From $\mathcal{N} = 2$ Supersymmetry to Adjoint QCD

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[2207.xxxxx]
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[2012.11843], [2208.xxxxx]
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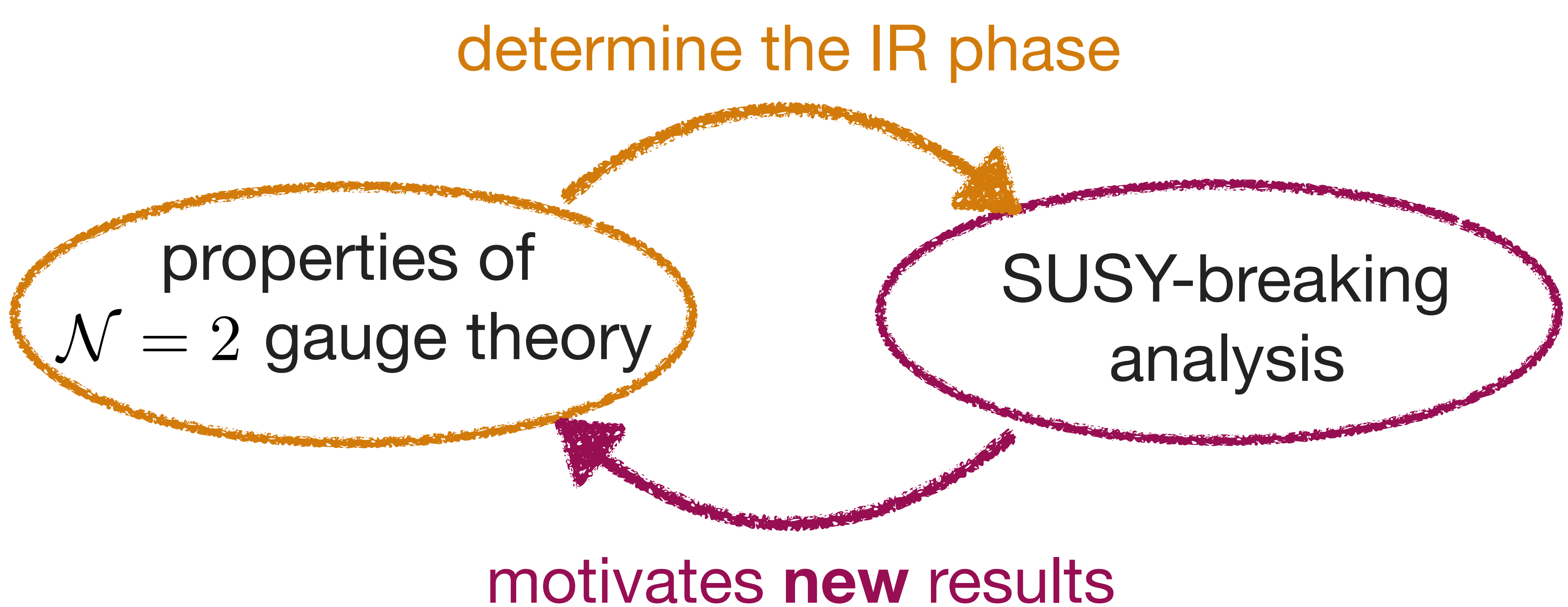
Goal: probing strong coupling dynamics of 4d gauge theories

- Today: **new perspectives** on the strong coupling dynamics of *non*-supersymmetric gauge theories.

"4d adjoint QCD"

$SU(N)$ gauge theory with $N_f = 2$ adjoint Weyl fermions $\lambda_\alpha^{i=1,2}$

- **Approach**: break $\mathcal{N} = 2$ supersymmetry.



I) Probing the IR phase of adjoint QCD

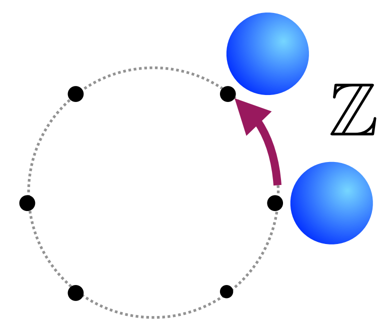
with $\mathcal{N} = \mathbf{2}$ *supersymmetry*

Standard lore for $N_f = 2$ adjoint QCD

Chiral symmetry breaking: $\langle \mathrm{tr}\lambda^i \lambda^j \rangle \neq 0$, leading to

$$SU(N_f = 2)_R \overset{\langle\lambda\lambda\rangle}{\to} U(1)^* \qquad U(1)_r \overset{\text{ABJ}}{\to} \mathbb{Z}_{4N} \overset{\langle\lambda\lambda\rangle}{\to} \mathbb{Z}_4$$

→ *N* copies of a $\mathbb{CP}^1 = \frac{SU(2)}{U(1)}$



$\mathbb{Z}_N \simeq \mathbb{Z}_{4N}/\mathbb{Z}_4$

Confinement: unbroken $\mathbb{Z}_N^{(1)}$ 1-form center symmetry.

also see: [Unsal '07] [Bi, Senthil '18] [Anber, Poppitz '18] [Cordova, Dumitrescu '18] …

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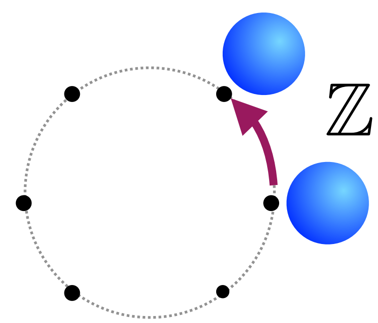

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* The $U(1)$ Cartan must be unbroken by the Vafa-Witten theorem.

[Vafa, Witten '84]

- Adding a small fermion mass in the $\mathbb{CP}^1$ vacua
  → the *N* confining vacua of $\mathcal{N} = 1$ SYM
- Nice parallels to the $\mathcal{N} = 2$ origin of confinement in the $\mathcal{N} = 1$ vacua. [Seiberg, Witten '94][Douglas, Shenker '95]

also see: [Unsal '07] [Bi, Senthil '18] [Anber, Poppitz '18] [Cordova, Dumitrescu '18] …

Strategy: breaking $\mathcal{N}=2$ supersymmetry

[Cordova, Dumitrescu '18][D'Hoker, Dumitrescu, Gerchkovitz, **EN** '22]

$\mathcal{N}=2$ super Yang Mills with G gauge group

UV

$$V_{\cancel{\text{SUSY}}} = M^2\mathcal{T}, \quad \mathcal{T} \sim \mathrm{tr}\,(\bar{\phi}\phi)$$

vector multiplet scalar

$$M/\Lambda \gg 1$$

IR

G gauge theory with 2 adjoint Weyl $\lambda_\alpha^{i=1,2}$

- $\mathcal{T}$ preserves all symmetries (except SUSY) and anomalies.

Can we use the supersymmetric embedding to explore the IR dynamics of *non*-supersymmetric $SU(N)$ adjoint QCD?

Tracking SUSY-breaking to the IR

- $\mathcal{T} \sim \mathrm{tr}\bar{\phi}\phi$ lies in the **protected** stress tensor multiplet, and is tracked on the Coulomb branch to the Kähler potential *K*,

[Luty, Rattazzi '99][Abel, Buican, Komargodski '11][Cordova, Dumitrescu '18]

$$\boxed{\mathcal{T} \longrightarrow K = \frac{1}{2\pi} \sum_{m=1}^{N-1} \mathrm{Im}\big(\bar{a}_m \, \underbrace{a_{Dm}}_{\text{SW periods}}\big)}$$

- For $M \ll \Lambda$ we can reliably analyze $V_{\cancel{\mathrm{SUSY}}} = M^2 \mathcal{T}$.

  $\ast$ This requires the **entire** $\mathcal{N} = 2$ low-energy effective action (K, τ_{mn},...) determined by the *SU*(*N*) Seiberg-Witten solution.

[Seiberg, Witten '94][Argyres, Faraggi '94][Klemm, Lerche, Theisen, Yankielowicz '94]

A dual description as a function of *M*

- For $M \gg \Lambda$ we flow to adjoint QCD.

  Key result: we construct a dual motivated/deduced from softly-broken Seiberg-Witten theory, which interpolates from small to large M.

  The dual leads to a compelling qualitative picture of the dynamics as a function of M, and predicts the $\mathbb{CP}^1$ phase of adjoint QCD.

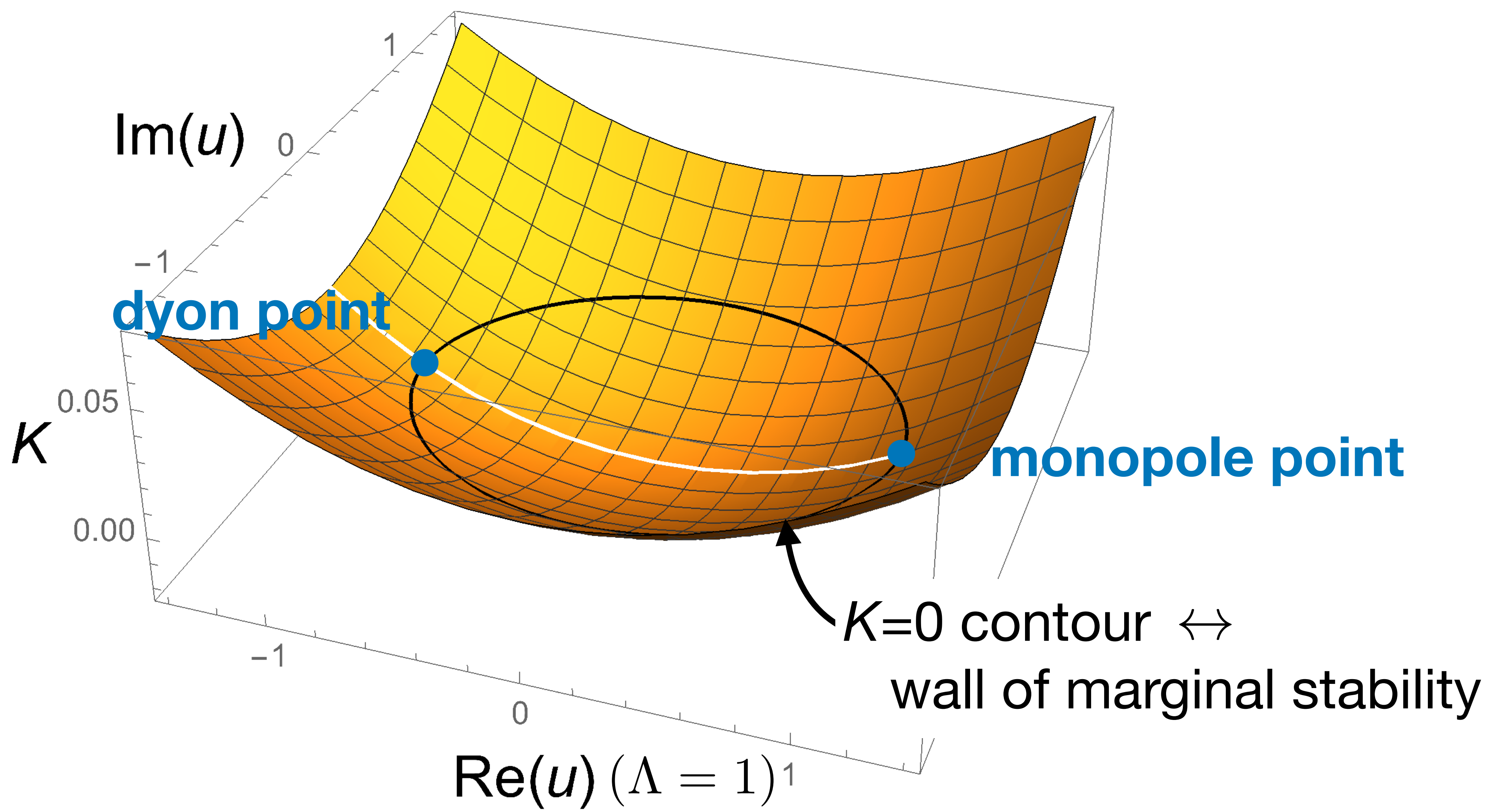
II) SUSY-breaking at small M

and the role of the Kähler potential

The Kähler potential for *SU*(2)

- $K(u) \sim \mathrm{Im}(\bar{a} a_D)$ is a known function of $u \sim \mathrm{tr}\phi^2$ (see next slide). It is convex with a single minimum at the origin, where $K(0) < 0$.

see [Luty, Rattazzi '99] [Cordova, Dumitrescu '18]



A new expansion for the *SU*(*N*) periods [D'Hoker, Dumitrescu, **EN** '22]

- The periods are functions of *N*-1 complex u_n. Explicit, integrated solutions are known for *N*=2,3.
  [Seiberg, Witten '94][Klemm, Lerche, Theisen '95][Ito, Yang '95]…

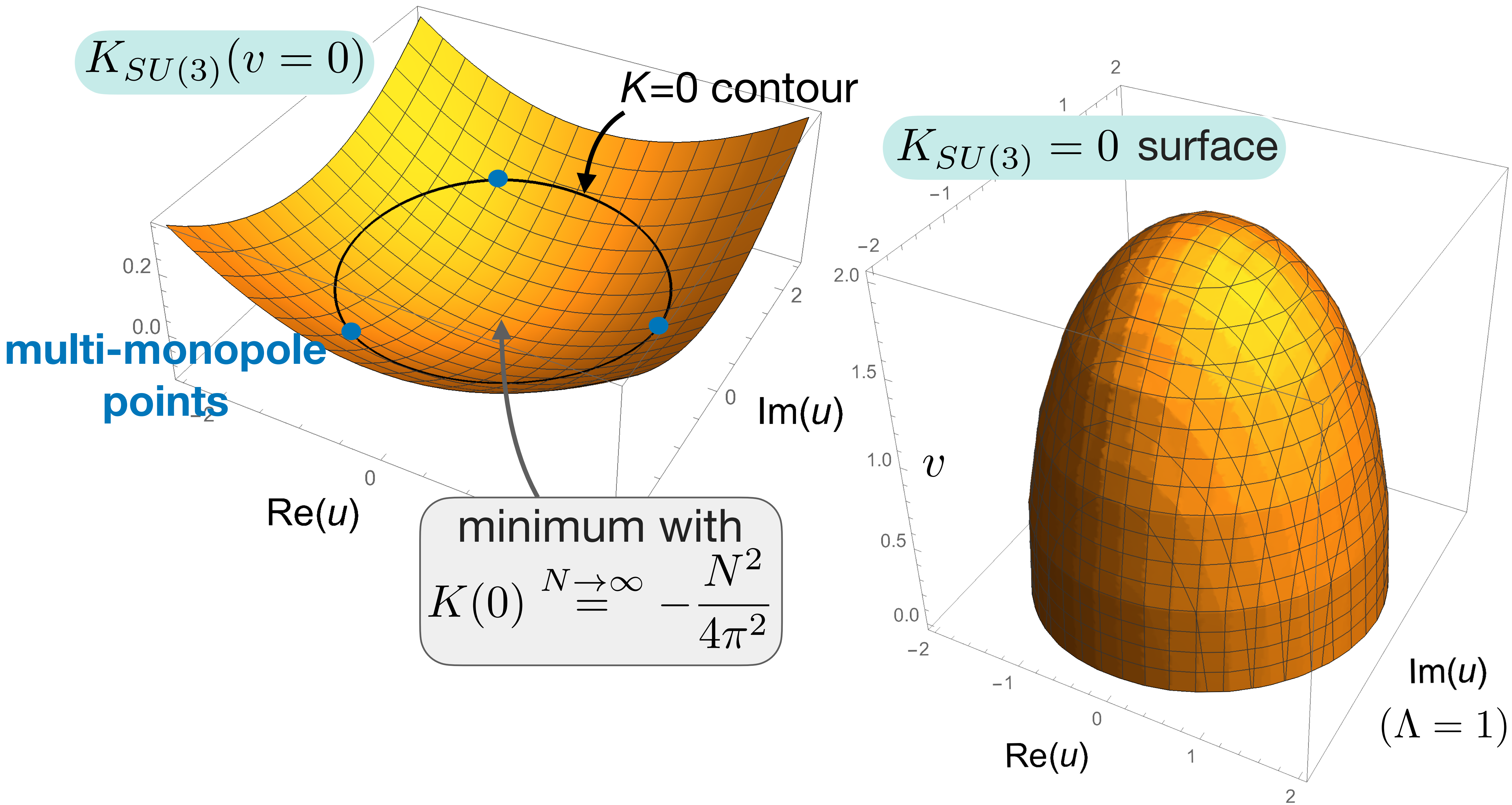
$$a_m, a_{Dm} \sim \oint_{A_m, B_m} \lambda \sim \begin{cases} {}_2F_1(u) & SU(2) \\ F_4(u, v) & SU(3) \end{cases}$$

- Result: we obtain a simple, **all-orders** Taylor series expansion for a_m, a_{Dm} around $u_n \sim 0$, given in terms of

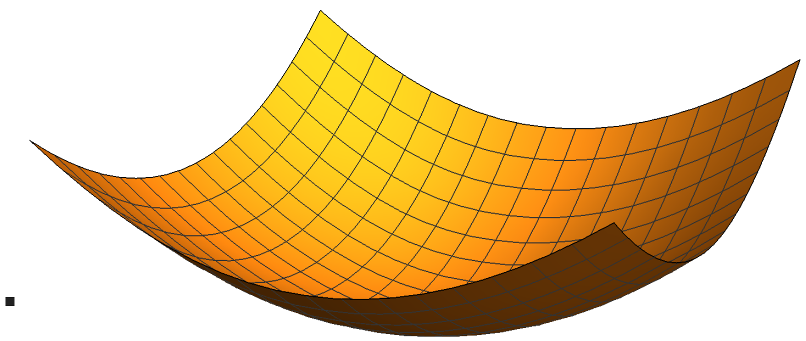
$$\int_0^{\xi} \lambda = \sum_{\{\ell_n\}=0}^{\infty} \underbrace{V_{\{\ell_n\}}(\xi)}_{\text{simple functions}} \, u_1^{\ell_1} \cdots u_{N-1}^{\ell_{N-1}} \qquad \xi = 2N \text{ root of unity}$$

Features of the *SU*(*N*) Kähler potential

- We have analytic and numerical evidence that the *SU*(2) picture remains qualitatively correct.



The physics of the small *M* vacuum



- $V_{\cancel{\mathrm{SUSY}}}$ only gives mass to the scalars.

- This vacuum is in the **Coulomb phase**, with unbroken $SU(2)_R \times \mathbb{Z}_{4N}$ chiral symmetry.

- This analysis is reliable at small M, but would be a surprising prediction for the phase of adjoint QCD!

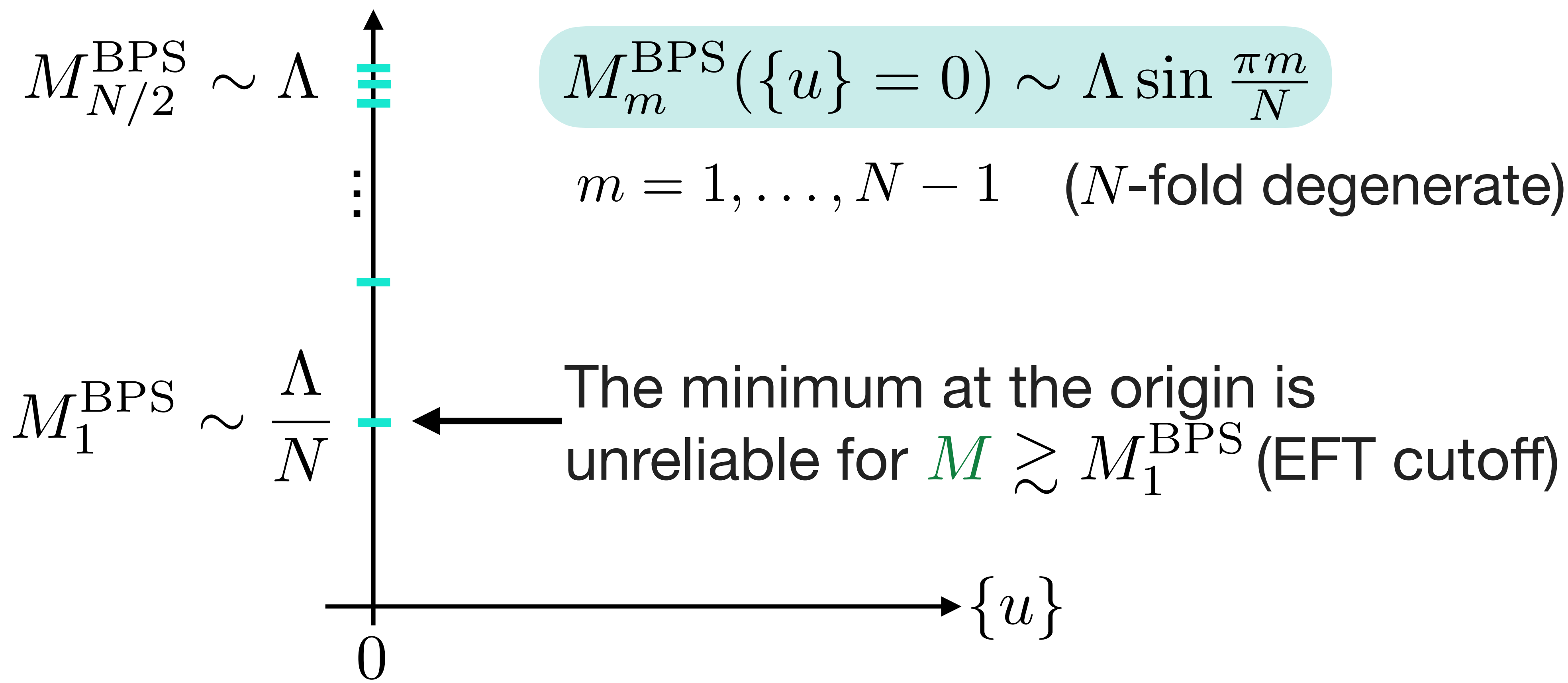
- *If* adjoint QCD confines and breaks chiral symmetry, the true vacuum requires at least one **phase transition** from this small M vacuum.

Interlude: Dialing up M

and the physics of the multi-monopole points

The BPS spectrum in the $SU(N)$ strong coupling chamber

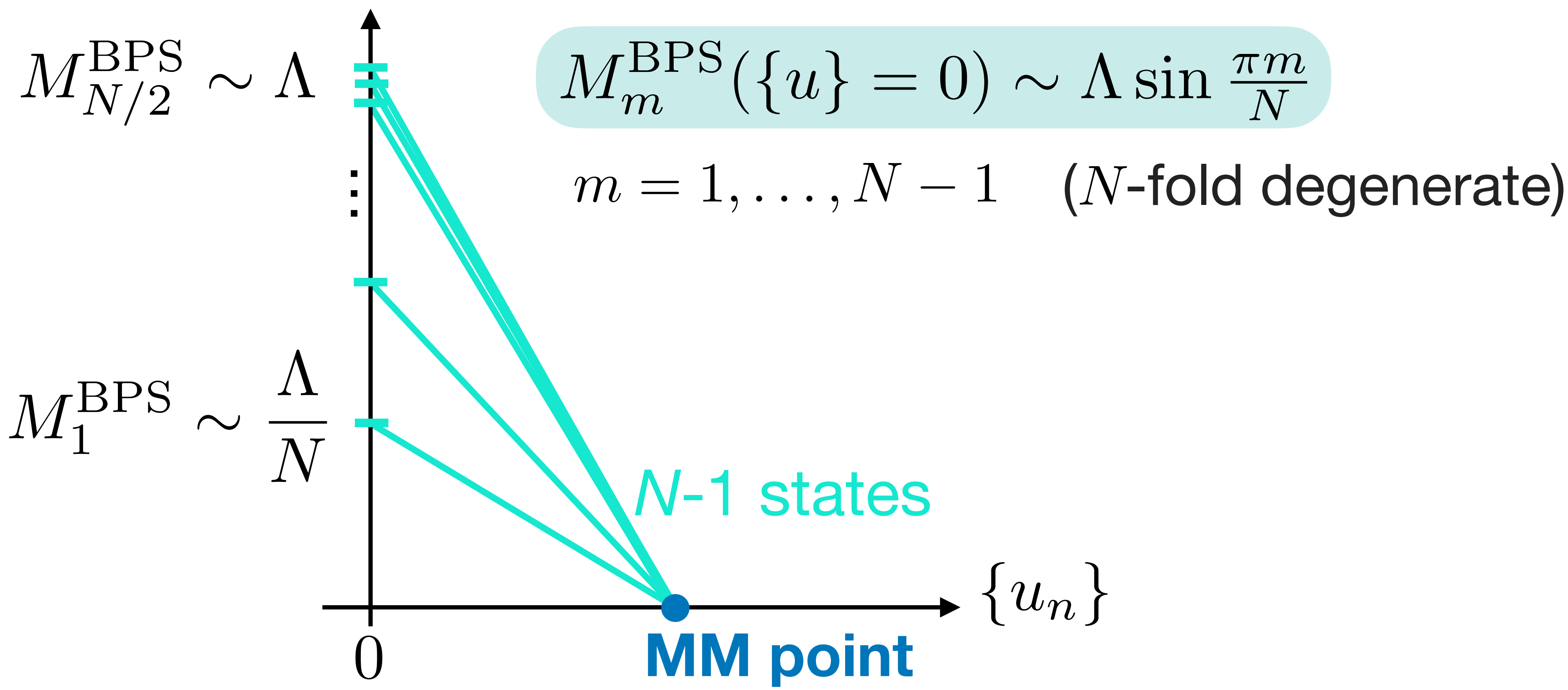
- There are N towers of N-1 mutually local dyons rotated by $\mathbb{Z}_N \subset \mathbb{Z}_{4N}$



[Lerche '00] [Alim, Cecotti, Cordova, Espahbodi, Rastogi, Vafa '11]
[Chuang, Diaconescu, Manschot, Moore, Soibelman '13] …

The multi-monopole points as a candidate dual

- One state from each tower becomes massless at each of the *N* **multi-monopole** points.



- **Strategy:** use the EFT of the light monopoles as a dual description that captures the physics of the massive BPS states.
  - ✳ Focus on 1 MM point—the rest are related by the $\mathbb{Z}_N$.

Exploring the dual as a function of *M*

- The dual description at the multi-monopole points is a $U(1)_D^{N-1}$ abelian Higgs model, with:

$$\text{vector multiplets } (a_{Dm}, \ldots) \;\leftrightarrow\; a_{Dm} = 0 \text{ at the MM point}$$

$$\text{hypermultiplets } (h_m^{i=1,2}, \ldots) \;\leftrightarrow\; \text{monopoles}$$

- Add $V_{\cancel{\mathrm{SUSY}}} = M^2 \mathcal{T}$ to the effective action, where $\mathcal{T}$ matches onto *K* upon integrating out the monopoles.

Key features of the scalar potential $V = V_{\text{SUSY}} + V_{\cancel{\text{SUSY}}}$

$$V_{\text{SUSY}} \sim \sum_m |a_{Dm}|^2 (\bar{h}^i h_i)_m + \sum_{m,n} (t^{-1})_{mn} \left[(\bar{h}^i h_j)_m (\bar{h}^j h_i)_n - \frac{1}{2} (\bar{h}^i h_i)_m (\bar{h}^j h_j)_n \right]$$

$$V_{\cancel{\text{SUSY}}} \sim M^2 \left(N \sum_m \mathrm{Im}(a_{Dm}) \sin \frac{\pi m}{N} + \sum_{m,n} t_{mn} a_{Dm} \bar{a}_{Dn} - \sum_m (\bar{h}^i h_i)_m + \dots \right)$$

The matrix of effective gauge couplings $\mathrm{Im}\tau_D = t$

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$$t_{mn}(\mu) \sim -\delta_{mn} \ln \tfrac{\mu}{\Lambda} + \ln \Lambda_{mn} + \ldots$$

couple the $U(1)_D$'s

$$\Lambda_{mm} = 16 N \sin^3 \tfrac{\pi m}{N} \,, \quad \Lambda_{m,n\neq m} = \frac{\sin^2 \frac{\pi(m+n)}{2N}}{\sin^2 \frac{\pi(m-n)}{2N}}$$

The detailed structure of the threshold corrections Λ_{mn} leads to interesting predictions for the dynamics.

* **We computed these** by analyzing the Seiberg-Witten periods near the MM point. [D'Hoker, Dumitrescu, Gerchkovitz, **EN** '20]

analysis initiated by [Douglas, Shenker '95], [D'Hoker, Phong '97]
also see [Edelstein, Mas '99],[Edelstein, Gomez-Reino, Marino '00],[Bonelli, Grassi, Tanzini '17]

Key features of the scalar potential

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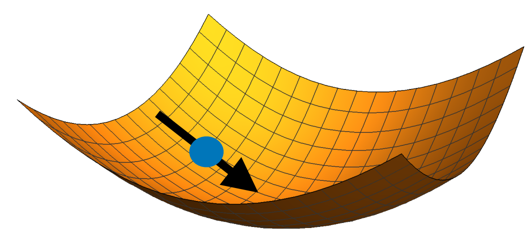
$$V_{\cancel{\text{SUSY}}} \sim M^2 \left(N \sum_m \mathrm{Im}(a_{Dm}) \sin \frac{\pi m}{N} + \sum_{m,n} t_{mn} a_{Dm} \bar{a}_{Dn} - \sum_m (\bar{h}^i h_i)_m + \ldots \right)$$

* Tadpole from matching onto *K*



* Positive a_{Dm} masses
* Tachyonic monopole masses $\Rightarrow$ *h*'s want to condense
* *D*-terms $\leftrightarrow$ ferromagnetic spin chain: $s^a_m = \bar{h}_m \sigma^a h_m$,

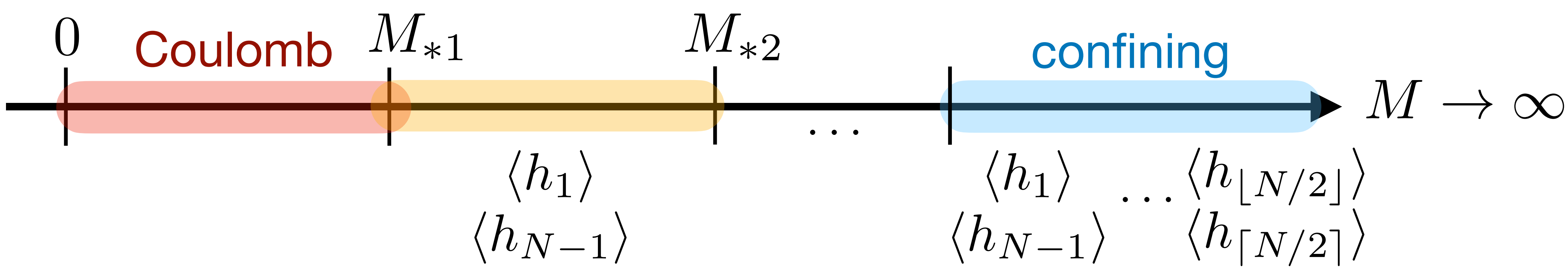
$$\sim \sum_{m<n} (t^{-1})_{mn} (\vec{s}_m \cdot \vec{s}_n) + \ldots \text{ where } (t^{-1})_{m,n\neq m} < 0 \;\Rightarrow\; \vec{s}_m \sim \vec{s}_n$$

III) A cascade of phase transitions

and the $\mathbb{CP}^1$ *at large M*

Result: a cascade of 1st order phase transitions

[D'Hoker, Dumitrescu, Gerchkovitz, **EN** '22]



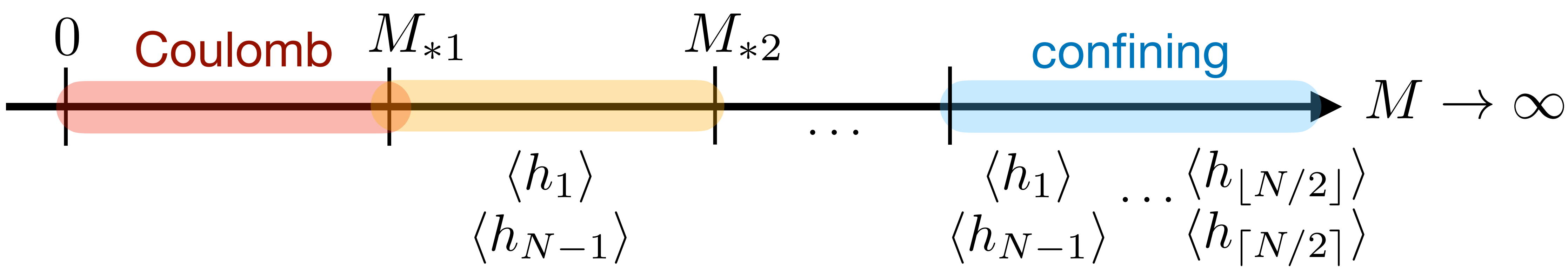
At small *M*: all $\langle h_m \rangle = 0$. Coulomb phase consistent with previous small-*M* analysis.

Increasing *M*: $\langle h_m \rangle, \langle h_{N-m} \rangle$ activate pair-wise (*C*-preserving) in a series of **1st order** phase transitions.

- $\ast$ The transitions approximately track: $M_{*m} \lesssim M_m^{\mathrm{BPS}} \sim \Lambda \sin \frac{\pi m}{N}$
- $\ast$ A **robust** feature of every higher phase: $SU(2)_R \to U(1)^*$ ($h_m^i \sim h_n^i$ due to *D*-terms) *consistent with VW

At large-*M*: all monopoles condense.

The large-*M* phase

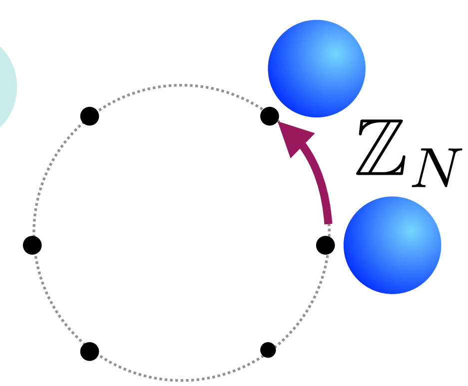


- $\langle h_m \rangle \neq 0 \;\leftrightarrow\; U(1)_{Dm}$ higgsed.

  All monopoles condense $\Rightarrow \mathbb{Z}_N^{(1)}$ unbroken → **confinement**

- All fluctuations get a mass except the 2 NGB from the $SU(2)_R$ chiral symmetry breaking (due to $t_{m,n\neq m}$) → a $\mathbb{CP}^1$

- 1 copy at each MM point → *N* $\mathbb{CP}^1$'s



Summary: the predicted phase of adjoint QCD from $\mathcal{N} = 2$

property of the large-*M* phase	**comes from?**
confinement	tachyonic *h*-mass in $\mathcal{T}$
$SU(N_f = 2) \to U(1)$*♜	ferromagnetic structure of *t* + SUSY D-terms
all fluctuations massive except 2 massless NGB	off-diagonal structure of *t*
$\mathbb{Z}_{4N} \to \mathbb{Z}_4$	symmetry relating MM points
CT unbroken*♜	tadpole in *K* + symmetry of MM points
C unbroken	dynamics
N-scaling of $\langle \lambda\lambda \rangle \sim N$	structure of *t* + SUSY relating $\langle \lambda\lambda \rangle \sim \langle hh \rangle$
N-scaling of vacuum energy $V \sim -N^2$	off-diagonal structure of *t*

match the $\mathbb{CP}^1$ phase

* consistent with VW for adjoint QCD

♜ true in *all* phases in the cascade (except first)

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Thank you!

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