IR dualities; $\mathscr{D} = 4$ $\mathscr{N} = 2, 4$ conformal dualities

Seiberg 94

See talk by Pasquetti



- $\mathscr{D} = 4$ $\mathscr{N} = 1$: huge variety of examples with weakly coupled conformal manifolds

Leigh, Strassler 95 *Green, Komargodski, Seiberg, Tachikawa, Wecht 10*

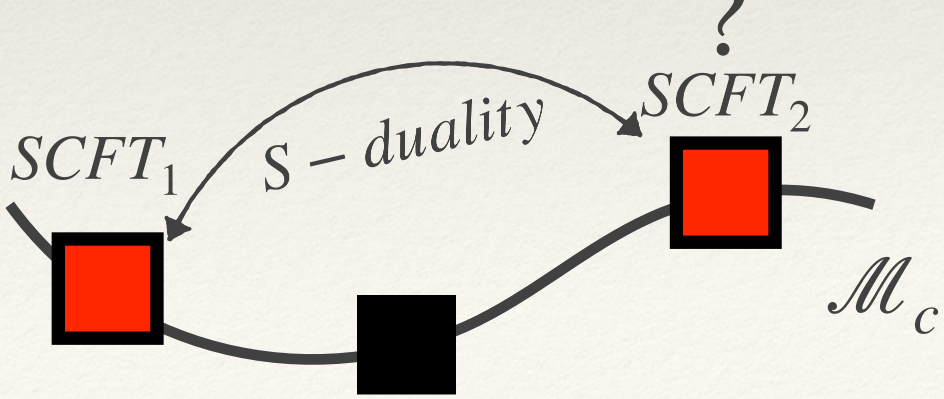
SR, Zafrir 18; SR, Sabag, Zafrir 20

Eg:

G_2 *SQCD with* $3 \times \mathbf{7} \oplus \mathbf{27}$

$SU(3) \times SU(2) \times SU(2)$ *quiver with*

$3 \times (\mathbf{2}, \mathbf{2}, \mathbf{1}) \oplus 2 \times (\mathbf{2}, \mathbf{1}, \mathbf{3}) \oplus 2 \times (\mathbf{1}, \mathbf{2}, \bar{\mathbf{3}}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{6}) \oplus (\mathbf{1}, \mathbf{1}, \bar{\mathbf{6}})$



❖ Q1: What is the S-duality structure of $\mathcal{N} = 1$ conformal manifolds ?

$G = SU(N)$

	Matter	$dim\,\mathcal{M}$	G_F^{free}	G_F^{gen}	$a,\ c$
1	$G = SU(4)$, $N_{\mathbf{20}} = 1, N_{\overline{S}} = 1$, $N_{AS} = 1$, $N_{\overline{F}} = 1, N_F = 2$	1	$U(1)^4 \times SU(2)$	$U(1)^2 \times SU(2)$	$a = \frac{61}{16}$, $c = \frac{31}{8}$
2	$G = SU(4)$, $N_{\mathbf{20'}} = 1, N_{Ad} = 1$	2	$U(1)$	$\emptyset$	$a = \frac{85}{24}$, $c = \frac{10}{3}$
3	$G = SU(4)$, $N_{\mathbf{20'}} = 1, N_{AS} = 4$	1	$U(1) \times SU(4)$	$SU(2)^2$	$a = \frac{179}{48}$, $c = \frac{89}{24}$
4	$G = SU(4)$, $N_{\mathbf{20'}} = 1, N_{AS} = 2$, $N_F = N_{\overline{F}} = 2$	2	$U(1)^3 \times SU(2)^3$	$SU(2)^2$ has a 1d subspace preserving $U(1) \times SU(2)^2$	$a = \frac{61}{16}$, $c = \frac{31}{8}$
5	$G = SU(6)$, $N_{\mathbf{20}} = 3, N_{Ad} = 1$, $N_{AS} = 1, N_{\overline{F}} = 2$	1	$U(1)^3 \times SU(2) \times SU(3)$	$SU(2)^2$	$a = \frac{437}{48}$, $c = \frac{227}{24}$
6	$G = SU(6)$, $N_{\mathbf{20}} = 2, N_{Ad} = 2$	3	$U(1) \times SU(2)^2$	$\emptyset$ has 1d subspace preserving $U(1)^2$	$a = \frac{425}{48}$, $c = \frac{215}{24}$
7	$G = SU(6)$, $N_{\mathbf{20}} = 2, N_{Ad} = 1$, $N_{AS} = 2, N_{\overline{F}} = 4$	3	$U(1)^3 \times SU(2)^2 \times SU(4)$	$U(1) \times SU(2)^2$ has a 1d subspace preserving $U(1)^2 \times USp(4)$	$a = \frac{37}{4}$, $c = \frac{39}{4}$
8	$G = SU(6)$, $N_{Ad} = 1, N_{\mathbf{20}} = 2$, $N_{AS} = 1$, $N_F = 3, N_{\overline{F}} = 5$	2	$U(1)^4 \times SU(2) \times SU(3) \times SU(5)$	$U(1)^2 \times SU(2)$ has 1d subspace preserving $U(1)^3 \times USp(4)$ also has 1d subspace preserving $U(1)^2 \times SU(2) \times SU(3)$	$a = \frac{151}{16}$, $c = \frac{81}{8}$

$G = E_N, F_4, G_2$

	Matter	$dim\,\mathcal{M}$	G_F^{free}	G_F^{gen}	$a,\ c$
1	$G = E_6$ $N_{\mathbf{27}} = N_{\overline{\mathbf{27}}} = 6$	41	$U(1) \times SU(6)^2$	$\emptyset$ has a 1d subspace preserving $SU(3)^2$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
2	$G = E_6, N_{\mathbf{27}} = 7$ $N_{\overline{\mathbf{27}}} = 5$	46	$U(1) \times SU(5) \times SU(7)$	$\emptyset$ has a 1d subspace preserving $SU(2) \times SU(3)$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
3	$G = E_6, N_{\mathbf{27}} = 8$ $N_{\overline{\mathbf{27}}} = 4$	61	$U(1) \times SU(4) \times SU(8)$	$\emptyset$ has a 1d subspace preserving $U(1)^2 \times SU(3)$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
4	$G = E_6, N_{\mathbf{27}} = 9$ $N_{\overline{\mathbf{27}}} = 3$	86	$U(1) \times SU(3) \times SU(9)$	$\emptyset$ has a 1d subspace preserving $U(1)^2 \times SU(3)^2$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
5	$G = E_6, N_{\mathbf{27}} = 10$ $N_{\overline{\mathbf{27}}} = 2$	121	$U(1) \times SU(2) \times SU(10)$	$\emptyset$ has a 1d subspace preserving $SU(3)^2$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
6	$G = E_6, N_{\mathbf{27}} = 11$ $N_{\overline{\mathbf{27}}} = 1$	166	$U(1) \times SU(11)$	$\emptyset$ has a 1d subspace preserving $SU(3)^2$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
7	$G = E_6, N_{\mathbf{27}} = 12$	221	$SU(12)$	$\emptyset$ has a 1d subspace preserving $U(1)^2 \times SU(3)^2$	$a = \frac{171}{8}$, $c = \frac{93}{4}$
8	$G = F_4, N_{\mathbf{26}} = 9$	85	$SU(9)$	$\emptyset$ has a 1d subspace preserving $SU(3)^2$	$a = \frac{117}{8}$, $c = \frac{65}{4}$
9	$G = G_2, N_{\mathbf{7}} = 12$	77	$SU(12)$	$\emptyset$ has a 1d subspace preserving $SU(3)^4$	$a = \frac{35}{8}$, $c = \frac{21}{4}$
10	$G = G_2, N_{\mathbf{27}} = 1$ $N_{\mathbf{7}} = 3$	3	$U(1) \times SU(3)$	$SU(2)$ has a 2d subspace preserving $SU(3)$	$a = \frac{29}{8}$, $c = \frac{15}{4}$

SR, Sabag, Zafrir 20

$G = SO(N)$

	Matter	$dim\,\mathcal{M}$	G_F^{free}	G_F^{gen}	$a,\ c$
1	$G = SO(7)$ $N_{\mathbf{8}} = 10,\ N_V = 5$	102	$U(1) \times SU(5) \times SU(10)$	$U(1)$ has a 1d subspace preserving $U(1)^5 \times SU(2)^5$	$a = \frac{19}{3}$, $c = \frac{89}{12}$
2	$G = SO(7),\ N_S = 1$ $N_{\mathbf{8}} = 2,\ N_V = 4$	1	$U(1)^2 \times SU(2) \times SU(4)$	$U(1) \times SU(2)^2$	$a = \frac{65}{12}$, $c = \frac{67}{12}$
3	$G = SO(7),\ N_S = 1$ $N_{\mathbf{8}} = 3,\ N_V = 3$	1	$U(1)^2 \times SU(3)^2$	$SU(2)$	$a = \frac{87}{16}$, $c = \frac{45}{8}$
4	$G = SO(7),\ N_S = 1$ $N_{\mathbf{8}} = 4,\ N_V = 2$	1	$U(1)^2 \times SU(2) \times SU(4)$	$U(1)^2 \times SU(2)^2$	$a = \frac{131}{24}$, $c = \frac{17}{3}$
5	$G = SO(7),\ N_{AS} = 1$ $N_{\mathbf{35}} = 1$	1	$U(1)$	$\emptyset$	$a = \frac{245}{48}$, $c = \frac{119}{24}$
6	$G = SO(7),\ N_{\mathbf{35}} = 1$ $N_{\mathbf{8}} = 4,\ N_V = 1$	2	$U(1)^2 \times SU(4)$	$U(1)^2$ has a 1d subspace preserving $U(1)^2 \times SU(2)$	$a = \frac{263}{48}$, $c = \frac{137}{24}$
7	$G = SO(7),\ N_{\mathbf{35}} = 1$ $N_{\mathbf{8}} = 5$	1	$U(1) \times SU(5)$	$USp(4)$	$a = \frac{11}{2}$, $c = \frac{23}{4}$
8	$G = SO(8),\ N_{\mathbf{8}_S} = 6$ $N_{\mathbf{8}_C} = 6,\ N_V = 6$	111	$U(1)^2 \times SU(6)^3$	$U(1)^2$ has a 1d subspace preserving $U(1)^4 \times SU(3)^2$	$a = \frac{33}{4}$, $c = \frac{19}{2}$
9	$G = SO(8)$, $N_S = 1,\ N_{AS} = 1$, $N_{\mathbf{8}_S} = 2$	1	$U(1)^2 \times SU(2)$	$SU(2)$	$a = \frac{331}{48}$, $c = \frac{163}{24}$
10	$G = SO(8),\ N_S = 1$, $N_{\mathbf{8}_S} = 2,\ N_{\mathbf{8}_C} = 2$, $N_V = 4$	4	$U(1)^3 \times SU(2)^2 \times SU(4)$	$U(1)$ has a 1d subspace preserving $U(1) \times SU(2)^2$	$a = \frac{117}{16}$, $c = \frac{61}{8}$
11	$G = SO(8),\ N_S = 1$, $N_{\mathbf{8}_S} = 1,\ N_{\mathbf{8}_C} = 1$, $N_V = 6$	1	$U(1)^3 \times SU(6)$	$U(1)^2 \times USp(4)$	$a = \frac{117}{16}$, $c = \frac{61}{8}$

$G = USp(2N)$

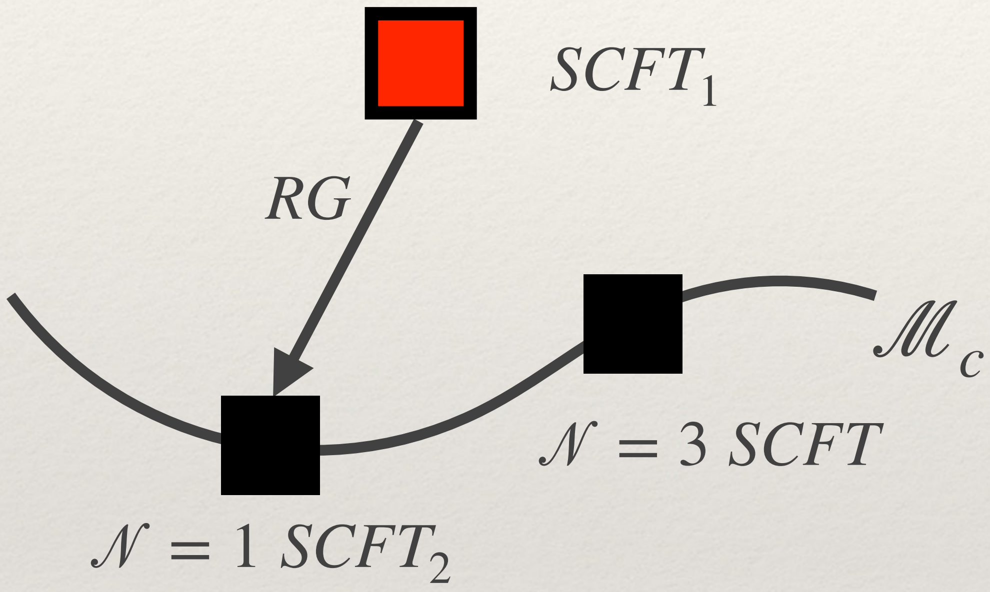
	Matter	$dim\,\mathcal{M}$	G_F^{free}	G_F^{gen}	$a,\ c$
1	$N_S = 1, N_{AS} = 1$, $N_F = 2N + 6$	$N + 4$ (5 for $N = 2$)	$U(1)^2 \times SU(2N + 6)$	$U(1)^{N+3}$ for $N > 2$ has a 1d subspace preserving $SO(2N + 6 - 2x) \times USp(2x)$ for $x < N - 1$ for $N \geq 2$ has a 1d subspace preserving $U(1) \times USp(2N - 2) \times SO(8)$	$a = \frac{26N^2+21N-1}{48}$, $c = \frac{14N^2+15N-1}{24}$
2	$N_S = 2, N_{AS} = 3$, $N = 2$	1	$U(1) \times SU(2) \times SU(3)$	$U(1) \times SU(2)$	$a = \frac{125}{48}$, $c = \frac{65}{24}$
3	$N_S = 2, N_{AS} = 2$, $N = 3$	1	$U(1) \times SU(2)^2$	$U(1)^2$	$a = \frac{259}{48}$, $c = \frac{133}{24}$
4	$N_S = 1, N_{AS} = 5$, $N_F = 2, N = 2$	1	$U(1)^2 \times SU(2) \times SU(5)$	$U(1) \times SU(2) \times USp(4)$	$a = \frac{133}{48}$, $c = \frac{73}{24}$
5	$N_S = 1, N_{AS} = 3$, $N = 5$	7	$U(1) \times SU(3)$	$\emptyset$ has a 1d subspace preserving $SU(2)$	$a = \frac{341}{24}$, $c = \frac{44}{3}$
6	$N_S = 1, N_{AS} = 3$, $N_F = 2, N = 4$	10	$U(1)^2 \times SU(2) \times SU(3)$	$U(1)$ has a 1d subspace preserving $SU(2)^2$	$a = \frac{457}{48}$, $c = \frac{241}{24}$
7	$N_S = 1, N_{AS} = 3$, $N_F = 4, N = 3$	19	$U(1)^2 \times SU(3) \times SU(4)$	$\emptyset$ has a 1d subspace preserving $U(1) \times SU(2) \times USp(4)$	$a = \frac{23}{4}$, $c = \frac{25}{4}$
8	$N_S = 1, N_{AS} = 3$, $N_F = 6, N = 2$	27	$U(1)^2 \times SU(3) \times SU(6)$	$\emptyset$ has a 1d subspace preserving $U(1) \times SU(2)^4$	$a = \frac{139}{48}$, $c = \frac{79}{24}$

Duality example: Strongly coupled SCFT from Weakly coupled SQFT

Garcia Etxebarria, Regalado 15

- $\mathscr{D} = 4$ $\mathscr{N} = 1$ IR dual to an $\mathscr{N} = 3$ SCFT

Zafrir 20



$G_{gauge} = SU(2) \times SU(2)$ *with matter in*

$(\mathbf{2},\mathbf{2}) \oplus (\mathbf{2},\mathbf{3}) \oplus (\mathbf{2},\mathbf{1}) \oplus (\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1})$

and W

$$\mathscr{M} = \mathbb{C}^3/\mathbb{Z}_3 \qquad a = c = \frac{5}{4}$$

Nishinaka, Tachikawa 16

- *Note that* $\mathscr{N} = 3$ *supersymmetry (conjecturally) emerges on some locus of* $\mathscr{M}_c$

- Surprisingly many strongly coupled $\mathscr{D} = 4$ $\mathscr{N} \geq 1$ zoom SCFTs can be described by $\mathscr{N} = 1$ Lagrangians; One can search for Lagrangians systematically starting from Skeletons using a variety of assumptions

See eg Maruyoshi, Song 16; SR, Zafrir 19, 20; Zafrir 19
Garcia Etxebarria, Heidenreich, Lotito, Sorout 21

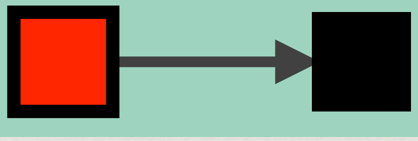
An "existential" question: Fixed $\mathscr{D}$

- *In the pre-history (the 90s) one discovered some exotic models by exploring moduli spaces (eg AD) or conformal manifolds (eg AS). Some properties of these models were known (the skeletons).*

  Argyres, Douglas 95; Argyres, Seiberg 07

- **Non-Lagrangian theories:** theories for which a Lagrangian construction is ***currently*** not known.

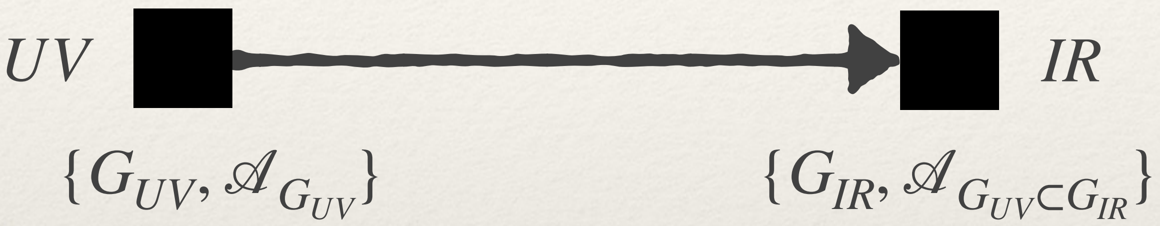
- Q2: For $\mathscr{D} \leq 4$ can all SCFTs be constructed deforming a free fixed point?



- Q2a: Given an SCFT in $\mathscr{D} \leq 4$ (the skeleton): are there obstructions to building a Lagrangian (supersymmetric or not supersymmetric)?

Obstructions for Lagrangians: fixed $\mathscr{D}$

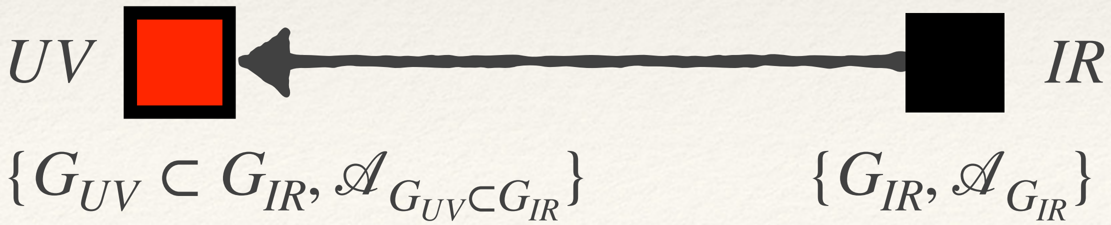
- \`t Hooft anomaly matching: *Anomalies are invariants of RG flow and thus if we compute them in UV constrain the physics in IR.*



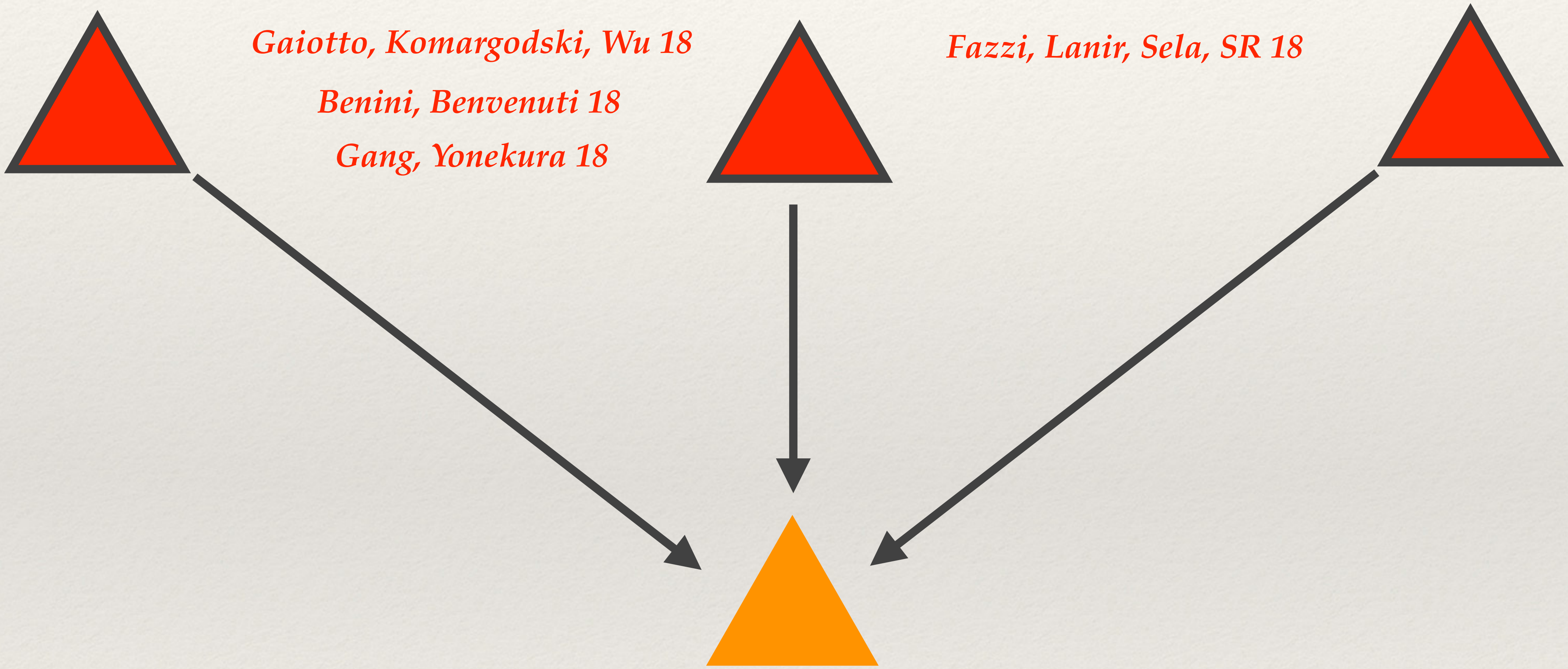
- *Generalizations with higher symmetries and anomalies*

See eg Gaiotto, Komargodski, Kapustin, Seiberg 17; Komargodski, Ohmori, Roumpedakis, Seifnashri 20; Cordova, Ohmori 19; Brennan, Cordova 20; Del Zotto, Ohmori 20

- Q2b: Given the full (generalized) symmetries and anomalies of an SCFT which sub-structure of this can in principle be realized by free fields with gauge and potential interactions?



$\mathscr{D} = 3$ Example



$\mathcal{N} = 1 \ \ WZ$

$\mathbf{G}_{\mathbf{UV}} = [\mathcal{N} = \mathbf{1}] \times \mathbf{T} \times [\mathbf{SU(3)}]$

Gaiotto, Komargodski, Wu 18

Benini, Benvenuti 18

Gang, Yonekura 18

$\mathcal{N} = 2 \ U(1) \ N_f = 2$

$\mathbf{G}_{\mathbf{UV}} = [\mathcal{N} = \mathbf{2}] \times \mathbf{T} \times [\mathbf{SU(2)} \times \mathbf{U(1)}]$

Fazzi, Lanir, Sela, SR 18

$\mathcal{N} = 2 \ SU(3) \ N_f = 3 \ CS$

$\mathbf{G}_{\mathbf{UV}} = [\mathcal{N} = \mathbf{2}] \times \cancel{\mathbf{T}} \times [\mathbf{SU(3)}]$

$\mathbf{G}_{\mathbf{IR}} = [\mathcal{N} = \mathbf{2}] \times \mathbf{T} \times [\mathbf{SU(3)}]$

- No Lagrangian manifesting the full symmetry known:
- Is there a fundamental obstruction or we need to work harder?

$\mathscr{D} = 4$ Example

- ⬛ *Eg $\mathcal{N} = 2$ MN E_6 SCFT has by now several different descriptions starting from free fixed point; Eg:*

- 🟥 $\mathcal{N} = 1$ $Spin(6)$ SQCD, 2 vectors, 5 spinors of both chiralities, and singlets: $G_{UV} = SU(2) \times SU(5) \times U(1) \subset SU(2) \times SU(6) \subset E_6 \times \cancel{U(1)}$

Zafrir 19

- 🟥 $\mathcal{N} = 1$ $SU(3) \times SU(2)$ SQCD with bi-fudundamental and fundamental matter, and singlets: $G_{UV} = U(1)^2 \times SU(5) \times U(1) \subset U(1) \times SO(10) \times U(1) \subset E_6 \times U(1)$

Garcia Etxebarria, Heidenreich, Lotito, Sorout 21

G_{UV} 🟥 ⟶ ⬛ $E_6 \times U(1)$

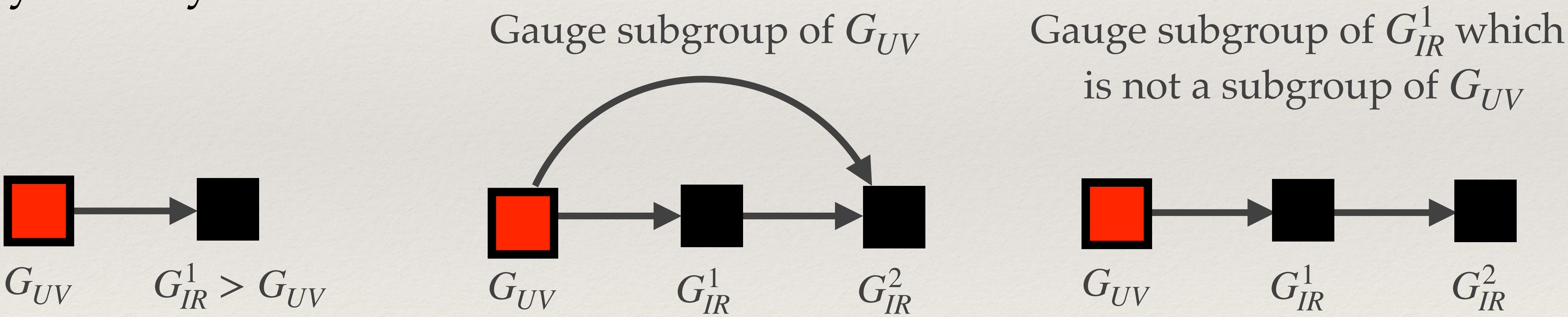
(See also Gadde, SR, Willett 15)

- The Lagrangians manifest part of the (super)symmetry, and the rest emerges in the IR

Gauging emergent symmetries: fixed $\mathscr{D}$

- Symmetries of the new fixed point can be larger than the symmetry of the original SCFT with the deformation

- If we have an SCFT constructed deforming free fixed point we can gauge a sub-group of the global symmetry including emergent symmetry

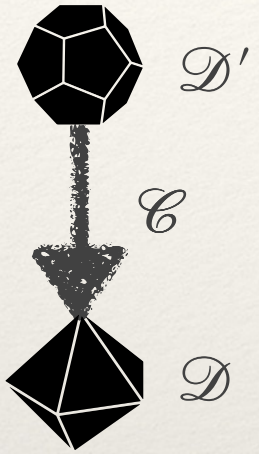


- Q2c: What is the subspace of SCFTs in $\mathscr{D} \leq 4$ that can be obtained by deforming free fixed points? Is it equal to the subspace when we allow gauging emergent symmetries?

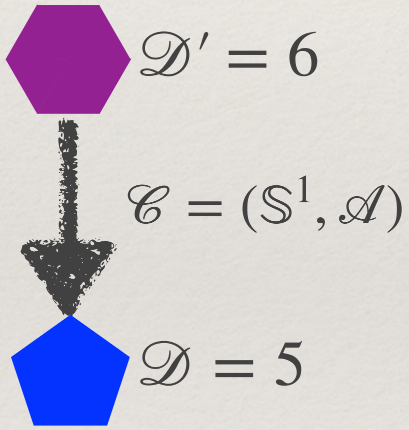
Plan:

- SQFTs from String Theory
- SQFTs from SQFTs
  - SQFTs from SQFTs in given $\mathscr{D}$
    - *Moduli spaces*
    - *In-$\mathscr{D}$ dualities*
    - *Lagrangians*
  - SQFTs from SQFTs across $\mathscr{D}$ ↓
    - *Across-$\mathscr{D}$ dualities*
    - $\mathscr{D} = 5$ *from* $\mathscr{D} = 6$
    - $\mathscr{D} = 4$ *from* $\mathscr{D} = 6$
    - *In-$\mathscr{D}$ dualities from Across-$\mathscr{D}$ dualities*
    - $\mathscr{D} = 6$ *Dualities*
  - SQFTs from SQFTs across $\mathscr{D}$ ↑
    - $\mathscr{D} = 6$ *SCFTs from* $\mathscr{D} < 6$ *SCFTs*

Relations III: SCFTs from SCFTs going across $\mathscr{D}$s

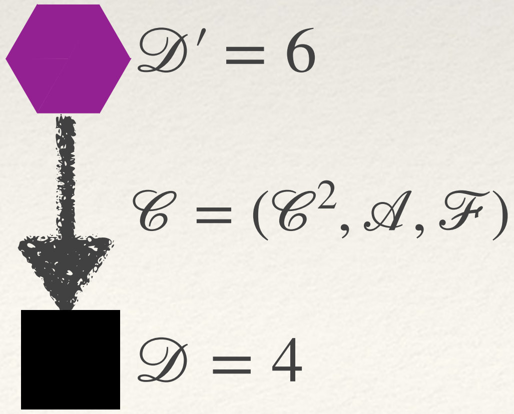


- For $\mathscr{D} \leq 5$ start from an SCFT in $\mathscr{D}' > \mathscr{D}$ and place on a $\mathscr{D}' - \mathscr{D}$ dimensional compact surface with background fields; At low energy obtain an effective theory in $\mathscr{D}$ dimensions
- *The resulting theory might be gapped, free, interacting SQFT*



- The resulting theory might be a (deformation of) an SCFT in $\mathscr{D}$ or UV completed only in $\mathscr{D}'$

A claim to fame: *Gaiotto 09*



- For $\mathscr{D} \leq 4$ this is a way to construct numerous lower dimensional SCFTs labeled by the compactification geometry (*$\mathscr{A}$ denotes holonomies around the cycles and $\mathscr{F}$ fluxes supported on the surface for global symmetries*)

Dualities across dimensions: $\mathscr{D} = 5$ and $\mathscr{D} = 6$

- In some cases starting with a theory in $\mathscr{D} = 6$ and placing it on a circle with holonomies the effective theory is a $\mathscr{D} = 5$ SCFT

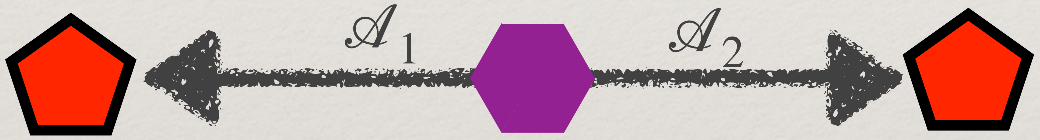
- In some cases starting with a theory in $\mathscr{D} = 6$ and placing it on a circle with holonomies the effective theory in $\mathscr{D} = 5$ is a gauge theory

- *We can view this situation as a duality across dimensions. This is analogous to Seiberg ``duality" of UV free and IR free theories (ie outside of the conformal window)*

- Q3: Which $\mathscr{D} = 5$ gauge theories are *across dimensions dual* to compactifications of $\mathscr{D} = 6$ SCFTs?

Dualities across dimensions: $\mathscr{D} = 5$ and $\mathscr{D} = 6$

- Q3a: Which $\mathscr{D} = 5$ gauge theories are UV completed by $\mathscr{D} = 6$ SCFTs and which are deformations of $\mathscr{D} = 5$ SCFTs?



Eg: ⬢ $=(D_{N+3}, D_{N+3})$ min. conf. matter

⬟ $G_{gauge}^{\mathscr{D}=5} = SU(2)^N \ or \ USp(2N) \ or \ SU(N+1)$

- Q3b: Can all $\mathscr{D} = 5$ SCFTs be obtained by circle compactifications of $\mathscr{D} = 6$ SCFTs?

See eg Jefferson, Kim, Vafa, Zafrir 17; Bhardwaj, Jefferson, Kim, Vafa 19;
Apruzzi, Lawrie, Lin, Schafer-Nameki, Wang 19; Apruzzi, Schafer-Nameki, Wang 19;
Hayashi, Kim, Lee, Taki, Yagi 15; Jefferson, Katz, Kim, Vafa 18; Apruzzi, Lawrie, Lin, Schafer-Nameki, Wang 19
Bhardwaj 19; Bhardwaj, Jefferson, Kim, Tarazi, Vafa 19; Bhardwaj, Zafrir 20

Kim Strings 17

Dualities across dimensions: $\mathscr{D} = 4$ and $\mathscr{D} = 6$

- In some cases starting with a theory in $\mathscr{D} = 6$, placing it on a surface with fluxes, the effective theory in $\mathscr{D} = 4$ is an interacting SCFT

- Unlike in $\mathscr{D} = 5$, in $\mathscr{D} = 4$ we can in principle explicitly construct interacting SCFTs starting from free fixed points

- Q4: Does a given geometric construction of $\mathscr{D} = 4$ interacting SCFT have an explicit construction *(dual across dimensions)* as a deformation of a free fixed point directly in $\mathscr{D} = 4$?

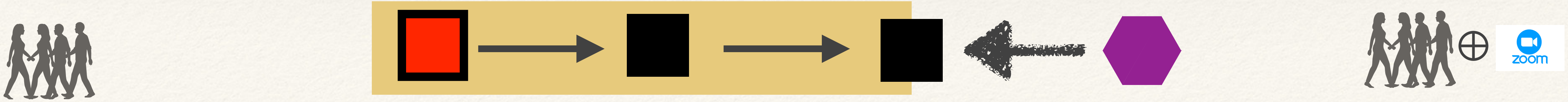
- *If such a description exists the situation is similar to Seiberg duality inside the conformal window: we have two different UV complete descriptions of the same fixed point.*

Dualities across dimensions: $\mathscr{D}=4$ and (2,0) $\mathscr{D}=6$

- A canonical example of across dimensions duality here is taking A_1 (2,0) theory on a genus g Riemann surface
- *This construction has a dual* $\mathscr{D}=4$ *description in terms of an* $SU(2)^{2g-2}$ *SQCD with tri-fundamental matter content* *Gaiotto 09*

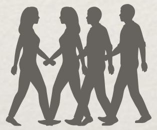
- Another example following recent progress is taking A_2 (2,0) theory on a genus g Riemann surface

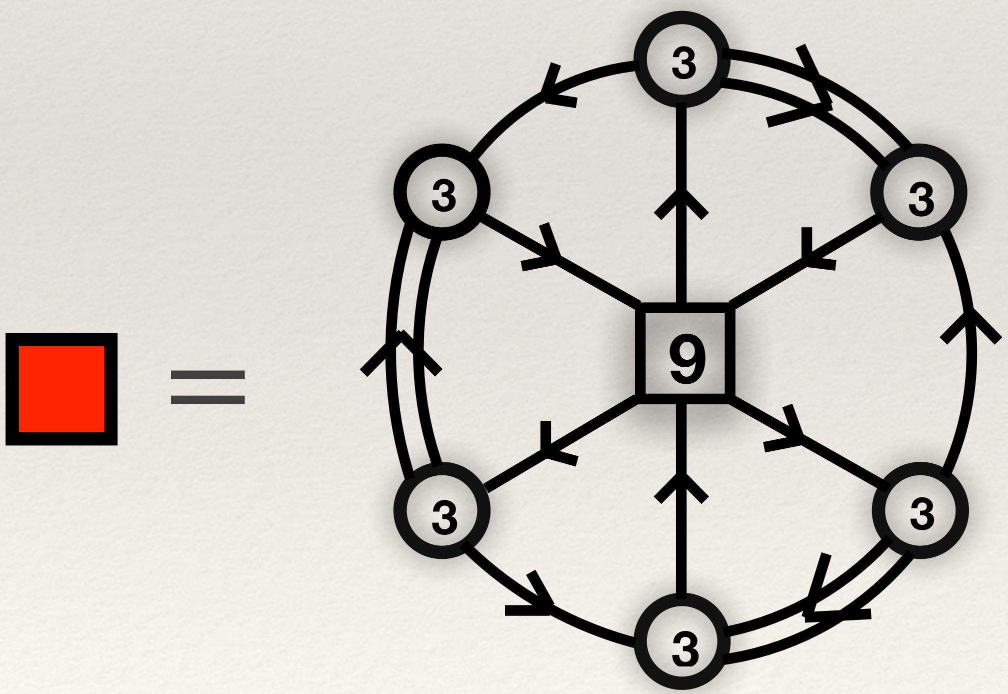
  Gadde, SR, Willett 15; Zafrir 19; Garcia Etxebarria, Heidenreich, Lotito, Sorout 21
- *This construction has a dual* $\mathscr{D}=4$ *description in terms of eg an* $Spin(6)^{2g-2}\times SU(3)^{3g-3}$ *gauge theory where one gauges* $SU(3)^3$ *subgroup of an emergent factors of* E_6 *symmetry*

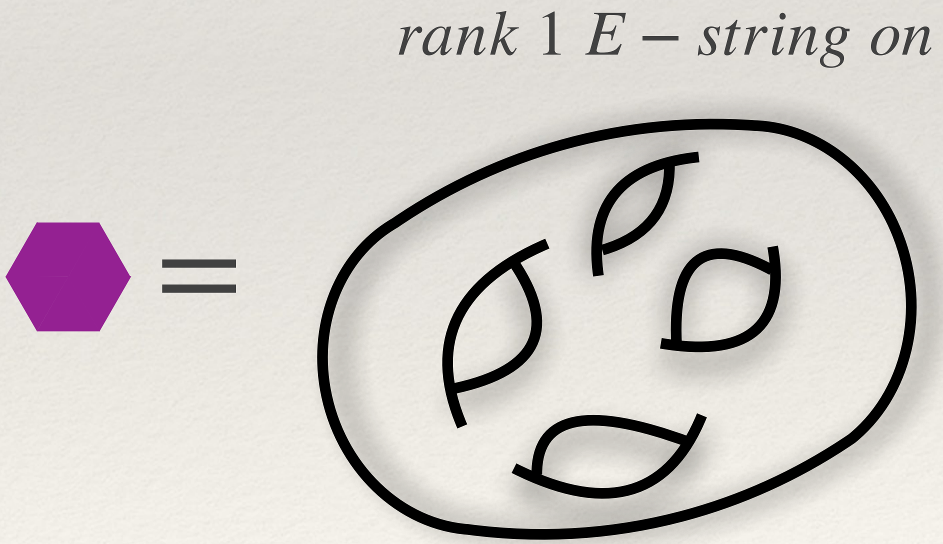


Dualities across dimensions: $\mathscr{D}=4$ and (1,0) $\mathscr{D}=6$

- Yet another example of across dimensions duality is taking a (1,0) SCFT, the rank one E-string theory, on a genus g Riemann surface
- *This construction has a dual $\mathscr{D}=4$ description in terms of an $SU(3)^{2g-2}$ SQCD with bi-fundamental and fundamental matter content* SR, Zafrir 19; SR, Sabag 20





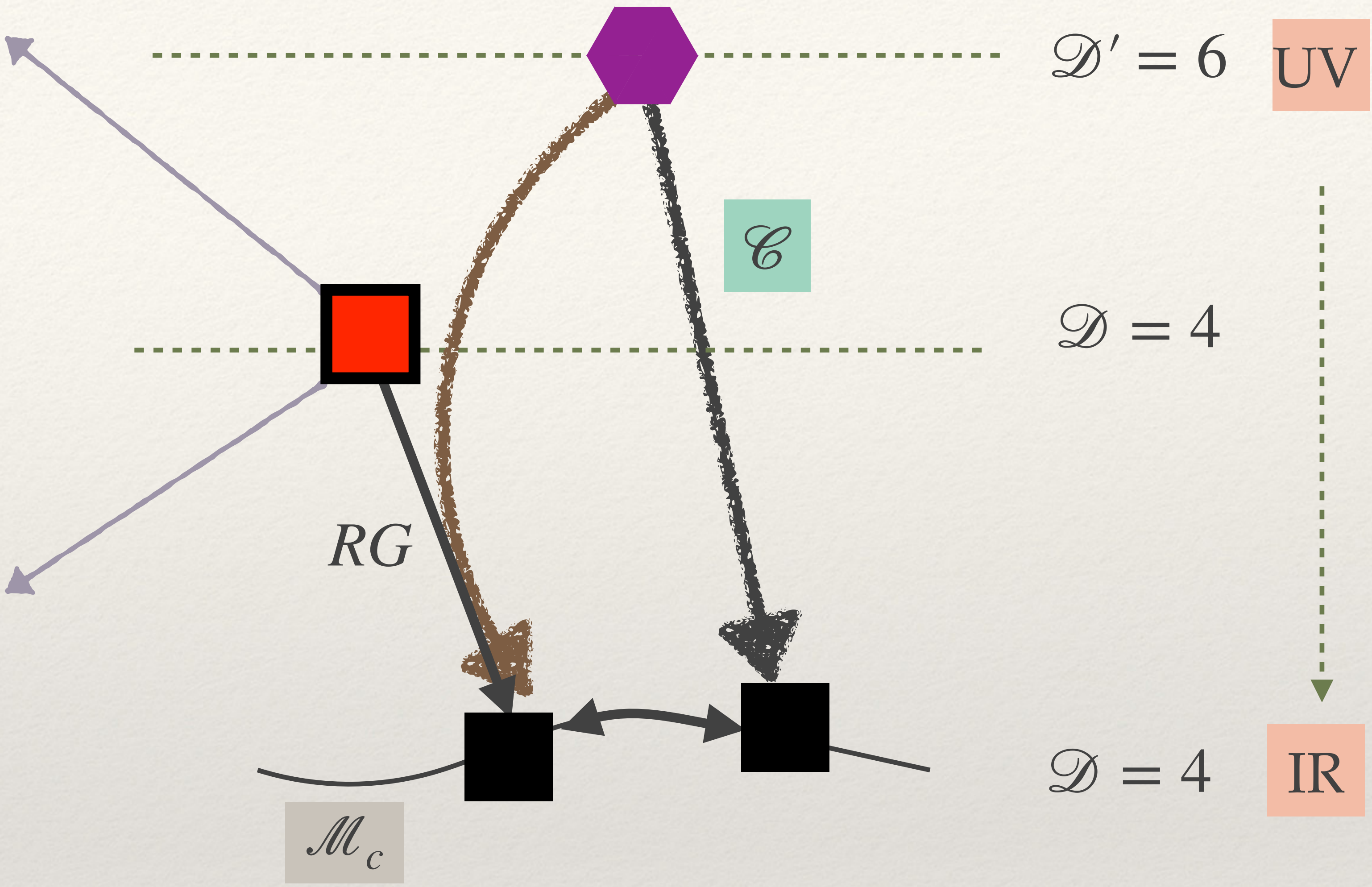


$G_{QFT} = SU(9) \;\rightarrow\; E_8\,(=G_{\mathscr{D}=6})$ on $\mathscr{M}_c$

$\dim \mathscr{M}_c = 3g-3+(g-1)\,248 = 3g-3+(g-1)\dim E_8$

- Is UV completed in $\mathscr{D} = 6$

- Might also have a UV completion in $\mathscr{D} = 4$



- 🟥 *oftentimes contains irrelevant superpotential interactions; Eg*

$$W = \underbrace{\phi}_{\text{Free field}} \; \underbrace{\mathcal{O}}_{eg\ \mathcal{O} = Q^N} + \cdots$$

Dualities across dimensions: status of $\mathscr{D} = 4$ and $\mathscr{D} = 6$

- Over time more and more across dimension dualities are discovered.

See eg Distler et al

Some (2,0) compactifications: Eg $AD, T_3, T_4, R_{0,4}, R_{2,5}, MN^{(2n)}_{E_6}$ *See eg SR, Zafrir 19; Maruyoshi, Song 16; Benvenuti, Giacomelli 18*

(1,0)

ADE conf. matter on a torus *Kim, Vafa, SR, Zafrir 17 and 18; Bah,Hanany, Maruyoshi, SR, Tachikawa, Zafrir 16*

(D_{N+3}, D_{N+3}) minimal conf. matter on any surface

(A_{k-1}, A_{k-1}) next to minimal conf. matter on any surface

SR, Sabag 19 and 20

$SU(3)$ and $SO(8)$ minimal SCFTs on any surface *SR, Zafrir 18*

Rank Q E-string on a torus/sphere *Pasquetti, SR, Sacchi, Zafrir 19; Hwang, SR, Sabag, Sacchi 21*

and more … *See eg Gaiotto, SR 15; Zafrir 18; SR, Sela, Zafrir 18; Sela, Zafrir 19*

- Q2d: Do all $\mathscr{D} = 4$ SCFTs constructed across $\mathscr{D}$ have an explicit construction as a deformation of a free fixed point directly in $\mathscr{D} = 4$?

Skeletons II: Generalized symmetries

- *The way one argues for all these across dimension dualities is by computing the skeleton in both constructions (symmetries, anomalies, BPS spectra, etc) and matching them; all these dualities are conjectures; consistency*

- Vigorous progress in understanding generalized notions of symmetry in a general QFT: higher form, higher group, non-invertible, discrete symmetries and anomalies, gauging

See eg Gaiotto, Kapustin, Seiberg, Willett 14; Tachikawa 17; Benini, Cordova, Hsin 18; Cordova, Dumitrescu, Intriligator 18; Gaiotto, Johnson-Freyd 19; many others

See talk by Ohmori
Shao Strings 21

- Can deduce the global symmetries and anomalies from the geometric and string theoretic constructions:

  - (Higher form) symmetries from String Theory

Bah, Bonetti, Minasian, Nardoni 18-19
Morrison, Schafer-Nameki, Willett 20;
Albertini, Del Zotto, Garcia Etxebarria, Hosseini 20;
Bhardwaj, Schafer-Nameki 20; Gukov, Hsin, Pei 20; and many more
Bergman, Tachikawa, Zafrir 20

  - Higher group symmetries from String Theory

See eg Cordova, Dumitrescu, Intriligator 20;
Apruzzi, Bhardwaj, Gould, Schafer-Nameki 22;
Del Zotto, Garcia Etxebarria, Schafer-Nameki 22;
Bhardwaj 21; and many more

See talk by Garcia Etxebarria

Higher $\mathscr{D}$ SQFTs: Holography

- Some of the constructions of lower $\mathscr{D}$ theories from higher $\mathscr{D}$ might come in families of growing central charges and satisfying all the demands to admit a Holographic dual. The Holographic duals, as usual, can be used to extract some information about lower dimension SCFTs
- Classification of AdS solutions

Eg Gauntlett, Martelli, Sparks, Waldram 04; Gaiotto, Maldacena 10; Ferrero, Gauntlett, Perez Ipina, Martelli, Sparks 20-21; Apruzzi, Fazzi, Passias, Tomasiello 14; Gaiotto, Tomasiello 14; Bergman, Rodriguez Gomez 12; D'Hoker, Gutperle, Uhlemann 16-17

- Eg: Generalized symmetries and anomalies from Holography

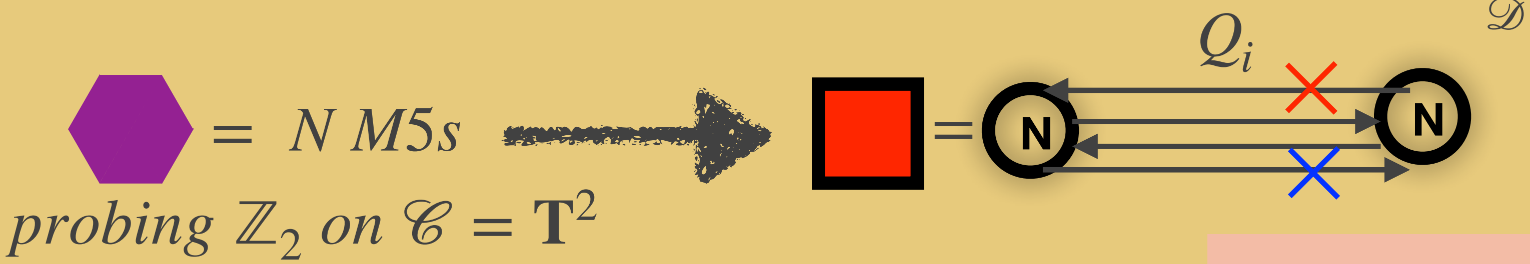
Eg Bah, Bonetti, Minasian, Nardoni 19; Bergman, Fazzi, Rodriguez-Gomez, Tomasiello 20; Apruzzi, van Beest,Gould, Schafer-Nameki 21; Bah, Bonetti, Minasian 20;

- Eg: Holographic duals of AD theories; Holographic RG flow; Insights into punctures

Bah, Bonetti, Minasian, Nardoni 21 *Anderson, Beem, Bobev, Rastelli 11* *Bah 15; Bah, Passias, Tomasiello 17; Gaiotto, Maldacena 09*

- Free fields from Geometry / Holography

Bah, Bonetti, Leung, Weck 21



$$W = Q_1Q_2Q_3Q_4 + \phi_1 Q_1^N + \phi_2 Q_2^N$$

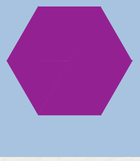
$\mathscr{D} = 4$ across dimensions dual to $\mathscr{D} = 6$ SCFT compactifications often consist of an SCFT and decoupled free fields

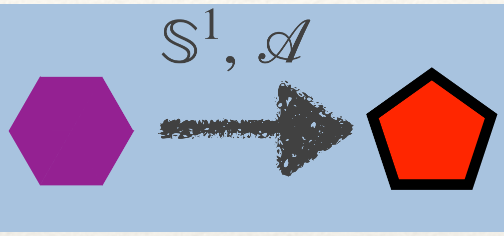
Existence of these fields is inferred first by matching `t Hooft anomalies between $\mathscr{D} = 4$ and $\mathscr{D} = 6$

Can be understood from Holography / Geometry without understanding in detail the $\mathscr{D} = 4$ SCFT

Systematics of Dualities Across Dimensions?

- *An approach integrating together various developments:*

Step 0: Choose 

Step 1: 

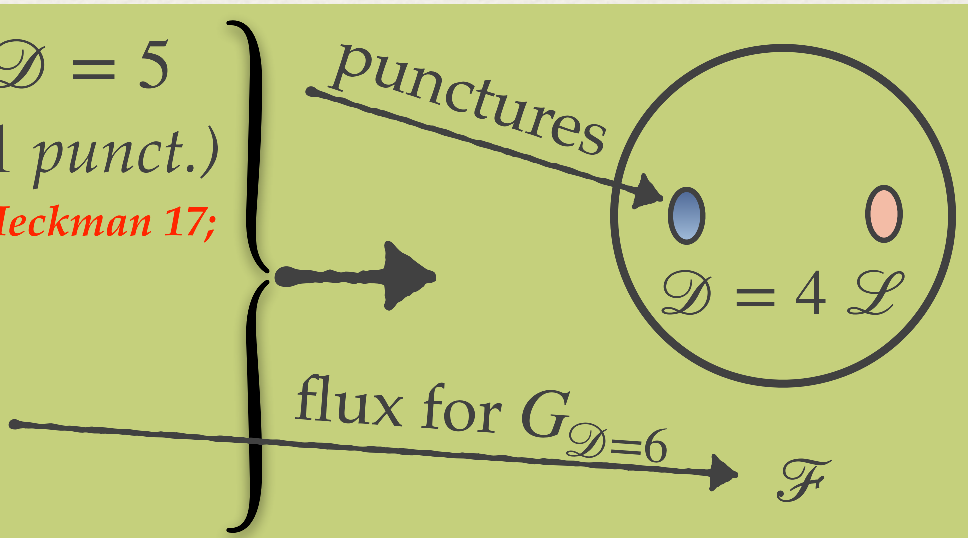
Step 2: Understand boundary conditions in $\mathscr{D} = 5$

(Even for (2,0) *a rich problem for* $\mathscr{N} = 1$ *punct.)*

See eg Heckman, Jefferson, Rudelius, Vafa 16; Hassler, Heckman 17; Wang, Xie 18

Step 3: Understand domain walls in $\mathscr{D} = 5$

See eg Gaiotto, Kim 15; Chan, Ganor, Krogh 00



(Defects, boundaries, and domain walls in more generality)

See eg Gaiotto, Witten 08; Dimofte, Gaiotto, Paquette 17; Wang 21

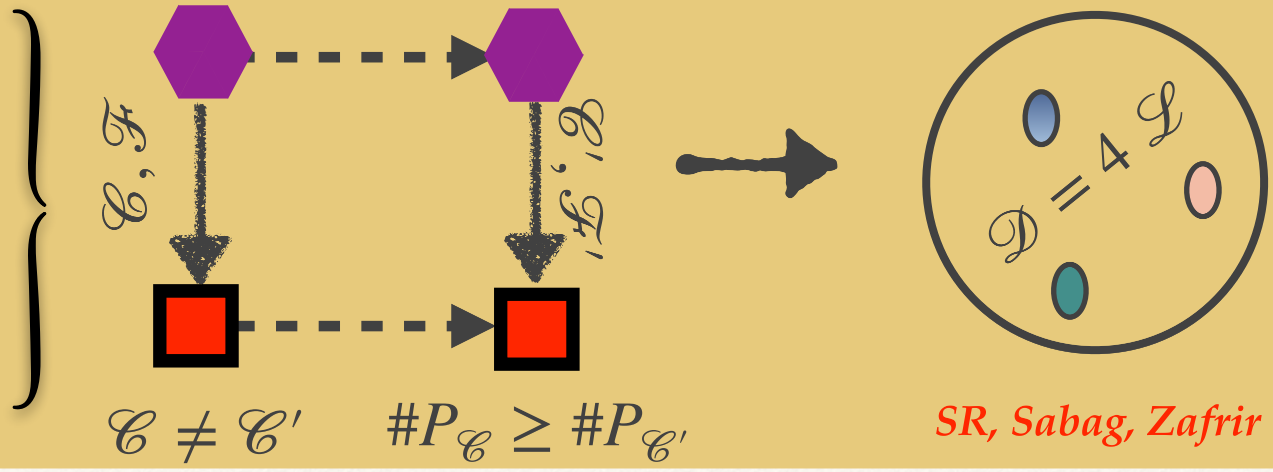
See talks by Komargodski, Sever

Kim, SR, Vafa, Zafrir 17

Step 4: Understand $\mathscr{D} = 6$ VEV flows

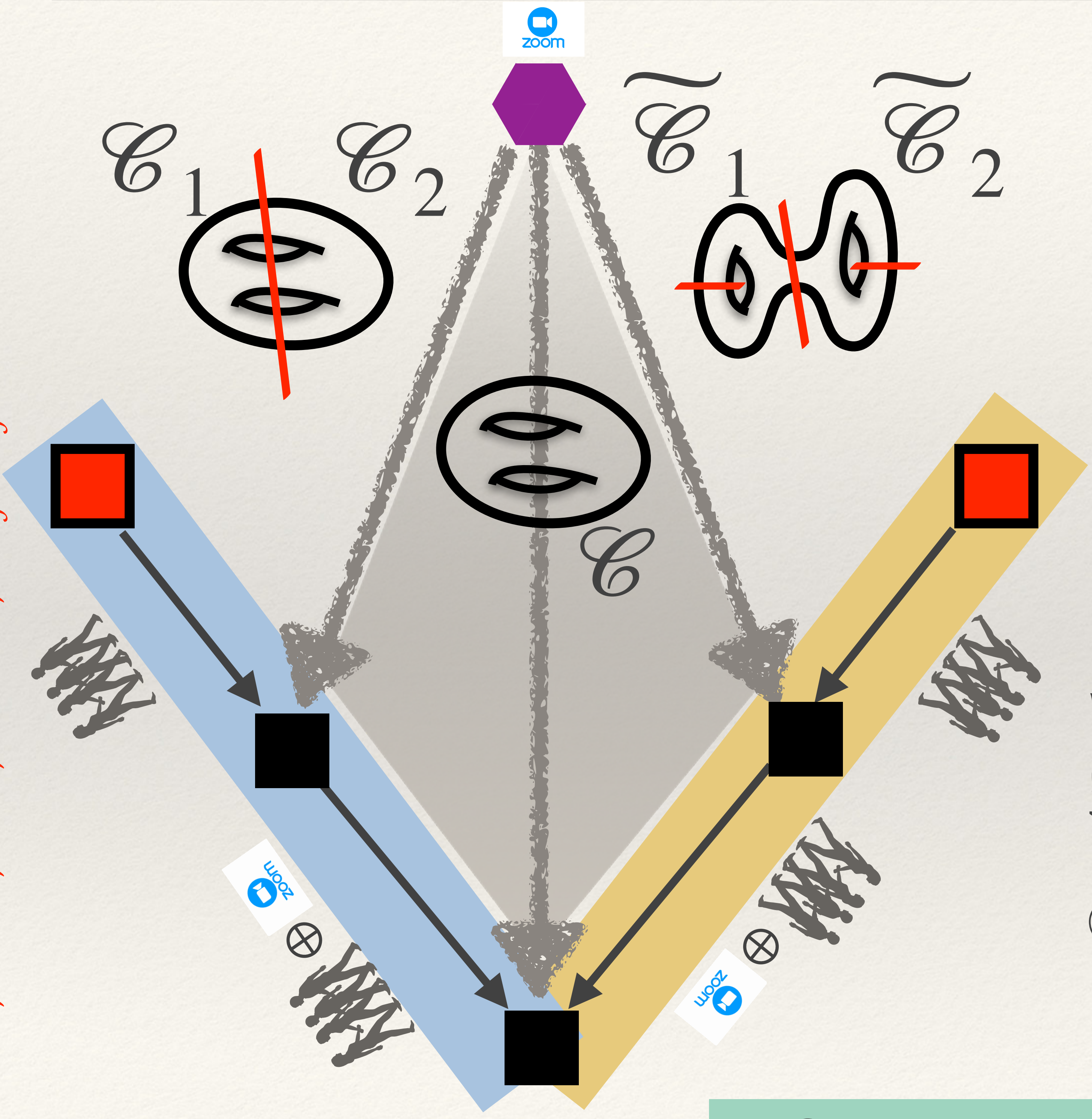
See eg Heckman, Rudelius, Tomasiello 16; Heckman, Kundu, Zhang 21; Cordova, Dumitrescu, Intriligator 15

Step 5: Understand interplay between compactification, flows, and $\mathscr{F}$



SR, Sabag, Zafrir 19

In $\mathscr{D}$ dualities from across $\mathscr{D}$ dualities



Gaiotto 09; Gaiotto, Moore, Neitzke 09; many many others

Geometric operation

$$\mathscr{C} = \oplus_i \mathscr{C}_i = \oplus_j \widetilde{\mathscr{C}}_j$$

$$T_{4d}[T_{6d}; \mathscr{C}] =$$

$\mathscr{D} = 4$ Duality

$$= \otimes_i T_{4d}[T_{6d}; \mathscr{C}_i]$$

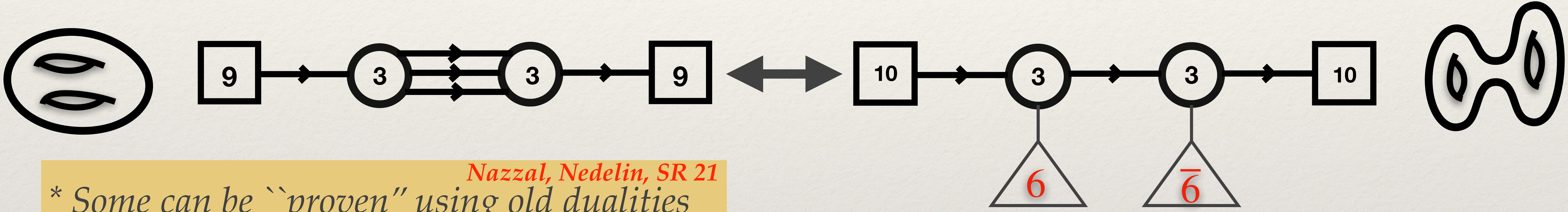
QFT operation (gauging, superpotential)

$$= \otimes_j T_{4d}[T_{6d}; \widetilde{\mathscr{C}}_i]$$

In $\mathscr{D}$ dualities follow from consistency of across $\mathscr{D}$ dualities

$\mathscr{D} = 4$ dualities from across dimension dualities

- Old dualities from Geometry: $\mathcal{N} = 1$ Seiberg dualities, $\mathcal{N} = 2$ S-duality, $\mathcal{N} = 1$ Intriligator-Pouliot dualities, …
- Novel looking dualities from Geometry: Eg:



Nazzal, Nedelin, SR 21

** Some can be ``proven" using old dualities*

** But for many dualities a ``proof" from fundamental ones is not known*

- Q5: Do all dualities in $\mathscr{D} \leq 4$ have a geometric explanation?
- Old and New dualities with no known Geometry: Eg ADE $\mathcal{N} = 1$ dualities with two adjoints; G_2 SQCD to quiver mentioned above

Eg Kutasov, Schwimmer 95; Intriligator, Wecht 03; Kutasov, Lin 14; Intriligator, Nardoni 16

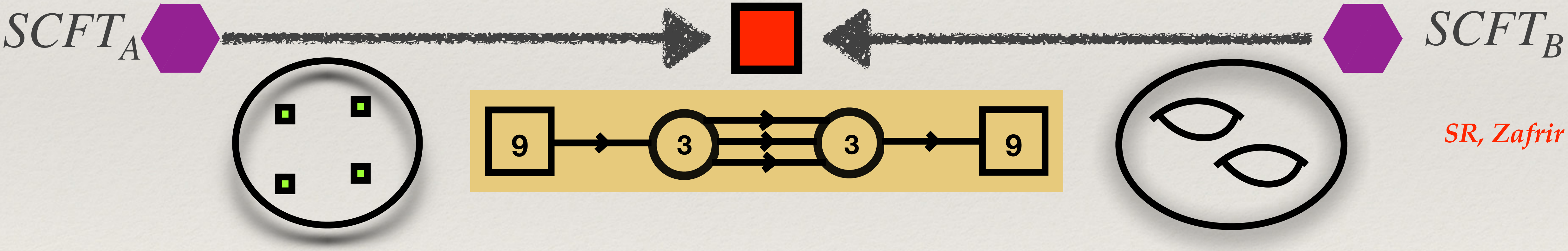
- Q5a: Is there a basic set of $\mathscr{D} \leq 4$ dualities?

See talk by Pasquetti

$\mathscr{D} = 6$ IR dualities

- Can start from different $\mathscr{D} = 6$ SCFTs; deform them by two different geometries; and flow to same $\mathscr{D} = 4$ SCFT $\rightarrow$ $\mathscr{D} = 6$ IR duality

- Eg: $SCFT_A$ is min. $SU(3)$ SCFT, $SCFT_B$ is rank one E-string; $\mathscr{C}_A$ four punctured sphere and $\mathscr{C}_B$ is genus two surface



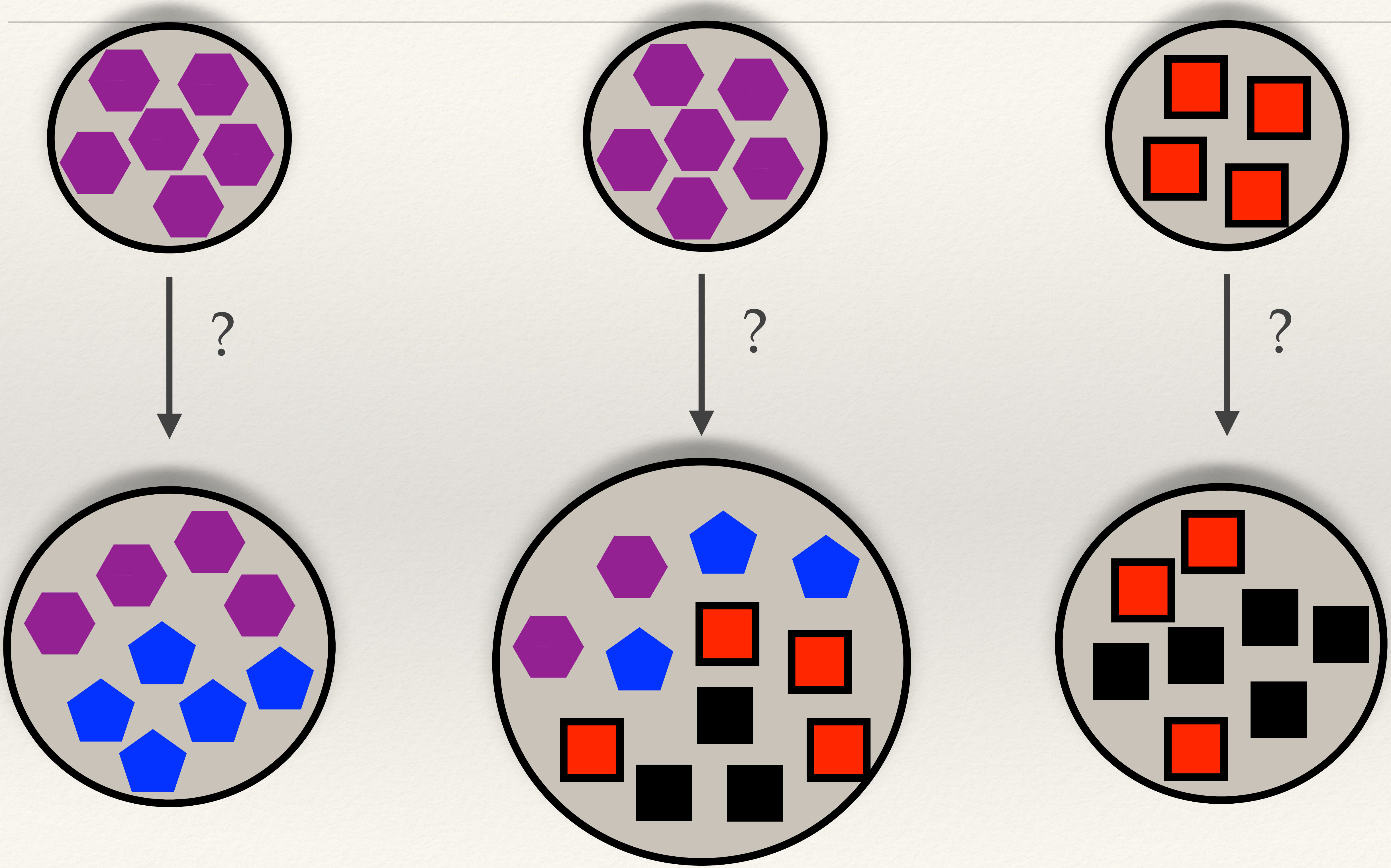
SR, Zafrir 18

- Many more examples

Ohmori, Shimizu, Yonekura, Tachikawa 15; Baume, Kang, Lawrie 21; Kim, SR, Vafa, Zafrir 18

- Q6: Can we explain all such $\mathscr{D} = 6$ IR dualities from string theory?

Summary of Classification Questions



$\mathscr{D} = 4$ $\mathscr{N} = 2$ (??) *Eg Bhardwaj, Tachikawa 13*

Plan:

- SQFTs from String Theory
- SQFTs from SQFTs
    - SQFTs from SQFTs in given $\mathscr{D}$
        - *Moduli spaces*
        - *In-$\mathscr{D}$ dualities*
        - *Lagrangians*
    - SQFTs from SQFTs across $\mathscr{D}$
        - *Across-$\mathscr{D}$ dualities*
        - $\mathscr{D} = 5$ *from* $\mathscr{D} = 6$
        - $\mathscr{D} = 4$ *from* $\mathscr{D} = 6$
        - *In-$\mathscr{D}$ dualities from Across-$\mathscr{D}$ dualities*
        - $\mathscr{D} = 6$ *Dualities*
    - SQFTs from SQFTs across $\mathscr{D}$
        - $\mathscr{D} = 6$ *SCFTs from* $\mathscr{D} < 6$ *SCFTs*

Skeletons III: (S)Partition functions from higher $\mathscr{D}$ SCFTs

- Given a Lagrangian numerous supersymmetric partition functions can be computed using localization
- These typically either are, or can be related to, various counting problems
- These partition functions are usually independent of continuous parameters and RG flows
- In the case of across dimensional dualities the partition functions can be used in two ways:

⬢ on $\mathscr{M}^{\mathscr{D}=4} \times \mathscr{M}^{\mathscr{D}'-\mathscr{D}=2}$ $\Bigg\{$

Deduce $\mathbf{Z}_{\mathscr{M}^{\mathscr{D}}}$ of $T_{\mathscr{M}^{\mathscr{D}'-\mathscr{D}}}$ from geometry

Obtain Skeletons of lower $\mathscr{D}$ strongly coupled SCFTs;
input into deriving across dimensions dualities

Alday, Gaiotto, Tachikawa 09; many many others; Le Floch 20

See eg talk by Paquette

Deduce $\mathbf{Z}_{\mathscr{M}^{\mathscr{D}} \times \mathscr{M}^{\mathscr{D}'-\mathscr{D}}}$ of ⬢ from across dimensions duality

Obtain Skeletons of higher $\mathscr{D}$ intrinsically strongly coupled SCFTs
from lower dimensional Lagrangians

Eg HC Kim, S Kim 12

- Eg given $\mathscr{D} = 4$ dual of $\mathscr{D} = 6$ compactification on $\mathscr{C}_i$ can compute partition function on $\mathscr{M}^{\mathscr{D}=4} = \mathscr{M}_\alpha \times \mathbb{S}^1$, $\mathbf{Z}_{\mathbb{S}^1 \times \mathscr{C}_i \times \mathscr{M}_\alpha}$, which encodes non-trivial information about ⬢

- Q7: What can we deduce about ⬢ scanning over all $\mathscr{C}_i$ and $\mathscr{M}_\alpha$?

Skeletons III: (S)Partition functions from higher $\mathscr{D}$ SCFTs

- The (s)partition functions $\mathbf{Z}$ are given in terms of a variety of special functions
- Expected physical properties (such as dualities, emergence of symmetry, RG flows) imply exact properties of $\mathbf{Z}$

Physics → Math

Math → Physics

- Given a conjectural physical statement test against the precise mathematical consequences

  Eg Dolan, Osborn 08; many others

- From known properties of special functions deduce putative physical statements

  Eg van de Bult → Benini, Closset, Cremonesi 11
  Rains → Pasquetti, SR, Sacchi, Zafrir 19
  Buican, Li, Nishinaka 19 → ?

- (S)Partition functions lead to numerous relations to Physics, Mathematics, and Mathematical Physics

Eg:

- *Integrable models*

  Eg Nekrasov, Shatashvili 09; Gaiotto, Rastelli, SR 12; SR 18; Ruijsenaars 20; Nazzal, Nedelin, SR 21; Chen, Haghighat, Kim, Lee, Sperling 21

  (Classification of integrable models/$\mathscr{D} = 6$ SCFTs)

- *Modularity*

  Eg Spiridonov, Vartanov 12; Gadde 20; Beem, Rastelli 19; Beem, SR, Singh 21; Pan, Peelaers 21;

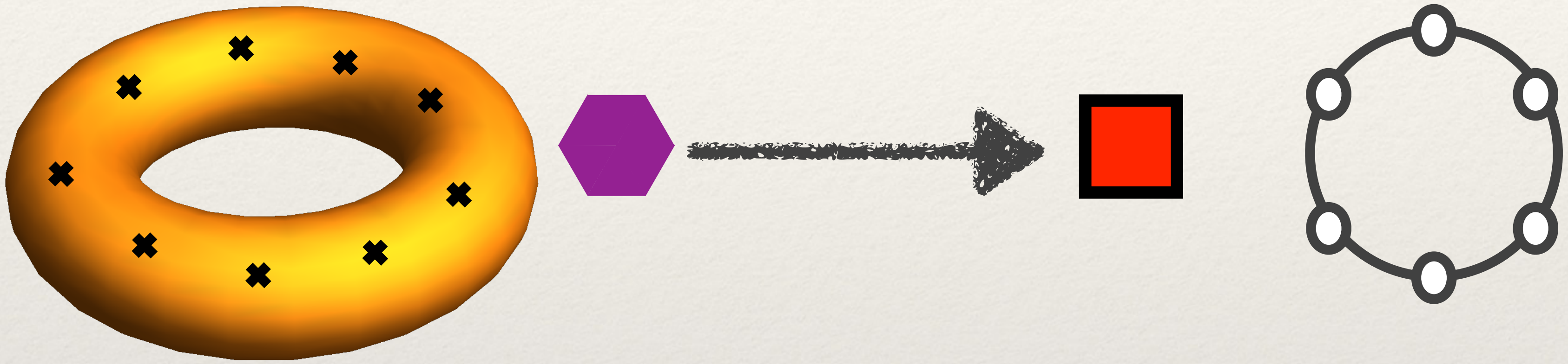
  (See eg Cheng, Dabholkar, Gukov, Murthy and many others for lower dimensions)

- *Gravity and Holography*

  See talk by Benini

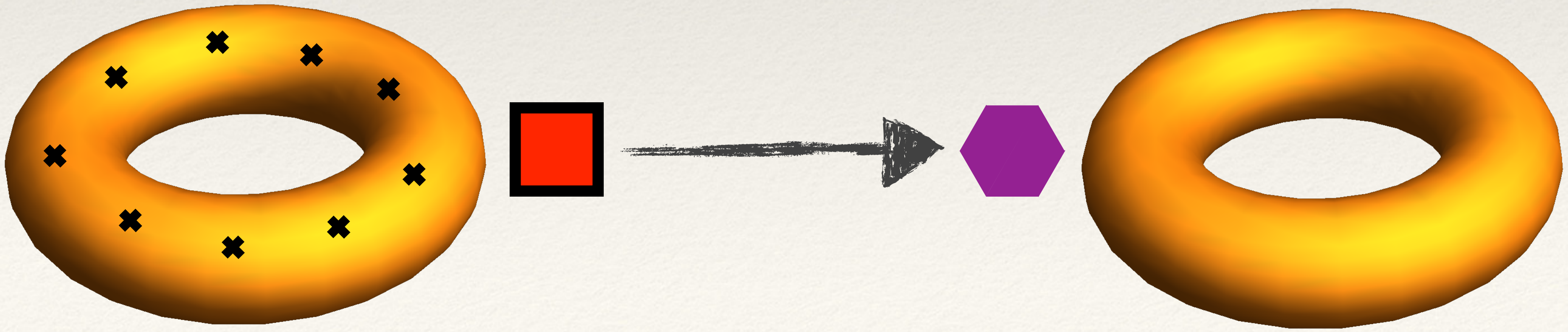
$\mathscr{D} = 6$ theories from $\mathscr{D} < 6$

- A_{N-1} (2,0) $\mathscr{D} = 6$ SCFT compactified on a torus with k minimal punctures is across dimensions dual to a circular quiver



Gaiotto 09

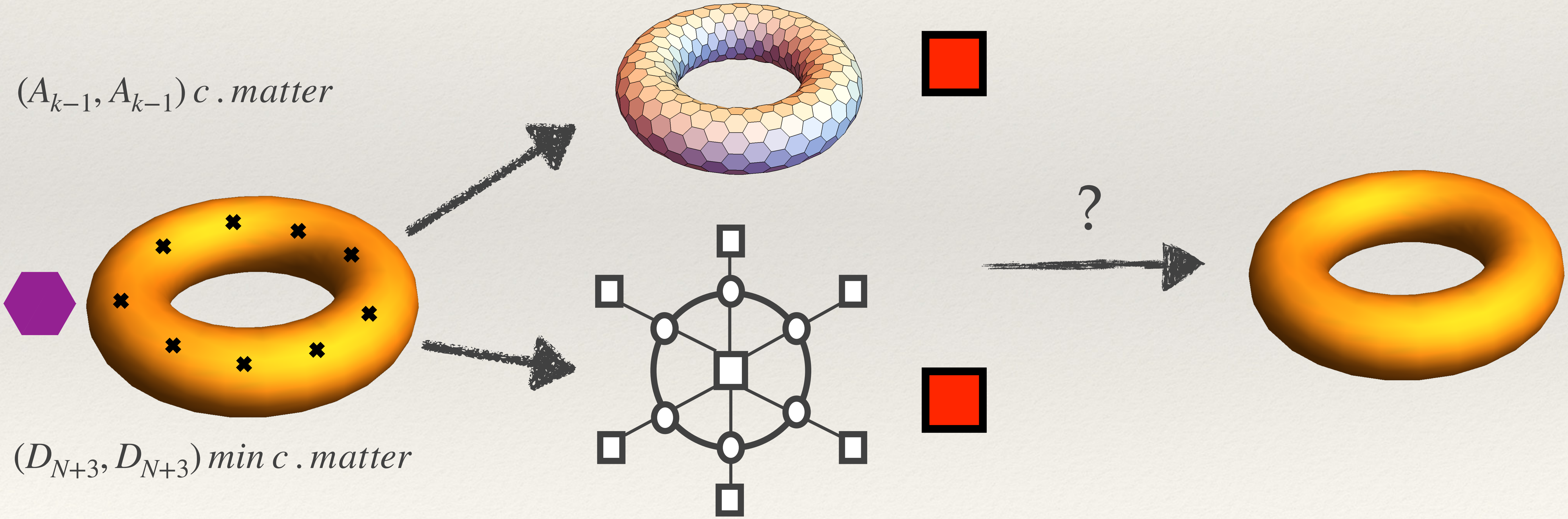
- *Conjecture (deconstruction):* Take a double scaling limit if large number of punctures and close them. Closing punctures is obtained by giving VEVs to certain operators. One then obtains the full $\mathscr{D} = 6$ SCFT on a finite size torus. **Arkani-Hamed, Cohen, Kaplan, Karch, Motl 03**



See also Hayling, Papageorgakis, Pomoni, Rodriguez-Gomez 17 ***See Strings 2018 Cordova***

More $\mathscr{D} = 6$ theories from $\mathscr{D} < 6$

- Consider a (1,0) $\mathscr{D} = 6$ SCFT compactified on a torus with k ``minimal'' punctures and find its across dimensions dual
- Take a double scaling limit of large number of punctures and close them. Does one then obtain the full $\mathscr{D} = 6$ SCFT on a finite size torus?



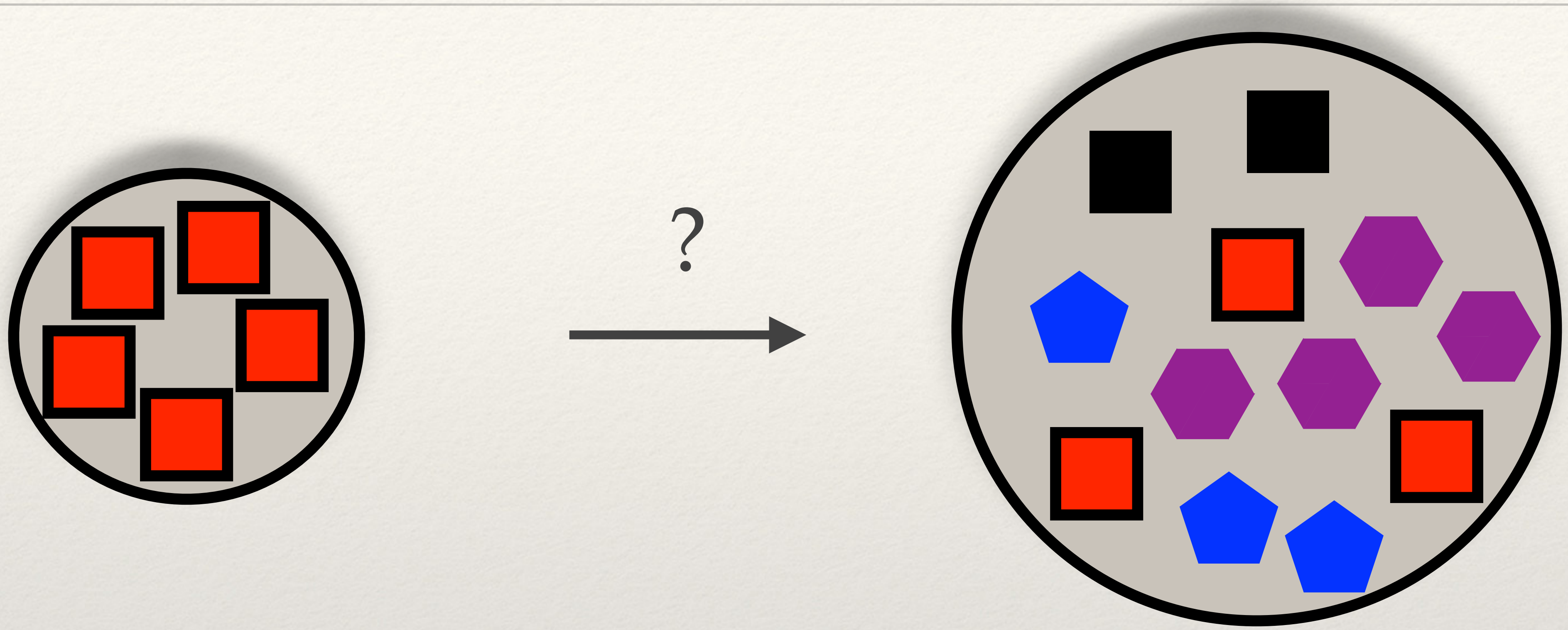
$(A_{k-1}, A_{k-1})\, c\,.\,matter$

$(D_{N+3}, D_{N+3})\, min\, c\,.\,matter$

?

- Q8: Can all $\mathscr{D} = 6$ SCFTs be deconstructed in terms of $\mathscr{D} = 4$ SCFTs?

Bolder question



- Q8a: Can we construct all SCFTs deforming free fixed points in $\mathscr{D} \leq 4$ and taking limits thereof?
- This entails constructing new SCFTs from descriptions which manifest part of global and space-time (super)symmetry
- *SCFTs encode aspects of string theory through holography so maybe this idea is not that crazy*

Additional comments

Eg Dimofte, Gukov, Gaiotto 11; Cho, Gang, Kim 20 *Eg Sacchi, Sela, Zafrir 21*

- Lower $\mathscr{D}$: Eg across dimension dualities $\mathscr{D} = 6 \to \mathscr{D} = 3$; $\mathscr{D} = 5 \to \mathscr{D} = 3$; lifting $\mathscr{D} = 3$ mirror symmetry to $\mathscr{D} = 4$; $\mathscr{D} = 5$ IR dualities ? …

Eg Hwang, Pasquetti, Sacchi 20

- Q9: Constructing Geometric tools to compute beyond Skeletons?

- Connecting with Bootstrap — the philosophy discussed here focuses on relations between theories while bootstrap focuses on fixed points.

- Q10: Existence of non-supersymmetric $\mathscr{D} > 4$ CFTs?

See Benetti Genolini, Honda, Kim, Tong, Vafa 20; Bertolini, Mignosa 21

Morris 04; De Cessare, Di Pietro, Serone 21

- What about non-supersymmetric $\mathscr{D} \leq 4$ CFTs? Can all these be obtained from higher $\mathscr{D}$ supersymmetric ones? *See talk by Nardoni*

- Can all CFTs be obtained from theories with eight supercharges?

Conclusions

- Higher $\mathscr{D}$ SQFTs are a very rich laboratory to test theoretical ideas in QFT and discover novel results and insights experimentally.

- The higher $\mathscr{D}$ world, being intrinsically interacting, forces us to look for new ways of thinking.

- The SCFTs in all $\mathscr{D}$ are intimately interrelated.

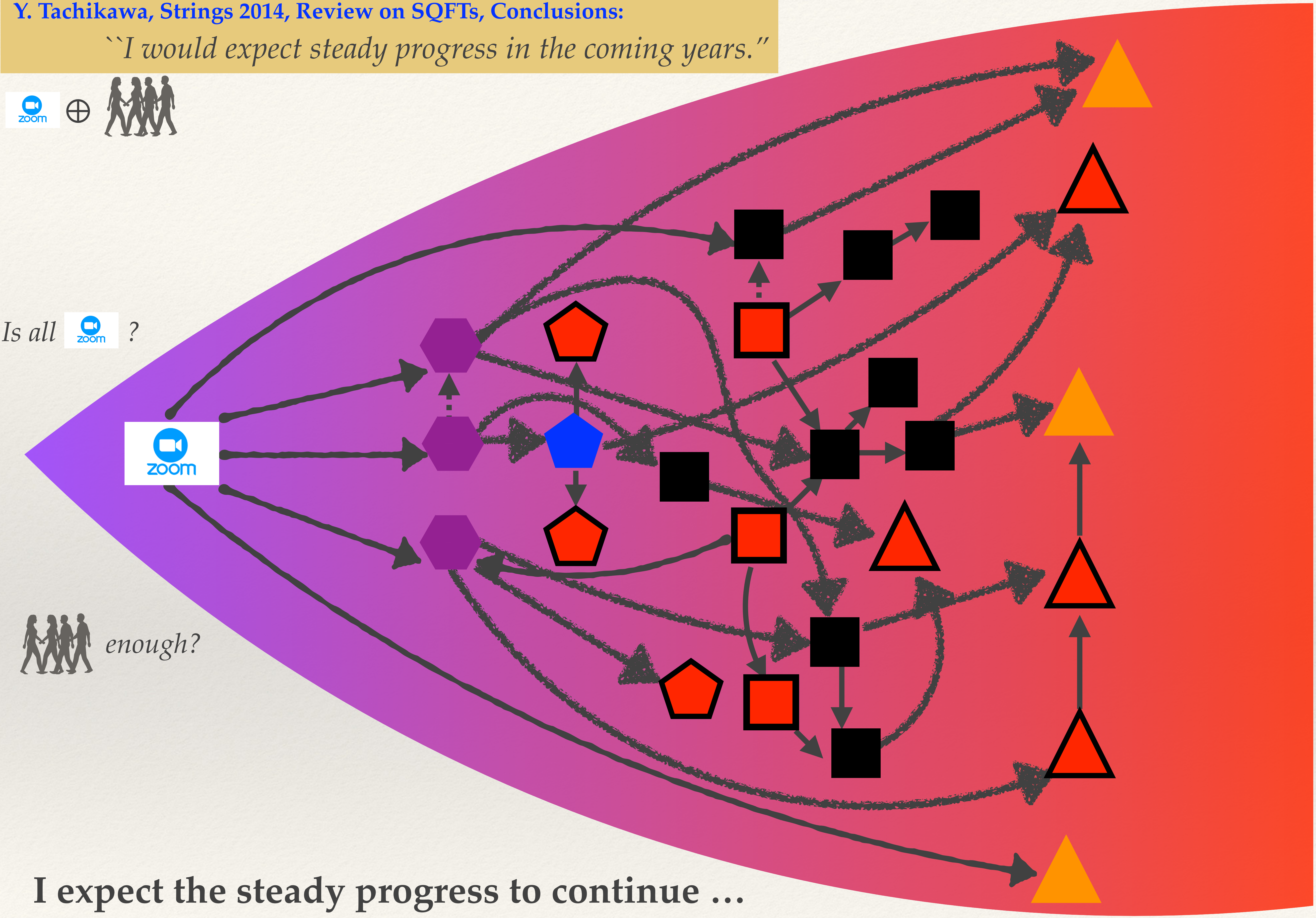
Y. Tachikawa, Strings 2014, Review on SQFTs, Conclusions:

``I would expect steady progress in the coming years.”

zoom ⊕ 👥

Is all zoom *?*

Is 👥 *enough?*



I expect the steady progress to continue …

Thank you for your attention!!