Fluxes, holography and the uses of exceptional generalised geometry

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Geometrical backgrounds are ubiquitous in string theory

phenomenology, swampland, holography, ...

and we have many tools for case without non-trivial fluxes

- Lie groups, cosets G/H, special holonomy (Calabi–Yau, G_2, Sasaki–Einstein etc), ...
- $\rightsquigarrow$ moduli, spectra, existence of solutions, ...

What about when there are (large) non-trivial fluxes?

exceptional generalised geometry is a framework to extend standard geometrical constructions to include fluxes

building on history of using G-structures and generalised complex geometry

Thank my collaborators

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Exceptional generalised geometry

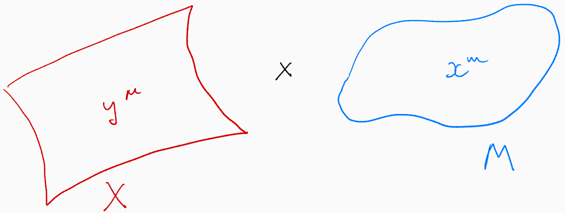
Supersymmetry and generalised G-structures

Consistent truncations

Holography

Exceptional generalised geometry

Set up: compactification geometry $X_D \times M_d$



- on-shell: X is Minkowski or AdS warped product

$$\mathrm{d}s^2 = \mathrm{e}^{2\Delta}\mathrm{d}s^2(X) + \mathrm{d}s^2(M) \qquad + \quad \text{flux on } M$$

  no-go theorems for Minkowski ⇒ need sources for flux [*Maldacena, Nunez 00; Ivanov, Papadopoulos 00; . . .*]

- off-shell: repackage full $(D + d)$-dim (or truncated) theory as theory on X

$$\text{scalars:} \quad g_{mn}(y, x), \quad A_{m_1 \ldots m_p}(y, x), \quad \ldots$$

$$\text{vectors:} \quad g_{\mu m}(y, x), \quad A_{\mu m_1 \ldots m_{p-1}}(y, x), \quad \ldots \qquad \textit{etc}$$

[*de Wit, H. Nicolai 86; . . .*]

- symmetries of the NSNS fields are diffeos ξ^μ and gauge transf λ_μ

$$\delta g_{mn} = (\mathcal{L}_\xi g)_{mn}, \qquad \delta B_{mn} = (\mathcal{L}_\xi B)_{mn} + (\mathrm{d}\lambda)_{mn}, \qquad \delta\phi = \mathcal{L}_\xi \phi$$

with the algebra $\xi'' = [\xi, \xi']$ and $\mathrm{d}\lambda'' = \mathcal{L}_\xi \mathrm{d}\lambda' - \mathcal{L}_{\xi'}\mathrm{d}\lambda$

- package into generalised tangent space $E \simeq TM \oplus T^*M$

$$V^M = \begin{pmatrix} \xi^m \\ \lambda_m \end{pmatrix} \in \Gamma(E) \qquad \text{generalised vector, } M = 1\dots, 2d$$

and choose integration to give generalised Lie derivative

$$V'' = L_V V' = \begin{pmatrix} [\xi, \xi'] \\ \mathcal{L}_\xi \lambda' - \imath_{\xi'}\mathrm{d}\lambda \end{pmatrix} \qquad \textit{why?}$$

[*Liu, Weinstein, Xu 97; Hitchin 02; Gualtieri 04*]

- preserves the natural $O(d,d)$ metric on E

$$\eta_{MN} V^M V^N = V^{\mathrm{T}} \begin{pmatrix} 0 & \frac{1}{2}\mathbb{1} \\ \frac{1}{2}\mathbb{1} & 0 \end{pmatrix} V = \tfrac{1}{2}\xi^m \lambda_m, \qquad L_V \eta = 0$$

- so can extend generalised Lie derivative L_V to

$$\text{generalized tensor} = \text{rep of } O(d,d) \times \mathbb{R}^+ \supset GL(d,\mathbb{R})$$

where $\mathbb{R}^+$ weight p counts powers of $(\det T^*M)^p$

Basic idea is to "geometrize the flux"

reformulate supergravity and supersymmetric background geometries in terms of generalised tensors

- generalised metric $G \in \Gamma(S^2 E^* \otimes \det T^* M)$

$$G_{MN} = \mathrm{e}^{-2\phi}\sqrt{g}\begin{pmatrix} g - Bg^{-1}B & -Bg^{-1} \\ g^{-1}B & g^{-1} \end{pmatrix}_{MN}$$

  invariant under $\mathrm{O}(d) \times \mathrm{O}(d)$ subgroup

- family of generalised Levi–Civita connections $DG = 0$ and $T(D) = 0$

$$(D_V W)^M = \xi^\mu \left(\partial_\mu W^M + \Omega_\mu{}^M{}_N W^N\right) + \lambda_\mu (\tilde{\Omega}^{\mu\, M}{}_N W^N)$$

  where $T(D) \in \Gamma(\Lambda^3 E \oplus E)$

- Ricci tensor is unique and gives NSNS equations of motion, $R_{MN} = 0$

$$\int_M \mathrm{vol}_G \, \mathrm{R} = \int_M \sqrt{g}\mathrm{e}^{-2\phi}\left(\mathcal{R} + 4(\partial\phi)^2 - \tfrac{1}{12}H^2\right),$$

other field RR $= \mathrm{O}(d,d) \times \mathbb{R}^+$ spinors; fermions $= \mathrm{Spin}(d) \times \mathrm{Spin}(d)$ spinors

[*Siegel 93; Hohm, Kwak 10; Jeon, Lee, Park 11; Coimbra, Strick.-Const. DW 13*]

Exceptional generalised geometry

[*Hull 07; Pacheco, DW 08; Berman, Perry 10; Coimbra, Strick.-Const. DW 13*]

How extend to RR sector? M-theory? $F = \mathrm{d}A$, $\tilde{F} = *F = d\tilde{A} - \frac{1}{2}A \wedge F$

$$\delta g = \mathcal{L}_{\xi} g, \qquad \delta A = \mathcal{L}_{\xi} A + \mathrm{d}\omega, \qquad \delta \tilde{A} = \mathcal{L}_{\xi} \tilde{A} + \mathrm{d}\sigma - \tfrac{1}{2}\mathrm{d}\omega \wedge A$$

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- for example $d = 6$: $E \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M$

$$L_V V' = [\xi, \xi'] + \left(\mathcal{L}_\xi \omega' - \imath_{\xi'} \mathrm{d}\omega\right) + \left(\mathcal{L}_\xi \sigma' - \imath_{\xi'} \mathrm{d}\sigma - \omega' \wedge \mathrm{d}\omega\right)$$

preserves $E_{6(6)} \times \mathbb{R}^+$ cubic invariant $c_{MNP} V^M V^N V^P$ and $E \sim 27_{1/2}$

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- gen metric G_{MN} invariant under $USp(8) \subset E_{6(6)} \times \mathbb{R}^+$

  bosonic supergravity on M $=$ generalised Einstein gravity

  fermions are reps of local USp(8) symmetry

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- extend to $\mathrm{E}_{d(d)}$ $(d \leq 7)$ and IIB by different $\mathrm{GL}(d-1, \mathbb{R}) \subset \mathrm{E}_{d(d)} \times \mathbb{R}^+$

  $$E \simeq TM \oplus 2T^*M \oplus \Lambda^3 T^*M \oplus 2\Lambda^5 T^*M$$

Formalism

- reformulation of full 11d M-theory on $X \times M$ [*Hohm, Samtleben 13*]

  scalars: $G_{MN}(x,y) \in \Gamma(S^2 E^* \otimes \det T^* M)$

  vectors: $A_\mu{}^M(x,y) = (g_\mu{}^n, A_{\mu mn}, \tilde{A}_{\mu m_1 \dots m_5}) \in \Gamma(T^* X \otimes E)$ etc

- $\mathrm{O}(d, d+n) \times \mathbb{R}^+$ description of heterotic
- DFT/ExFT: extend spacetime ($T_p\mathcal{M} = T_pX \oplus E_p$), locally same formalism [*Hull, Zwiebach 09*] [*Hohm, Samtleben 13; . . .*]
- $d > 7$? [*Hohm, Samtleben 14; Bossard, Ciceri, Inverso, Kleinschmidt, Samtleben 19,21*]

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Higher-derivative corrections? difficult: need to modify L_V for α' and M-theory [*Hohm, Zweibach 14; Marques, Nuñez 15, ...;*] [*Coimbra, Minasian 17; Coimbra 19*][*Bossard, Kleinschmidt 15*]

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Why this structure? "G-algebroid", descends from L_∞ symmetry of closed SFT [*Bugden, Hulik, Valach, DW 21*] [*Sen 16; Arvanitakis, Hohm, Hull, Lekeu 20,21;. . .*]

Supersymmetry and generalised G-structures

Supersymmetric bkgrd $\Rightarrow$ new geometric structure on M eg cplx structure

- topological: "almost complex structure"

$$T_{\mathbb{C}}M = T^{1,0} \oplus T^{0,1} \qquad \Longleftrightarrow \qquad \text{global tensor } I^m{}_n$$

reduction of structure group of TM to $\mathrm{GL}(n, \mathbb{C}) \subset \mathrm{GL}(2n, \mathbb{R})$

- differential: "integrable complex structure"

$$\left[T^{1,0}, T^{1,0}\right] \subset T^{1,0} \qquad \text{involutive}$$

$$\Leftrightarrow \quad N_{mn}{}^p = I^p{}_q \partial_m I^q{}_n - I^p{}_q \partial_n I^q{}_m - I^q{}_m \partial_q I^p{}_n + I^q{}_n \partial_q I^p{}_m = 0$$

or, there exists a connection ∇ such that

$$\nabla I = 0 \quad \text{"compatible"} \qquad\qquad T(\nabla) = 0 \quad \text{"torsion-free"}$$

Supersymmetric backgrounds

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- no flux $\Rightarrow$ special holonomy $\Rightarrow$ integrable $G \subset \text{SO}(d)$ structure e.g.

$$\begin{aligned} \text{SU}(n) &\subset \text{SO}(2n) && \text{Calabi–Yau} && \mathrm{d}\omega = \mathrm{d}\Omega = 0 \\ G_2 &\subset \text{SO}(7) && \text{Joyce} && \mathrm{d}\phi = \mathrm{d} * \phi = 0 \qquad \text{etc.} \end{aligned}$$

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$$\mathrm{d}(\text{structure form}) = \text{flux} \qquad \text{“intrinsic torsion”}$$

$\rightsquigarrow$ classification, new solutions, but e.g. moduli hard [*Gauntlett, Martelli, Pakis, DW 02; Gutowski, Hull, Pakis, Reall 02, . . .*]

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- for type II: $\mathrm{O}(d,d) \times \mathbb{R}^+$ geometrizes NSNS flux

$$\mathrm{d}\Phi^{\pm} = 0, \qquad \mathrm{d}\Phi^{\mp} = \text{RR flux}$$

[*Hitchin 02; Gualtieri 04; Graña, Minasian, Petrini, Tomasiello 04,05; . . .*]

[*Coimbra, Strick.-Const., DW 14; Coimbra, Strick.-Const. 16, 17*]

By analogy, generalised G-structure on E for $G \subset \mathrm{E}_{d(d)} \times \mathbb{R}^+$

Theorem: generic supersymmetric flux backgrounds (M-theory, type II, $d \leq 7$) are equivalent to

Minkowski $\Rightarrow G \subset H_d$ integrable structure

AdS $\Rightarrow G \subset H_d$ structure with singlet intrinsic torsion

where G is the stabliser group of the Killing spinor(s)

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for example $D = 4$: susy parameter ϵ in 8 of $H_7 = \mathrm{SU}(8) \subset \mathrm{E}_{7(7)} \times \mathbb{R}^+$

$$\mathcal{N} = 1 \qquad G = \mathrm{Stab}(\epsilon_1) = \mathrm{SU}(7)$$

$$\mathcal{N} = 2 \qquad G = \mathrm{Stab}(\epsilon_1, \epsilon_2) = \mathrm{SU}(6) \qquad \text{etc}$$

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Analogue of special holonomy $\rightsquigarrow$ classification, new solutions, moduli

Example: "generalising G_2" $\mathcal{N} = 1$, $D = 4$ in M-theory

[Ashmore, Strick.-Const., Tennyson, DW 19] [c.f. Lukas, Saffin 04]

For SU(7), generalised invariant tensor in $912_{3/2}$,

$$\psi^{MNP} \in \Gamma(W_{\mathbb{C}}) \qquad W \simeq \mathbb{R} \oplus \Lambda^3 T^*M \oplus (T^*M \otimes \Lambda^5 T^*M) \oplus \dots$$

viewing 11d M-theory as 4d $\mathcal{N} = 1$ on X with chiral matter ψ

$$\mathcal{Z} = \{\text{SU(7) structures } \psi\} \qquad \infty\text{-dim Kähler manifold}$$

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- F-terms: from superpotential $\Leftrightarrow$ involutive SU(7)-inv sub-bundle

$$L_{C_3} C_3 \subset C_3 \qquad \begin{aligned} E_{\mathbb{C}} &\simeq C_3 \oplus C_{-1} \oplus C_{-3} \oplus C_1 \\ 56 &= 7 \oplus 21 \oplus \bar{7} \oplus \bar{21} \end{aligned}$$

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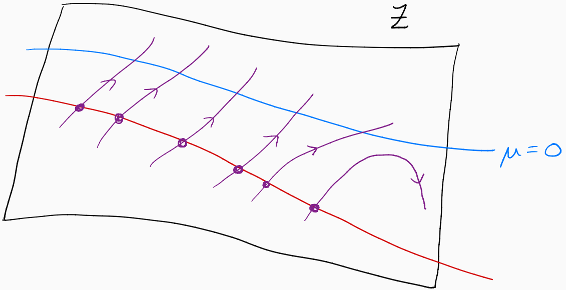
- D-terms: moment map for GDiff symmetry acting on $\mathcal{Z}$ by $\delta\psi = L_V \psi$

$$\mu(V) = 0, \qquad \forall V \in \Gamma(E) \simeq \mathfrak{gdiff}$$

Typical of supersymmetry conditions: first solve F-terms (holomorphic)



- $\mathcal{Z}$ is Kähler (infinite-dimensional) with group action G
- orbits of $G_{\mathbb{C}}$ intersect $\mu = 0$ if "stable" – algebraic condition

Kähler–Einstein, Sasaski–Einstein, Hermitian Yang-Mills, ...

[*Yau; Tian; Donaldson, ...*]

Moduli

Involutive structure defines a complex

$$\cdots \xrightarrow{\mathrm{d}_C} \Lambda^p C_3^* \xrightarrow{\mathrm{d}_C} \Lambda^{p+1} C_3^* \xrightarrow{\mathrm{d}_C} \cdots$$

- deforming ψ but *not flux sources* gives

$$\begin{aligned}\text{local moduli space} &\simeq H^3(\Lambda^* C_3^*, \mathrm{d}_C) \oplus H^6(\Lambda^* C_3^*, \mathrm{d}_C) \\ &\simeq H^3_{\mathrm{dR}}(M, \mathbb{C}) \oplus H^6_{\mathrm{dR}}(M, \mathbb{C})\end{aligned}$$

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Can extend to type II (e.g. GMTP backgrounds)

- other new results e.g. moduli of Graña–Polchinksi background (matches naive superpotential expectation) [*Ashmore, Stric.-Const., Tennyson, DW 19; Smith, Tennyson, DW w.i.p.*]

Further directions and extensions

Other dimensions and amounts of supersymmetry

- $\frac{1}{4}$-susy: "exceptional Calabi–Yau" including moduli [*Ashmore, DW 15*];
  $\frac{1}{2}$-susy: (Mink. and AdS) [*Malek 17*]
- heterotic Hull–Strominger system including moduli [*Ashmore, Strick.-Const., Tennyson, DW 19*] [*cf de la Ossa, Svanes 14; Garcia-Fernandez, Rubio, Tipler 19*]

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Kähler potential on $\mathscr{Z}$ gives "exceptional Hitchin functionals"

- SU(7) structure extends G_2, ECY extends cplx-struct functional
- quantisation and topological theories? (heterotic [*Svanes, Tennyson w.i.p.*])

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Existence of solutions from stability?

- for extended G_2: $\mathrm{d}\phi = 0$ then vary in $H^3_{\mathrm{dR}}(M)$ for $\mathrm{d} * \phi = 0$
- toric backgrounds for type II and ECY?

Consistent truncations

Basic idea

Solutions of truncated theory are solutions of full theory

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}M^2\phi^2 + \tfrac{1}{2}(\partial\pi)^2 - \tfrac{1}{2}m^2\pi^2 - \lambda\phi\pi^2$$

$$(\partial^2 + M^2)\phi = -\lambda\pi^2 \qquad (\partial^2 + m^2)\pi = -2\lambda\phi\pi$$

- truncate to ϕ ✓, truncate to π ✗, even if $M \gg m$
- symmetry: keep singlets under $\mathbb{Z}_2$ where $\phi \to \phi$, $\pi \to -\pi$

Usually fields come from Kaluza–Klein modes in compactification

- gives consistent "uplift" of dimensionally reduced theory
- closed sector at large N in holography
- (partial) check of stability (AdS swampland conjecture)

Long history searching for suitable ansätze

- Scherk–Schwarz: $M = \mathcal{G}$, expand in (left-)invariant objects on M

$$g^{\mu\nu} = \phi^{ab}(y)\, \hat{e}_a^\mu(x) \hat{e}_b^\nu(x), \quad \text{etc} \qquad\qquad [\hat{e}_a, \hat{e}_b] = f_{ab}{}^c \hat{e}_c$$

  giving theory with maximal susy [*Scherk, Schwarz 79*]
- "mysterious spheres": S^4 and S^7 in M-theory, S^5 in IIB, complicated ansatz, maximal susy [*de Wit, Nicolai 87; Natase, Vaman, van Nieuwenhuizen 99*]
- conv. G-structure with constant singlet torsion [*Gauntlett, Kim, Varela, DW 09; Cassani, Dall'Agata, Faedo 10; Gauntlett, Varela 10; ...*]

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General picture?

Theorem: Given a generalised G-structure on M with constant, singlet intrinsic torsion, keeping all G-invariant fields gives consistent truncation of M-theory or type II on M [*Cassani, Josse, Petrini, DW 19*]

Maximal susy: "generalised Scherk–Schwarz", $G = \mathbb{1}$ "trivial structure"

E is parallelisable, admits global frame, $M = G_E/H_E$ where $\mathfrak{g}_E = \mathfrak{a}/\mathfrak{i}$

invariant gen. tensors: $\hat{E}_A \in \Gamma(E)$ basis for E

singlet torsion: $L_{\hat{E}_A}\hat{E}_B = X_{AB}{}^C\hat{E}_C$ Leibniz alg. $\mathfrak{a}$

scalars $G^{MN} = \phi^{AB}(y)\hat{E}^M_A\hat{E}^N_B$, vectors $A^M_{\hat{\mu}} = a^A_{\hat{\mu}}(y)\hat{E}^M_A$ etc.

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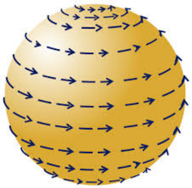
- "mysterious spheres" are generalised parallelisable [Lee, Strick.-Const., DW 14]
- $\rightsquigarrow$ gauged maximal supergravity, embedding tensor $X_{AB}{}^C$

[de Wit, Nicolai 87; Hull, Reid-Edwards 05; Geissbuhler 11; Graña, Marquéz 12; Berman, Musaev, Thompson 12; Godazgar, Godazgar, Nicolai 13; . . .]



For generalised Scherk–Schwarz

- full consistency of IIB S^5 truncation (and massive IIA S^6) [*Baguet, Hohm, Samtleben 15; ...*] [*also Ciceri, de Wit, Varela 14*]
- reduction to algebraic problem [*Inverso 17; Bugden, Hulik, Valach, DW 21*]
  $\rightsquigarrow$ classification of all compact simple gaugings [*Valach, DW w.i.p.*]
- Poisson–Lie U-duality: $\{\hat{E}_A\}$ and $\{\hat{E}'_A\}$ give same algebra $\mathfrak{a}$ [*Sakatani 20; Malek, Thompson 20; Bugden, Hulik, Valach, DW 21*]

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Mapping the landscape

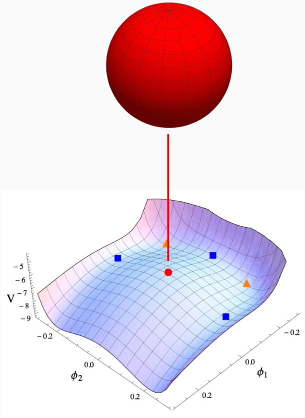
- no dyonic $\mathcal{N} = 8$ SO(8) gaugings, but other dyonic gaugings possible [*Lee, Strick.-Const., DW 15; Guarino, Jafferis, Varela 15; Inverso, Samtleben, Trigiante 16*]
- all $\frac{1}{2}$-susy $D = 5, 6, 7$ gaugings [*Malek, Samtleben 19; Malek, Vall Camell 20*];
  all $\frac{1}{4}$-susy $D = 5$ gaugings [*Josse, Malek, Petrini, DW 21*]
- proof of "pure supergravity" conjecture of [*Gauntlett, Varela 19*]

also $d > 7$ and many other cases (Maldacena–Nunez, β-deformed, etc ...)

Kaluza–Klein spectroscopy

[*Malek, Samtleben 19,20*]

Can you do more? Complete spectrum?? Still determined by $\mathfrak{a}$??



- potential $V(\phi)$ for truncation scalars ϕ^{AB} has several AdS extrema
- $M = \mathrm{SO}(d+1)/\mathrm{SO}(d) \Rightarrow$ expand fluctuations in $\mathrm{SO}(d+1)$ reps $\boldsymbol{r}_i$; only mass eigenstates for round sphere
- *however* mass matrix only depends on $\boldsymbol{r}_i$, ϕ^{AB} and $X_{AB}{}^C$
- gives full spectrum at any extremum (in BPS multiplets if supersymmetric)
- includes example with no isometries [*Cesaro, Larios, Varela 21*]

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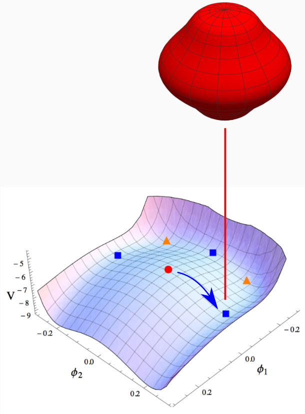

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Implications and new directions

AdS swampland conjecture: no stable, non-susy AdS bkgds [*Ooguri, Vafa 16*]

- S^7 in M-theory SO(3) $\times$ SO(3) extremum: unstable to higher KK modes [*Malek, Nicolai, Samtleben 20*]
- S^6 in massive IIA G_2 extremum: perturbatively stable! [*Guarino, Malek, Samtleben 20,21*]
- IIB S-fold gauging: conformal manifold of non-susy perturb. stable vacua! [*Giambrone, Guarino, Malek, Samtleben, Sterckx, Trigiante 21*]

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Full spectrum of conformal dimensions in holographic dual $\rightsquigarrow$ topology of conformal manifold [*Giambrone, Malek, Samtleben, Trigiante 21; GGMSST 21*]

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Cubic interactions? Consistent truncations with less susy? susy breaking deformations? . . . many new possibilities

Holography

Old problem: $\mathcal{N} = 1$ marginal deformations for $\mathcal{N} = 4$

Superpotential deformation [*Leigh, Strassler 95*]

$$\Delta\mathcal{W} = \tfrac{1}{2}\lambda_1 \,\mathrm{tr}\, \Phi^1\Phi^2\Phi^3 + \tfrac{1}{6}\lambda_2 \,\mathrm{tr}\left[(\Phi^1)^3 + (\Phi^2)^3 + (\Phi^3)^3\right]$$

Gravity dual? deforming S^5 and adding fluxes

- $\lambda_2 = 0$: "β-deformation", $U(1)^3$ isometry, exact dual [*Lunin, Maldacena 05*]
- $\lambda_1 \neq 0, \lambda_2 \neq 0$: tour de force to 2nd/3rd order [*Aharony, Kol, Yankielowicz 02*] only $U(1)_R$ isometry, as hard as finding explicit Calabi–Yau metrics

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But ... much of field theory quite simple, since only depends on holomorphic structure. Is there a generic supergravity analogue?

Can we understand the dual geometry beyond classic Sasaki–Einstein examples?

[*Ashmore, Petrini, DW 16*]

Inv. tensors define $USp(6) \subset E_{6(6)} \times \mathbb{R}^+$ structure $\Rightarrow g_{mn}, B^i_{mn}, C_{mnpq}, \phi, \chi, \Delta$

V structure, K $\quad E \simeq TM \oplus 2T^*M \oplus \Lambda^3 T^*M \oplus 2\Lambda^5 T^*M$

H structure, X $\quad \operatorname{ad} \tilde{F}_{\mathbb{C}} \otimes \det T^*M \simeq T^*M \oplus 2\Lambda^3 T^*M \oplus \dots$

Differential conditions: singlet intrinsic torsion

- F-terms: X defines involutive sub-bundle

$$E_{\mathbb{C}} \simeq C_+ \oplus C_- \oplus C_0, \qquad L_{C_+} C_+ \subset C_+$$

- D-terms: moment map for GDiff, generated by L_V

$$\mu(V) = -\frac{\mathrm{i}}{4} \int_M \frac{\operatorname{tr} X(L_V \bar{X})}{(\operatorname{tr} X\bar{X})^{1/2}} = \int_M c(K, K, V) \qquad \forall V \in \mathfrak{gdiff}$$

- R-charges: $L_K X = 3\mathrm{i}X$ and $L_K K = 0$

- For Sasaski–Einstein writing $\tau = \chi + \mathrm{i}e^{-2\phi}$ and $u^i = \tau_2^{-2}(\tau, 1)^i$

$$X = -\tfrac{1}{2}\mathrm{i}u^i \, \mathrm{e}^{C + \frac{1}{4}\mathrm{i}\Omega\wedge\bar{\Omega}} \cdot \sigma \wedge \Omega, \qquad \text{“Cauchy–Riemann structure”}$$
$$K = \mathrm{e}^{C} \cdot (\xi - \sigma \wedge \omega)\,, \qquad \text{“contact structure”}$$

- Universal form of central charge

$$a^{-1} \propto \int_M c(K, K, K)$$

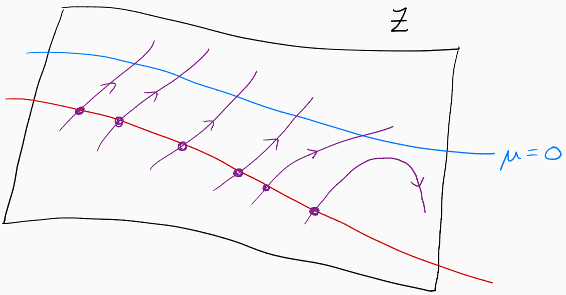
- SCFT result [*Kol 02,03; Green, Komargodski, Seiberg, Tachikawa, Wecht 10*]
  all marginal deformations are in the superpotential and are all exactly marginal unless there is a global symmetry

  follows directly from moment map structure of ESE [*Ashmore, Gabella, Graña, Petrini, DW 16*]

What about the missing deformed solutions? Analogue with CY theorem ...

[*Ashmore, Petrini, Tasker, DW 21*]

Using the GIT picture



[*Ashmore, Petrini, Tasker, DW 21*]

Using the GIT picture

- "Exceptional Sasaki" = relax D-term (n.b. Sasaki $\subset$ ES)
- $\mathrm{GDiff}_{\mathbb{C}}$ orbit generated by $\delta X = L_V X$ with $V \in \Gamma(E_{\mathbb{C}}) \simeq \mathfrak{gdiff}_{\mathbb{C}}$ and intersects moment map condition on ESE background

Physically

$$\text{orbit } [X] = \{X' : X' = \mathrm{GDiff}_{\mathbb{C}} \cdot X\} \text{ encodes superpotential } \mathcal{W}$$

- $\delta X = L_V X$ part of long vector deforming Kähler potential
- orbit is renormalisation flow of Kähler potential; class $[X]$ does not change for domain wall flow – non-renormalization of $\mathcal{W}$

$$X' = -\tfrac{1}{3}\mathrm{i} L_K X, \qquad K^{*\prime} = \mu \qquad \text{where } K^*_M = c_{MNP} K^N K^P$$

New solution for finite marginal deformation of any Sasaki–Einstein I

We find new family of Exceptional Sasaki solutions with $\mathcal{L}_\xi f = 3\mathrm{i}f$

$$K = \mathrm{e}^{C} \cdot (\xi - \sigma \wedge \omega)$$
$$X = \mathrm{e}^{b^i(\tau,f)+C} \cdot \Big(\mathrm{d}f + v^i(\tau, f)\, \sigma \wedge \Omega\Big)$$

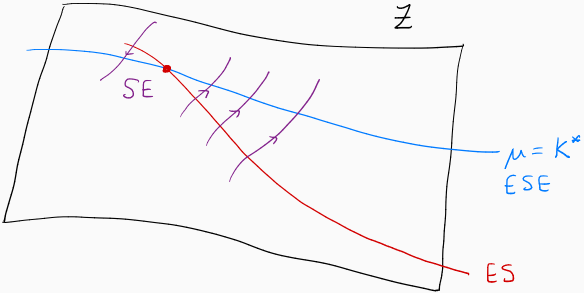
with $b^i \in \Gamma(\Lambda^2 T^*_{\mathbb{C}} M)$ linear in f and v^i quadratic in f

- complicated deformed metric g, axion-dilaton and fluxes
- valid for marginal deformation of any Sasaki–Einstein
- for S^5 matches to 2nd order [*Aharony et al 02*] with

$$f = \tfrac{1}{6} d_{ijk} z^i z^j z^k \qquad \text{on CY cone over SE}$$

  and same discrete symmetries as $\Delta\mathcal{W}$ [*cf Baggio, Bobev, Chester, Lauria, Pufu 17*]
- for $f = z^1 z^2 z^3$ on S^5 gives $\mathrm{GDiff}_{\mathbb{C}}$ of LM solution

ESE solution exists in open neighbourhood of Sasaki–Einstein point



- moment map $\tilde{\mu} = \mu - K^*$ for GDiff_K (preserves K)
- stable points form open set in $\mathcal{Z}$ (Kempf–Ness + no additional sym)
- Monge–Ampère-type equation?

Spectrum of single trace chiral operators

Count single-trace mesonic operators $\operatorname{tr}\Phi^{i_1}\cdots\Phi^{i_n}$ of R-charge $\frac{2}{3}k$

- deformations of C_+, counted by cohomology of

$$\cdots \xrightarrow{\mathrm{d}_C} \Lambda^p C_+^* \xrightarrow{\mathrm{d}_C} \Lambda^{p+1} C_+^* \xrightarrow{\mathrm{d}_C} \cdots$$

independent of choice of X in class $[X]$

i.e. holomorphic data and can calculate at ES point

- for SE gives "transverse Dolbeault cohomology" [*Eager, Schmude, Tachikawa 12*]
- if $\eta = \mathrm{d}f$ nowhere vanishing (not β-def, not $Y^{p,q}$) gives "η-cohomology"

$$\cdots \xrightarrow{\mathrm{d}} \eta \wedge \Lambda^p T^*_{\mathbb{C}} M \xrightarrow{\mathrm{d}} \eta \wedge \Lambda^{p+1} T^*_{\mathbb{C}} M \xrightarrow{\mathrm{d}} \cdots$$

can calculate using transverse Dolbeault [*Tasker 21*]

- universal result for Hilbert series, counting chirals with R-charge $\frac{2}{3}k$

$$\tilde{H}(t) = \sum_k (\#\text{ of chiral ops.})\, t^{2k} = 1 + \mathcal{I}_{\text{s.t.}}(t) - \frac{t^6}{1-t^6}$$

where $\mathcal{I}_{\text{s.t.}}(t)$ is single trace index

- for regular Sasaki–Einstein

$$S^5: \quad \tilde{H}(t^{1/2}) = \frac{(1+t)^3}{1-t^3}$$ matches math $HC_0(k)$ [van den Bergh]

$$T^{1,1}: \quad \tilde{H}(t^{1/3}) = \frac{1+4t+2t^2}{1-t^2}$$ prediction (checked to level 7)

$$\mathrm{dP}_6: \quad \tilde{H}(t^{1/6}) = \frac{1+7t}{1-t}$$ prediction

Key point is that $[X]$ captures holomorphic information:

- same formalism for M-theory (MN, BBBW, ...): count chirals?
- 3d $\mathcal{N} = 2$ theories and SE_7 deformations; $d > 7$ and relation to spindles?
- superconformal index from $(\Lambda^p C_+^*, \mathrm{d}_C)$ complex via localisation on M [*Ashmore, Smith, Tasker, Tennyson, DW w.i.p.*]

should reduce to holomorphic structure of probe geometry

From moment map/GIT:

- general picture of dual of a-max/$\mathcal{F}$-max [*Ashmore, Petrini, DW w.i.p.*]
- extension of "K-stability" of SE and existence of solutions; relation to flow in QFT? [*cf Collins, Xie, Yau 16; Fazzi, Tomasiello 19*]

Summary

Exceptional generalised geometry $\rightsquigarrow$ new results for flux backgrounds

- moduli for flux compactifications, but need more examples
- mapping out landscape of consistent truncations
- powerful new Kaluza–Klein spectroscopy
- new holomorphic structures in holography

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Thank you!