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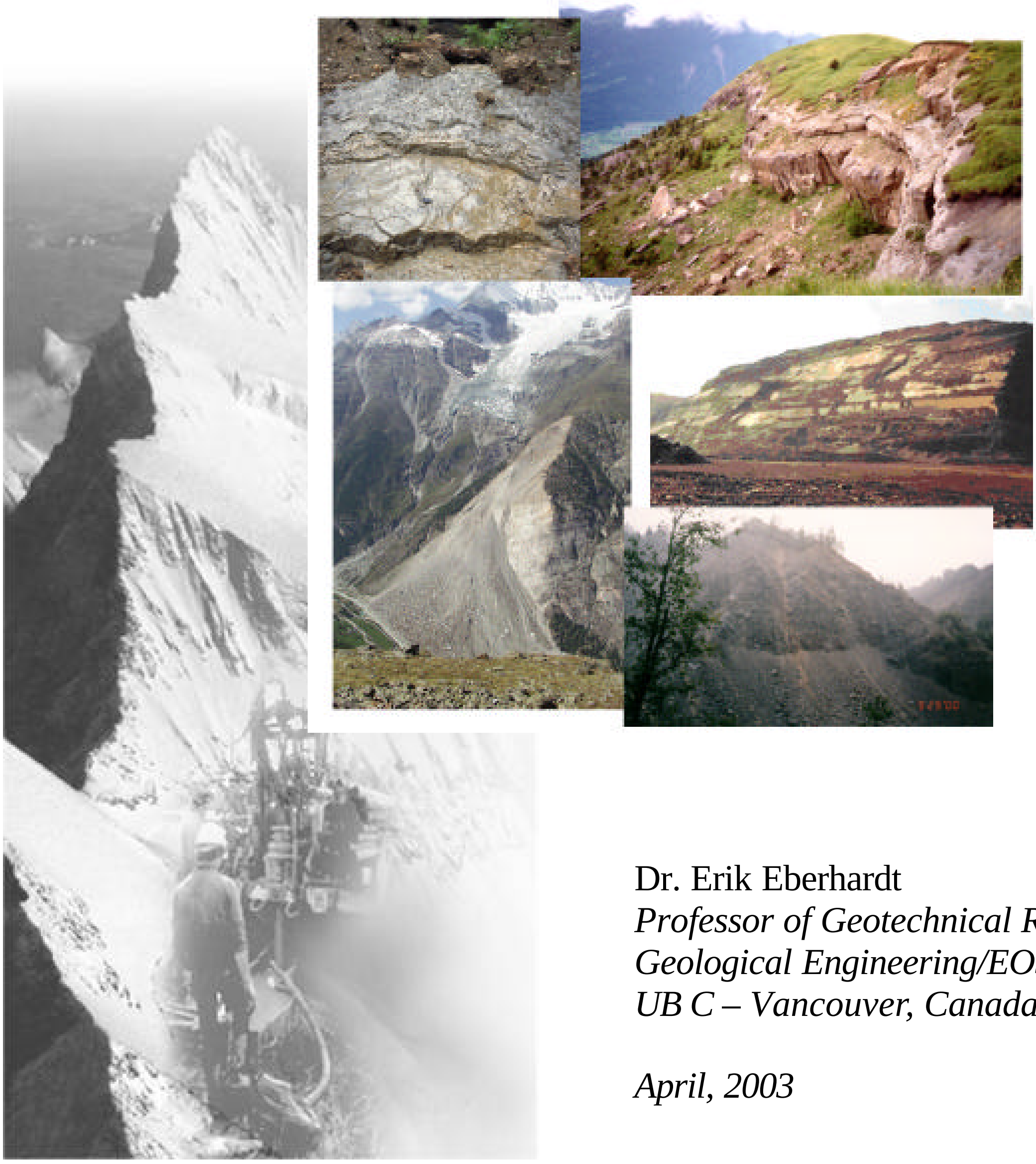
Rock Slope Stability Analysis – Utilization of Advanced Numerical Techniques



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Abstract

Despite improvements in recognition, prediction and mitigative measures, landslides still exact a heavy social, economic and environmental toll in mountainous regions. This is partly due to the complexity of the processes driving slope failures and our inadequate knowledge of the underlying mechanisms. Ever increasingly, experts are called upon to analyse and predict the stability of a given slope, assessing its risk, potential failure mechanisms and velocities, areas endangered, and possible remedial measures.

These lecture notes introduces the field of rock slope stability analysis and the purpose such analyses serve in the investigation of potential slope failure mechanisms. Advancements in and the evolution of computer based slope analysis techniques are discussed, first with respect to commonly applied conventional methods. The determination of kinematic feasiblity for several common modes of failure are presented in addition to the corresponding analytical and limit equilibrium solutions for factors of safety against slope failure.

The second part introduces numerical modelling methods and their application to rock slope stability analysis. The discussion concentrates on advancements in and the use of continuum and discontinuum numerical modelling codes. The incorporation and influence of pore pressures and dynamic loading are also presented. The steps taken in performing a numerical analysis are reviewed, with emphasis being placed on the importance of good modelling practice.

When properly applied and constrained, numerical modelling can significantly assist in the design process by providing key insights into potential stability problems and failure mechanisms. Yet it must also be emphasized that numerical modelling is a tool and not a substitute for critical thinking and judgement. As such, numerical modelling is most effective when applied by an experienced and cautious user.

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1. Introduction

Rock slope stability analyses are routinely performed and directed towards assessing the safe and functional design of excavated slopes (e.g. open pit mining, road cuts, etc.) and/or the equilibrium conditions of natural slopes. The analysis technique chosen depends on both site conditions and the potential mode of failure, with careful consideration being given to the varying strengths, weaknesses and limitations inherent in each methodology. In general, the primary objectives of rock slope stability analyses are:

- to determine the rock slope stability conditions;
- to investigate potential failure mechanisms;
- to determine the slopes sensitivity/susceptibility to different triggering mechanisms;
- to test and compare different support and stabilization options; and
- to design optimal excavated slopes in terms of safety, reliability and economics.

A site investigation study should precede any stability study and includes elements of geological and discontinuity mapping to provide the necessary input data for the stability analysis. The collection of data ideally involves rock mass characterization and the sampling of rock materials for laboratory analysis (i.e. strength and constitutive behaviour determination), field observations and *in situ* measurements. *In situ* monitoring of spatial and temporal variations in pore pressures, slope displacements, stresses and subsurface rock mass deformations, provide valuable data for constraining and validating the stability analysis undertaken.

In order to properly conduct such investigations, and to analyse and evaluate the potential hazard relating to an unstable rock slope, it is essential to understand the processes and mechanisms driving the instability. Landslide movements may be considered as falls, topples, slides, spreads or flows (Cruden & Varnes 1996), and in some cases involve different combinations of several failure modes (referred to as composite slides). These mechanisms are often complex and act at depth, making the investigation and characterization of contributing factors difficult. This poses a problem in the analysis stage of the investigation as uncertainties arise concerning the analysis technique to be employed and what input data is required (Fig. 1).

Today, a vast range of slope stability analysis tools exist for both rock and mixed rock-soil slopes; these range from simple infinite slope and planar failure limit equilibrium techniques to sophisticated coupled finite-/distinct-element codes. It is important to remember that it has only been 25 years since most rock slope stability calculations were performed either graphically or using a hand-held calculator, one exception being advanced analyses involving critical surface searching routines performed on a mainframe computer and Fortran cards. The great majority of early stability analysis programs were in-house with very little software being available commercially. Today, every engineer and geologist has access to a personal computer that can undertake with relative ease complex numerical analyses of rock slopes.

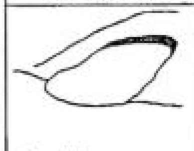
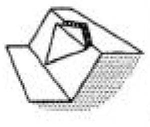
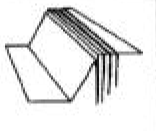
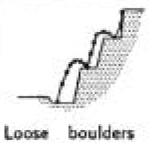
STRUCTURE	TYPICAL PROBLEMS	CRITICAL PARAMETERS	ANALYSIS METHODS	ACCEPTABILITY CRITERIA
 Landslides.	Complex failure along a circular or near circular failure surface involving sliding on faults and other structural features as well as failure of intact materials.	• Presence of regional faults. • Shear strength of materials along failure surface. • Groundwater distribution in slope, particularly in response to rainfall or to submergence of slope toe. • Potential earthquake loading.	Limit equilibrium methods which allow for non-circular failure surfaces can be used to estimate changes in factor of safety as a result of drainage or slope profile changes. Numerical methods such as finite element or discrete element analysis can be used to investigate failure mechanisms and history of slope displacement.	Absolute value of factor of safety has little meaning but rate of change of factor of safety can be used to judge effectiveness of remedial measures. Long term monitoring of surface and subsurface displacements in slope is the only practical means of evaluating slope behaviour and effectiveness of remedial action.
 Soil or heavily jointed rock slopes.	Circular failure along a spoon-shaped surface through soil or heavily jointed rock masses.	• Height and angle of slope face. • Shear strength of materials along failure surface. • Groundwater distribution in slope. • Potential surcharge or earthquake loading.	Two-dimensional limit equilibrium methods which include automatic searching for the critical failure surface are used for parametric studies of factor of safety. Probability analyses, three-dimensional limit equilibrium analyses or numerical stress analyses are occasionally used to investigate unusual slope problems.	Factor of safety > 1.3 for "temporary" slopes with minimal risk of damage. Factor of safety > 1.5 for "permanent" slopes with significant risk of damage. Where displacements are critical, numerical analyses of slope deformation may be required and higher factors of safety will generally apply in these cases.
 Jointed rock slopes.	Planar or wedge sliding on one structural feature or along the line of intersection of two structural features.	• Slope height, angle and orientation. • Dip and strike of structural features. • Groundwater distribution in slope. • Potential earthquake loading. • Sequence of excavation and support installation.	Limit equilibrium analyses which determine three-dimensional sliding modes are used for parametric studies on factor of safety. Failure probability analyses, based upon distribution of structural orientations and shear strengths, are useful for some applications.	Factor of safety > 1.3 for "temporary" slopes with minimal risk of damage. Factor of safety > 1.5 for "permanent" slopes with significant risk of damage. Probability of failure of 10 to 15% may be acceptable for open pit mine slopes where cost of clean up is less than cost of stabilization.
 Vertically jointed rock slopes .	Toppling of columns separated from the rock mass by steeply dipping structural features which are parallel or nearly parallel to the slope face.	• Slope height, angle and orientation. • Dip and strike of structural features. • Groundwater distribution in slope. • Potential earthquake loading.	Crude limit equilibrium analyses of simplified block models are useful for estimating potential for toppling and sliding. Discrete element models of simplified slope geometry can be used for exploring toppling failure mechanisms.	No generally acceptable criterion for toppling failure is available although potential for toppling is usually obvious. Monitoring of slope displacements is the only practical means of determining slope behaviour and effectiveness of remedial measures.
 Loose boulders on rock slopes.	Sliding, rolling, falling and bouncing of loose rocks and boulders on the slope.	• Geometry of slope. • Presence of loose boulders. • Coefficients of restitution of materials forming slope. • Presence of structures to arrest falling and bouncing rocks.	Calculation of trajectories of falling or bouncing rocks based upon velocity changes at each impact is generally adequate. Monte Carlo analyses of many trajectories based upon variation of slope geometry and surface properties give useful information on distribution of fallen rocks.	Location of fallen rock or distribution of a large number of fallen rocks will give an indication of the magnitude of the potential rockfall problem and of the effectiveness of remedial measures such as draped mesh, catch fences and ditches at the toe of the slope.

Figure 1. Typical problems, critical parameters, methods of analysis and acceptability criteria for rock slopes (from Hoek 1991).

Given the wide scope of numerical applications available today, it has become essential for the practitioner to fully understand the varying strengths and limitations inherent in each of the different methodologies. For example, limit equilibrium methods still remain the most commonly adopted solution method in rock slope engineering, even though most failures involve complex internal deformation and fracturing which bears little resemblance to the 2-D rigid block assumptions required by most limit equilibrium back-analyses. Initiation or trigger mechanisms may involve sliding movements which can be analyzed as a limit equilibrium problem, but this is followed by or preceded by creep, progressive deformation and extensive internal disruption of the slope mass. The factors initiating eventual failure may be complex and not easily allowed for in simple static analysis. Not withstanding the above comments, limit equilibrium analyses may be highly relevant to simple block failure along discontinuities. It thus follows that where applicable, limit equilibrium techniques should be used in conjunction with numerical modelling to maximize the advantages of both.

In this sense the practitioner today, if he is to demonstrate due-diligence, must show he has used both all the tools at his disposal and, more importantly, the correct tools. An argument for the use of all relevant available slope analysis techniques in a design or back-analysis is emphasized by the observation of Chen (2000),

> *"In the early days, slope failure was always written off as an act of God. Today, attorneys can always find someone to blame and someone to pay for the damage – especially when the damage involves loss of life or property".*

The design of a slope using a limit equilibrium analysis alone may be completely inadequate if the slope fails by complex mechanisms (e.g. progressive creep, internal deformation and brittle fracture, liquefaction of weaker soil layers, etc.). Furthermore, within slope engineering design and analysis, increased use is being made of hazard appraisal and risk assessment concepts. A risk assessment must address both the consequence of slope failure and the hazard or probability of failure; both require an understanding of the failure mechanism in order that the spatial and temporal probabilities can be addressed.

In the following sections, rock slope stability analysis techniques will be reviewed concentrating on the development of numerical modelling methods. A review of conventional methods of stability analysis will precede these sections to highlight recent developments in limit-equilibrium based computer programs designed to enhance visualization of simple slope stability problems.

2. Conventional Methods of Rock Slope Analysis

Conventional methods of rock slope analysis can be generally broken down into kinematic and limit equilibrium techniques. In addition, analytical computer-based methods have been developed to analyze discrete rock block falls (commonly referred to as rockfall simulators). Table 1 provides a summary of those techniques that are routinely applied together with their inherent advantages and limitations.

Table 1. Conventional methods of rock slope analysis (after Coggan *et al.* 1998).

Analysis Method	Critical Parameters	Advantages	Limitations
Kinematic (using stereographic interpretation)	Critical slope and discontinuity geometry; representative shear strength characteristics.	Relatively simple to use; give initial indication of failure potential; may allow identification and analysis of critical key-blocks using block theory; links are possible with limit equilibrium methods; can be combined with statistical techniques to indicate probability of failure.	Only really suitable for preliminary design or design of non-critical slopes; critical discontinuities must be ascertained; must be used with representative discontinuity/joint shear strength data; primarily evaluates critical orientations, neglecting other important joint properties.
Limit Equilibrium	Representative geometry and material characteristics; soil or rock mass shear strength parameters (cohesion and friction); discontinuity shear strength characteristics; groundwater conditions; support and reinforcement characteristics.	Wide variety of commercially available software for different failure modes (planar, wedge, toppling, etc.); can analyse factor of safety sensitivity to changes in slope geometry and material properties; more advanced codes allow for multiple materials, 3-D, reinforcement and/or groundwater profiles.	Mostly deterministic producing single factor of safety (but increased use of probabilistic analysis); factor of safety gives no indication of instability mechanisms; numerous techniques available all with varying assumptions; strains and intact failure not considered; probabilistic analysis requires well-defined input data to allow meaningful evaluation.
Physical Modelling	Representative material characteristics; appropriate scaling factors.	Mechanisms clearly portrayed and results of analysis are a useful constraint for numerical modelling; centrifuge models able to investigate effects of time on failure mechanisms.	Simplistic groundwater simulation especially in rock; techniques do not allow for the effects of scale and *in situ* stress; centrifuges can be expensive.
Rockfall Simulators	Slope geometry; rock block sizes and shapes; coefficient of restitution.	Practical tool for siting structures; can utilize probabilistic analysis; 2-D and 3-D codes available.	Limited experience in use relative to empirical design charts.

2.1 Kinematic analysis

Kinematic methods concentrate on the feasibility of translational failures due to the formation of "daylighting" wedges or planes. As such, these methods rely on the detailed evaluation of rock mass structure and the geometry of existing discontinuity sets that may contribute to block instability. This assessment may be carried out by means of stereonet plots and/or specialized computer codes which focus on planar and wedge formation. For example, the program DIPS (Rocscience 2001a) allows for the visualization and determination of the kinematic feasibility of rock slopes using friction cones, daylight and toppling envelopes, in addition to graphical and statistical analysis of the discontinuity properties (Fig. 2).

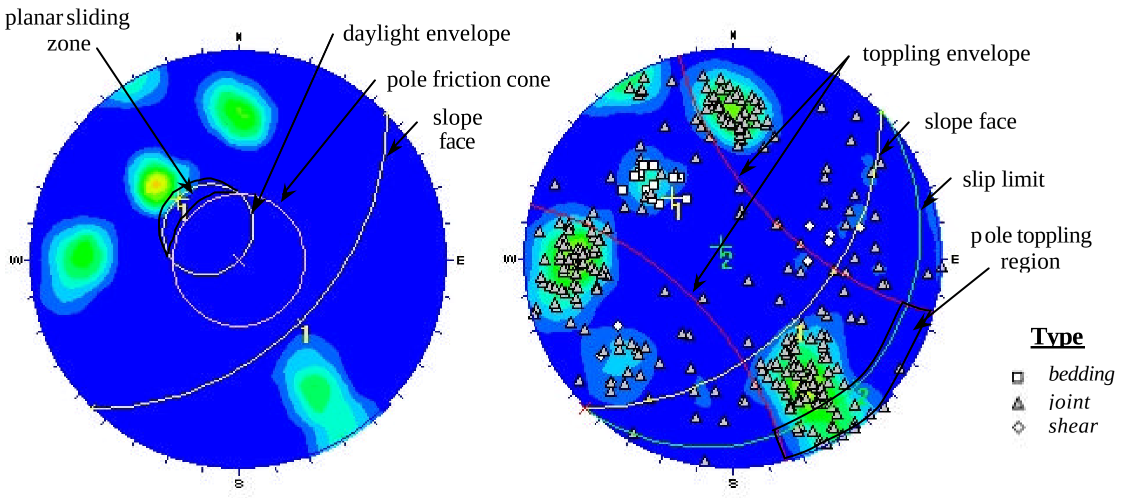


Figure 2. Planar (LEFT) and toppling (RIGHT) kinematic feasibility and stability analyses using stereographic constructions.

It is essential that the user is aware that such approaches only recognize potential sliding failures involving single discontinuities or discontinuity intersections. They do not cater for failure involving multiple joints/joint sets or internal deformation and fracture. Discontinuity data and joint set intersections defined in DIPS can though, be imported into companion limit equilibrium codes (e.g. SWEDGE - Rocscience 2001b) to assess the factor of safety against wedge failure. These programs often incorporate probabilistic tools, in which variations in joint set properties and added support measures can be assessed for their influence on the factor of safety. Computer-based wedge feasibility analysis can also be performed based on key block theory (Goodman & Shi 1985). The stability of such keyblocks is then assessed using limit equilibrium methods such as in the SAFEX program (Windsor & Thompson 1993) and KBSLOPE (Pantechnica 2001).

2.2 Limit equilibrium analysis

Limit equilibrium techniques are routinely used in the analysis of landslides where translational or rotational movements occur on distinct failure surfaces. Analyses are undertaken to provide either a factor of safety or, through back-analysis, a range of shear strength parameters at failure. In general, these methods are the most commonly adopted solution method in rock slope engineering, even though many failures involve complex internal deformation and fracturing which bears little resemblance to the 2-D rigid block assumptions required by limit equilibrium analyses. However, limit equilibrium analyses may be highly relevant to simple block failure along discontinuities or rock slopes that are heavily fractured or weathered (i.e. behaving like a soil continuum).

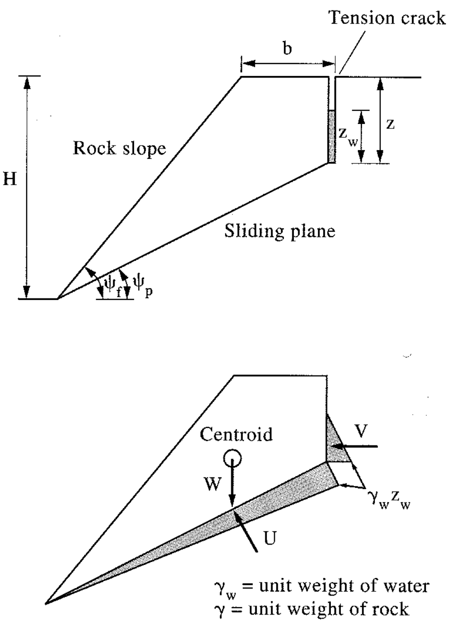
All limit equilibrium techniques share a common approach based on a comparison of resisting forces/moments mobilized and the disturbing forces/moments. Methods may vary, however, with respect to the slope failure mechanism in question (e.g. translational or rotational sliding), and the assumptions adopted in order to achieve a determinate solution. Considerable advances in commercially available limit equilibrium computer codes have taken place in recent years. These include:

- Integration of 2-D limit equilibrium codes with finite-element groundwater flow and stress analyses (e.g. Geo-Slope's SIGMA/W, SEEP/W and SLOPE/W - Geo-Slope 2000).
- Development of 3-D limit equilibrium methods (e.g. Hungr *et al.* 1989; Lam & Fredlund 1993).
- Development of probabilistic limit equilibrium techniques (e.g. SWEDGE - Rocscience 2001b; ROCPLANE – Rocscience 2001c).
- Ability to allow for varied support and reinforcement.
- Incorporation of unsaturated soil shear strength criteria.
- Greatly improved visualisation, and pre- and post-processing graphics.

2.2.1 Translational analysis

Limit equilibrium solutions for planar and wedge failures have been widely used to assess discontinuity-controlled rock slope instabilities. These techniques, largely based on solutions introduced by Hoek & Bray (1991), assume translational sliding of a rigid body along a plane or along the intersection of two planes in the case of a wedge. Since the sliding block does not undergo any rigid body rotations, all forces pass through the centroid of the block. Furthermore, as in all limit equilibrium solutions, it is assumed that all points along the sliding plane(s) are on the verge of failure.

These assumptions make the problem statically determinate, permitting the simple calculation of the ratio of resisting forces and driving forces (i.e. factor of safety). Resisting forces are provided by the shear strength of the sliding surface (e.g. cohesion and friction), and driving forces generally consist of the down-slope weight component of the sliding block and water pressures along the boundaries of the block. These forces, and their corresponding factor of safety formulation, are illustrated for simple planar and wedge stability problems in Figures 3 and 4, respectively. Adaptations to these solutions include those to account for planes with non-vertical tension cracks and non-horizontal upper slope surfaces (Sharma *et al.* 1995), stepped sliding surfaces (Kovari & Fritz 1984) and wedges with cohesive strength and water pressures (Hoek & Bray 1991).

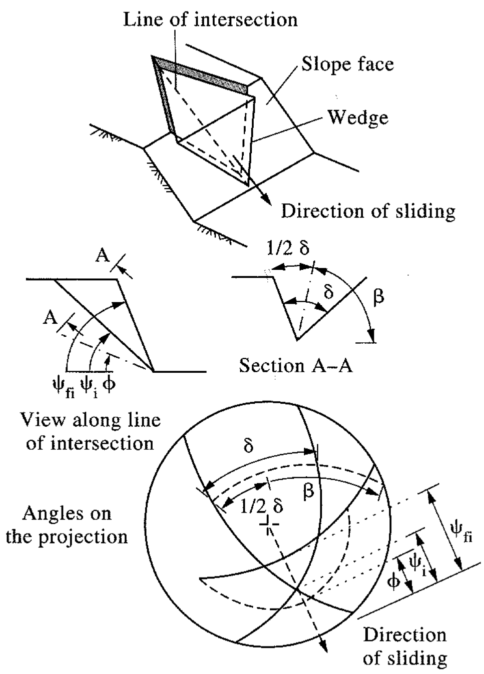


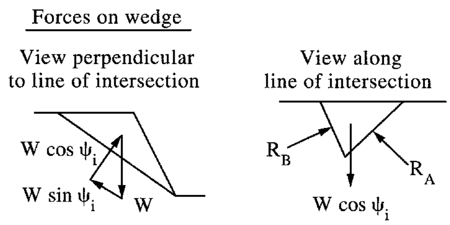
$$SF = \frac{c'(H-z)\csc y_p + (W\cos y_p - U - V\sin y_p)\tan f'}{V\cos y_p + W\sin y_p}$$

Where:

SF = safety factor
c' = effective cohesion
f' = effective friction angle
y_p = dip of slide plane
W = weight of block
U = uplift force due to water pressure along slide plane
V = force due to water pressure in tension crack
H = slope height
z = tension crack depth

Figure 3. Limit equilibrium solution for planar failure (after Hudson & Harrison 1997).





$$SF = \frac{(R_A + R_B)\tan f}{W\sin y_i}$$

and $R_A + R_B = \dfrac{W\cos y_i \sin b}{\sin(0.5d)}$

Where: SF = safety factor
f = friction angle
y_i = dip of the line of intersection
W = weight of block
b, d = wedge geometry factors

Figure 4. Limit equilibrium solution for wedge failure under dry conditions and with frictional strength only (after Hudson & Harrison 1997).

Computer programs based on these solutions, such as SWEDGE (Rocscience 2001b), provide a quick and interactive means to evaluate the geometry and stability of surface wedges defined by two intersecting discontinuity planes and a slope surface; similar programs exist for planar analysis (e.g. ROCPLANE - Rocscience 2001c). An added advantage to applying computer-based solutions is that they often incorporate probabilistic tools, in which variations in joint properties and added rock bolt support can be assessed for their influence on the factor of safety (Fig. 5). Similarly, fuzzy logic routines designed to manage uncertainty in the input parameters can be likewise incorporated (Faure & Maiolino 2000).

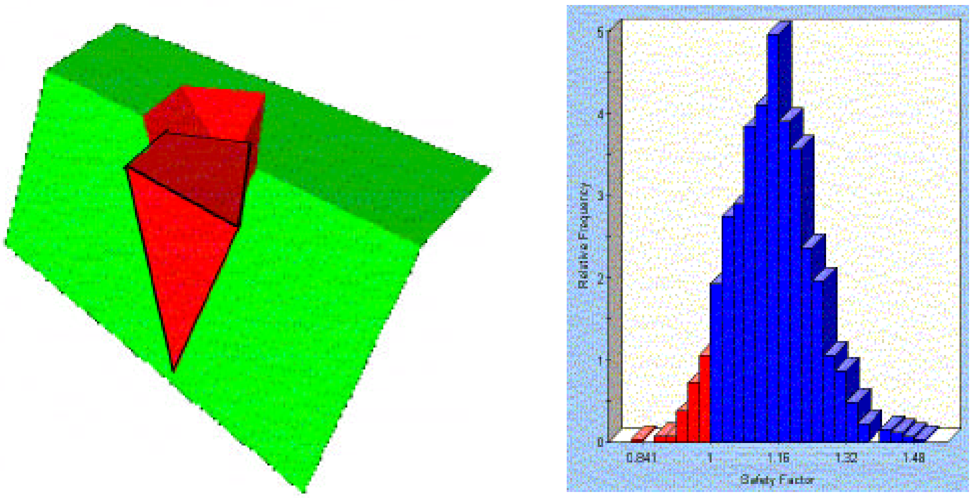


Figure 5. Probabilistic limit equilibrium wedge analysis. Relative frequency refers to the number of valid wedges formed by the Monte Carlo sampling of the input data.

2.2.2 Toppling analysis

Tools also exist for direct toppling modes of failure (similarly, solutions exist for flexural toppling but since these failures involve internal block deformations they are poorly treated using limit equilibrium techniques). Direct toppling occurs when the centre of gravity of a discrete block lies outside the outline of the base of the block, with the result that a critical overturning moment develops. Other considerations include the possibility that the block will slide, or that both sliding and toppling will occur simultaneously (Fig. 6).

Limit equilibrium analysis of toppling failure must therefore consider both the possibility of toppling and/or sliding. Figure 7 shows the acting forces and limit equilibrium conditions for toppling and sliding of a single 2-D block on a stepped base. Solution procedures, such as those outlined by Hoek & Bray (1991), are then extended to consider the equilibrium condition of the overall system of blocks. These typically show a set of sliding blocks in the toe region, stable blocks at the top, and a set of toppling blocks in-between. These equations are easily programmed and as such, provide quick computer-based calculations and visualization of sliding and toppling potential (Fig. 8).

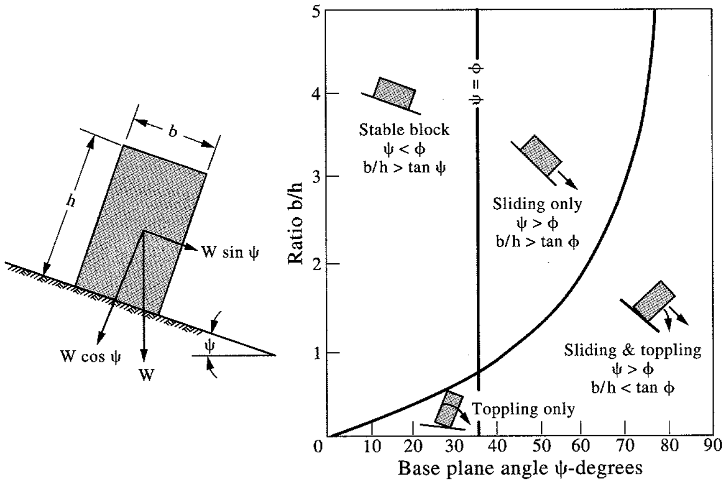
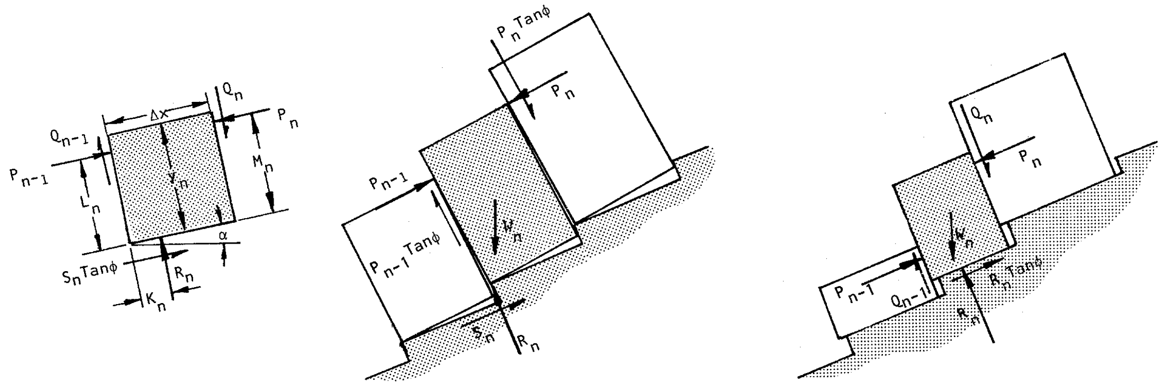


Figure 6. Sliding and toppling instability of a block on an inclined plane (from Hoek & Bray 1991).



$$P_{n-1} = \frac{P_n\,(M_n - \Delta x \tan f) + (W_n/2)(y_n \sin a - \Delta x \cos a)}{L_n} \qquad P_{n-1} = P_n - \frac{W_n\,(\tan f \cos a - \sin a)}{1 - \tan^2 f}$$

Figure 7. Limit equilibrium conditions for toppling and sliding, with input variables illustrated in the corresponding diagrams (after Hoek & Bray 1991).

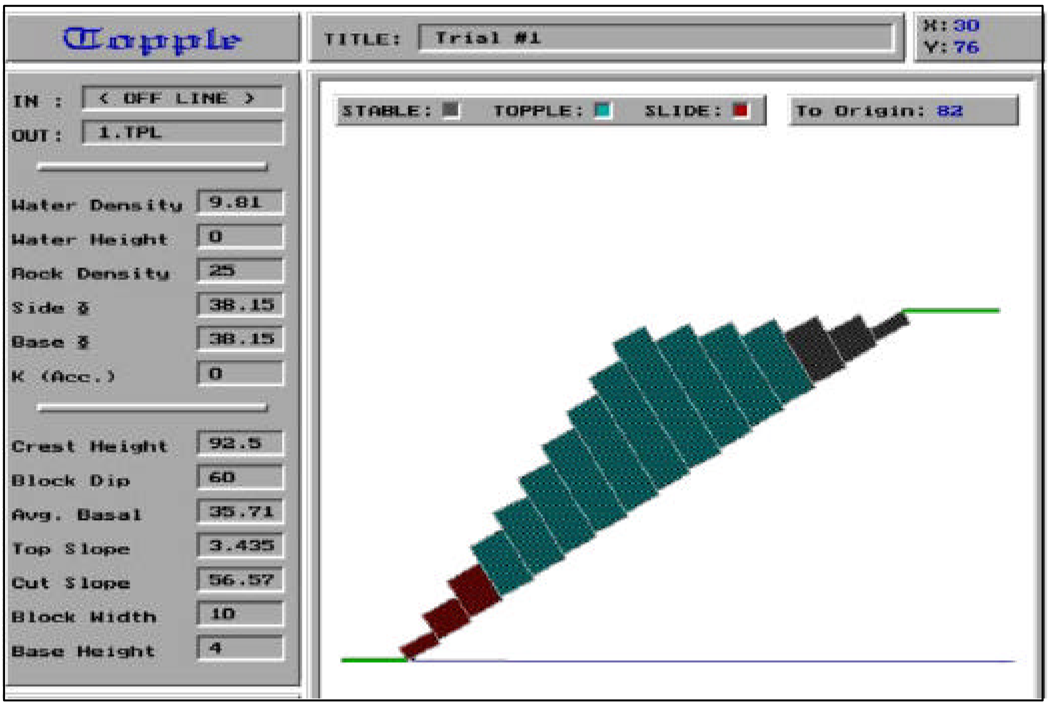


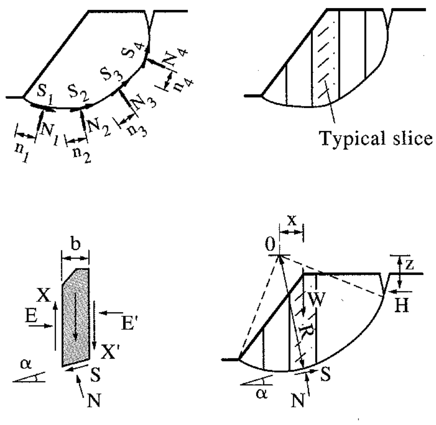
Figure 8. Computer-based limit equilibrium analysis of toppling and sliding potential in rock slopes.

2.2.3 Rotational analysis

For very weak rock, where the intact material strength is of the same magnitude as the induced stresses, the structural geology may not control stability and failure modes such as those observed in soils may occur. These are generally referred to as circular failures, rotational failures or curvilinear slips.

In analyzing the potential for failure, consideration must be given to the location of the critical slip surface and the determination of the factor of safety along it. Iterative procedures are used, each involving the selection of a potentially unstable slide mass, the subdivision of the mass into slices (i.e. *Method of Slices*), and consideration of the force and moment equilibrium acting on each slice (Fig.9).

Several methods exist (e.g. Ordinary, Bishop Simplified, Janbu, etc.), with each differing in terms of the underlying assumptions taken to make the problem determinate. These methods are summarized in Table 2.



$$SF = \frac{\sum SF\,(S\sec a)}{\sum (W\sin a) + Hz/R}$$

Where: SF = safety factor
S = effective shear strength (i.e. $S/\Delta b = c' + \sigma_n \tan\phi'$)
a = dip of base of slice
W = weight of slice
H = hydrostatic thrust from tension crack
z = depth of tension crack (relative to O)
R = length of moment arm.

Figure 9. Limit equilibrium solution for circular failure (after Hudson & Harrison 1997).

Table 2. Characteristics and assumptions adopted in commonly used methods of limit equilibrium analysis for rotational slope failures (from Duncan 1996).

METHOD	LIMITATIONS, ASSUMPTIONS, AND EQUILIBRIUM CONDITIONS SATISFIED
Ordinary method of slices (Fellenius 1927)	Factors of safety low—very inaccurate for flat slopes with high pore pressures; only for circular slip surfaces; assumes that normal force on the base of each slice is $W \cos \alpha$; one equation (moment equilibrium of entire mass), one unknown (factor of safety)
Bishop's modified method (Bishop 1955)	Accurate method; only for circular slip surfaces; satisfies vertical equilibrium and overall moment equilibrium; assumes side forces on slices are horizontal; $N+1$ equations and unknowns
Force equilibrium methods	Satisfy force equilibrium; applicable to any shape of slip surface; assume side force inclinations, which may be the same for all slices or may vary from slice to slice; small side force inclinations result in values of F less than calculated using methods that satisfy all conditions of equilibrium; large inclinations result in values of F higher than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns
Janbu's simplified method (Janbu 1968)	Force equilibrium method; applicable to any shape of slip surface; assumes side forces are horizontal (same for all slices); factors of safety are usually considerably lower than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns
Modified Swedish method (U.S. Army Corps of Engineers 1970)	Force equilibrium method, applicable to any shape of slip surface; assumes side force inclinations are equal to the inclination of the slope (same for all slices); factors of safety are often considerably higher than calculated using methods that satisfy all conditions of equilibrium; $2N$ equations and unknowns
Lowe and Karafiath's method (Lowe and Karafiath 1960)	Generally most accurate of the force equilibrium methods; applicable to any shape of slip surface; assumes side force inclinations are average of slope surface and slip surface (varying from slice to slice); satisfies vertical and horizontal force equilibrium; $2N$ equations and unknowns
Janbu's generalized procedure of slices (Janbu 1968)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes heights of side forces above base of slice (varying from slice to slice); more frequent numerical convergence problems than some other methods; accurate method; $3N$ equations and unknowns
Spencer's method (Spencer 1967)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that inclinations of side forces are the same for every slice; side force inclination is calculated in the process of solution so that all conditions of equilibrium are satisfied; accurate method; $3N$ equations and unknowns
Morgenstern and Price's method (Morgenstern and Price 1965)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that inclinations of side forces follow a prescribed pattern, called $f(x)$; side force inclinations can be the same or can vary from slice to slice; side force inclinations are calculated in the process of solution so that all conditions of equilibrium are satisfied; accurate method; $3N$ equations and unknowns
Sarma's method (Sarma 1973)	Satisfies all conditions of equilibrium; applicable to any shape of slip surface; assumes that magnitudes of vertical side forces follow prescribed patterns; calculates horizontal acceleration for barely stable equilibrium; by prefactoring strengths and iterating to find the value of the prefactor that results in zero horizontal acceleration for barely stable equilibrium, the value of the conventional factor of safety can be determined; $3N$ equations, $3N$ unknowns

Detailed analyses based on these methods can be quickly and efficiently performed when done so through computer-based calculations. Such analyses permit thorough searches for the critical slip surface (Fig. 10), a procedure that is extremely time consuming when performed by hand. Limit-equilibrium programs such as the two-dimensional SLIDE (Rocscience 2001d) and SLOPE/W (Geo-Slope 2000), and the three-dimensional CLARA (Hungr 1992) have the ability to model heterogeneous soil-type behaviours, complex stratigraphic and slip surface geometries and variable pore-water pressure conditions.

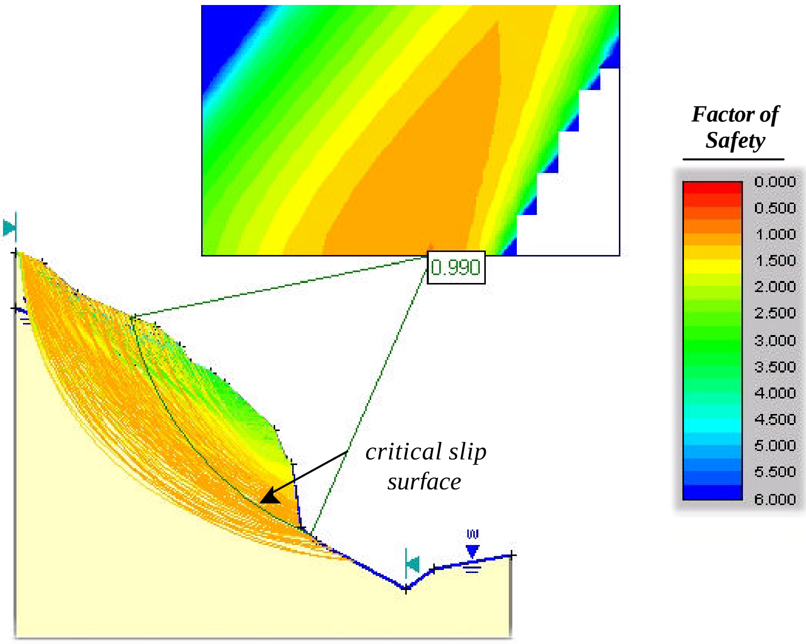


Figure 10. Limit equilibrium analysis of a rock slope performed using a critical surface search routine.

2.3 Rockfall simulators

One objective of rock slope stability analysis is to devise remedial measures to prevent rock mass movements. In the case of rockfalls, it is generally impossible to secure all blocks and therefore consideration must also be given to the design of protective measures near or around structures endangered by the falling blocks. The problem of rockfall protection work, therefore, largely involves the determination of travel paths and trajectories of unstable blocks that have detached from a rock slope face.

Analytical solutions, as described by Hungr & Evans (1988), treat the rock block as a point with a mass and velocity that moves on a ballistic trajectory while in the air, and bounces, rolls or slides when in contact with the slope surface. This is done by reversing and reducing the normal and tangential components of velocity upon contact through coefficients of normal and tangential restitution. The two restitution coefficients are taken as bulk measures of all impact characteristics, incorporating deformational work, contact sliding and transfers of rotational to translational momentum and vice versa. As a result, the coefficient must depend on fragment shape, slope surface roughness, momentum and deformational properties and, to a large extent, on the chance of certain conditions prevailing in a given impact.

The incorporation of these solutions into computer-based programs make up what are referred to as rockfall simulators. Programs such as ROCFALL (Rocscience 2001e) analyse the trajectory of falling blocks based on changes in velocity as rock blocks roll, slide and bounce on various materials that form the slope. Other factors solved for include block velocity, bounce height and endpoint distance, which can be analysed statistically over a repeated number of simulations to aid in a risk assessment (Fig. 11). Rockfall simulators can also assist in determining remedial measures by calculating the kinetic energy and location of impact on a barrier, which in turn can be defined in terms of capacity, size and location.

More recent developments in rockfall simulators include the use of different shaped rock 'elements' (Spang & Sönser 1995) and extensions into three-dimensions (Leroi *et al.* 1996). In the latter, models can include the 3-D topography based on digital elevation models (Fig. 12), the geomechanical characteristics of the material involved (geology of the blocks, lithology and vegetation of the ground), several common physical laws (stress-deformation curves, hydraulic friction, Coulomb friction) and the real geometry of the blocks.

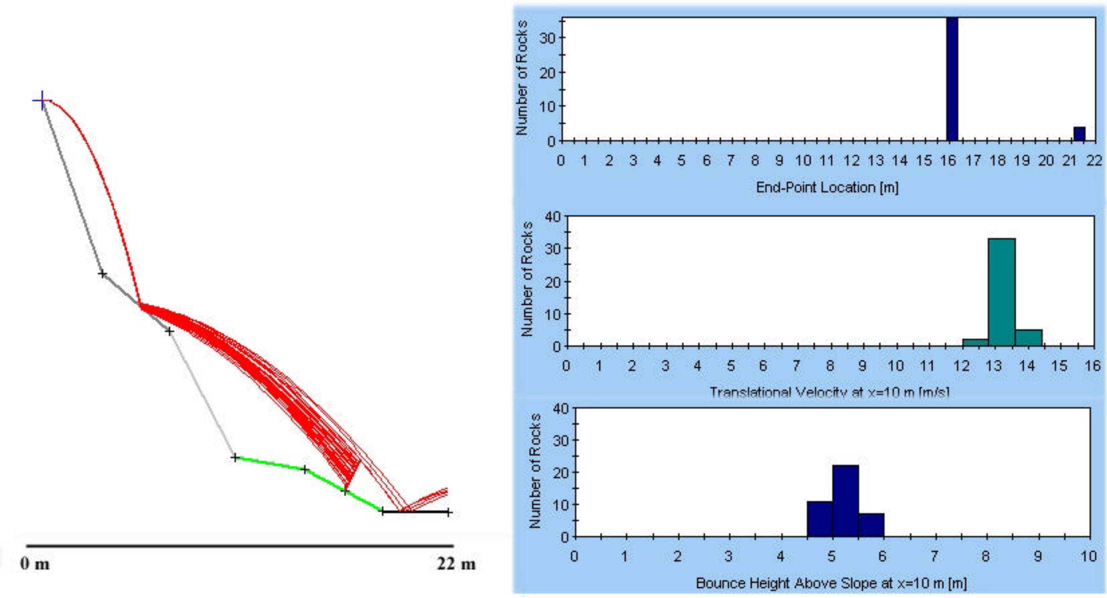


Figure 11. Rockfall analysis showing the trajectory paths for 40 simulated rockfalls and the corresponding end distances, velocities and bounce heights.

Furthermore, the interaction between several blocks, impact with buildings or other structures, and randomized initial conditions and rebound parameters can be included to help delimit hazard areas. Other variations that deal with failed rock blocks and rapid debris slides include Hungr's (1995) DAN code, which proposes a dynamic analysis tool suited for the prediction of flow and runout behaviour.

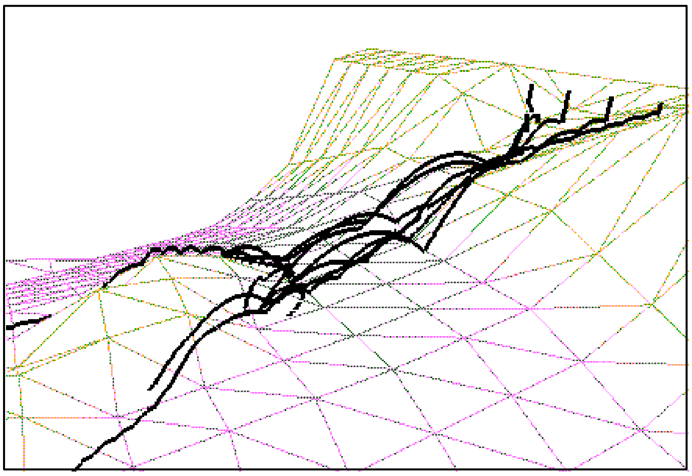


Figure 12. Three-dimensional rockfall simulation.

3. Numerical Methods of Rock Slope Analysis

Conventional forms of analysis are limited to simplistic problems in their scope of application, encompassing simple slope geometries and basic loading conditions, and as such, provide little insight into slope failure mechanisms. Many rock slope stability problems involve complexities relating to geometry, material anisotropy, non-linear behaviour, *in situ* stresses and the presence of several coupled processes (e.g. pore pressures, seismic loading, etc.).

To address these limitations, numerical modelling techniques have been forwarded to provide approximate solutions to problems, which otherwise, would not have been possible to solve using conventional techniques. Advances in computing power and the availability of relatively inexpensive commercial numerical modelling codes means that the simulation of potential rock slope failure mechanisms could, and in many cases should, form a standard component of a rock slope investigation.

Numerical methods of analysis used for rock slope stability investigations may be divided into three approaches:

- continuum modelling;
- discontinuum modelling
- hybrid modelling.

Continuum modelling is best suited for the analysis of slopes that are comprised of massive, intact rock, weak rocks, and soil-like or heavily jointed rock masses. Discontinuum modelling is appropriate for slopes controlled by discontinuity behaviour. Figure 13 demonstrates the use of these two approaches as applied to the same rock slope stability problem (that of complex buckling failure along an open pit coal mining slope). Hybrid codes involve the coupling of these two techniques (i.e. continuum and discontinuum) to maximize their key advantages. Table 3 briefly summarizes the advantages and limitations inherent in these different numerical modelling approaches.

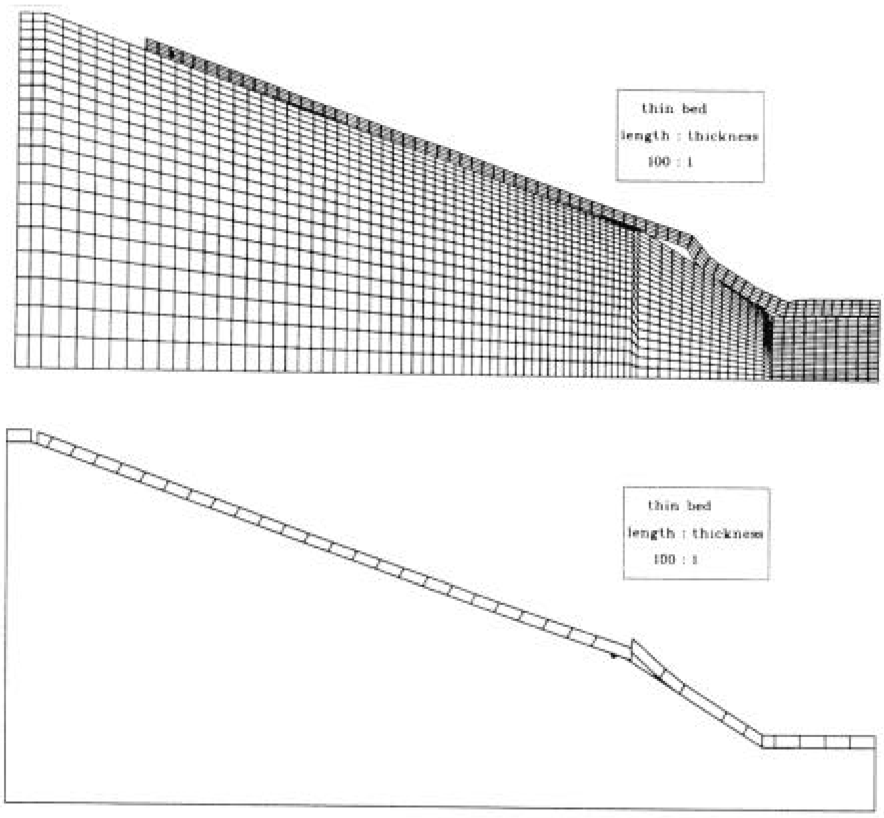


Figure 13. Continuum (TOP) and discontinuum (BOTTOM) modelling approaches applied to the analysis of buckling type failures in surface coal mine slopes (after Stead & Eberhardt 1997).

Table 3. Numerical methods of rock slope analysis (after Coggan *et al*. 1998).

Analysis Method	Critical Parameters	Advantages	Limitations
Continuum Modelling (e.g. finite-element, finite-difference)	Representative slope geometry; constitutive criteria (e.g. elastic, elasto-plastic, creep, etc.); groundwater characteristics; shear strength of surfaces; *in situ* stress state.	Allows for material deformation and failure (factor of safety concepts incorporated); can model complex behaviour and mechanisms; 3-D capabilities; can model effects of pore pressures, creep deformation and/or dynamic loading; able to assess effects of parameter variations; computer hardware advances allow complex models to be solved with reasonable run times.	Users must be well trained, experienced and observe good modelling practice; need to be aware of model and software limitations (e.g. boundary effects, meshing errors, hardware memory and time restrictions); availability of input data generally poor; required input parameters not routinely measured; inability to model effects of highly jointed rock; can be difficult to perform sensitivity analysis due to run time constraints.
Discontinuum Modelling (e.g. distinct-element, discrete-element)	Representative slope and discontinuity geometry; intact constitutive criteria; discontinuity stiffness and shear strength; groundwater characteristics; in situ stress state.	Allows for block deformation and movement of blocks relative to each other; can model complex behaviour and mechanisms (combined material and discontinuity behaviour coupled with hydro-mechanical and dynamic analysis); able to assess effects of parameter variations on instability.	As above, user required to observe good modelling practice; general limitations similar to those listed above; need to be aware of scale effects; need to simulate representative discontinuity geometry (spacing, persistence, etc.); limited data on joint properties available (e.g. jk_n, jk_s).
Hybrid Modelling	Combination of input parameters listed above for stand-alone models.	Coupled finite-/distinct-element models able to simulate intact fracture propagation and fragmentation of jointed and bedded rock.	Complex problems require high memory capacity; comparatively little practical experience in use; requires ongoing calibration and constraints.

3.1 Continuum approach

Continuum approaches used in slope stability analysis include the finite-difference and finite-element methods. In both these methods the problem domain is divided (discretized) into a set of sub-domains or elements (Fig. 14). A solution procedure may then be based on numerical approximations of the governing equations, i.e. the differential equations of equilibrium, the strain-displacement relations and the stress-strain equations, as in the case of the finite-difference method (FDM). Alternatively, the procedure may exploit approximations to the connectivity of elements, and continuity of displacements and stresses between elements, as in the finite-element method (FEM). The salient advantages and limitations of these two methods are discussed by Hoek *et al*. (1993), and both have found widespread use in slope stability analysis.

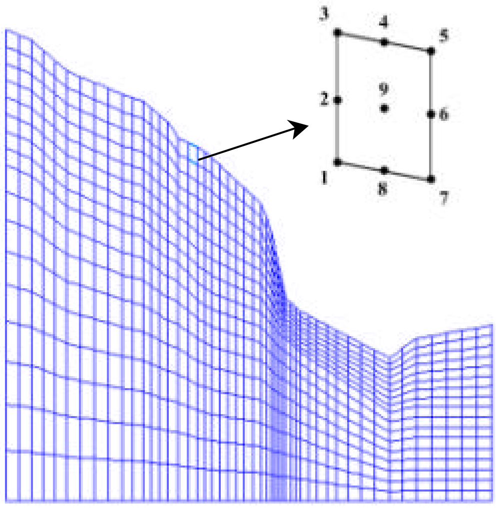


Figure 14. Finite-element mesh of a natural rock slope using 9-noded elements.

Continuum methods are best suited for the analysis of rock slopes that are comprised of massive intact rock, weak rocks, or heavily fractured rock masses. For the most part, earlier studies were often limited to elastic analyses and as such were limited in their application. Most continuum codes, however, now incorporate a facility for including discrete fractures such as faults and bedding planes. Numerous commercial codes are available, which often offer a variety of constitutive models including elasticity, elasto-plasticity, strain-softening and elasto-viscoplasticity (allowing for the modelling of time-dependent behaviour). Figure 15 illustrates the use of an elasto-plastic constitutive criterion to model translational slide movements associated with the 1903 Frank Slide, Canada.

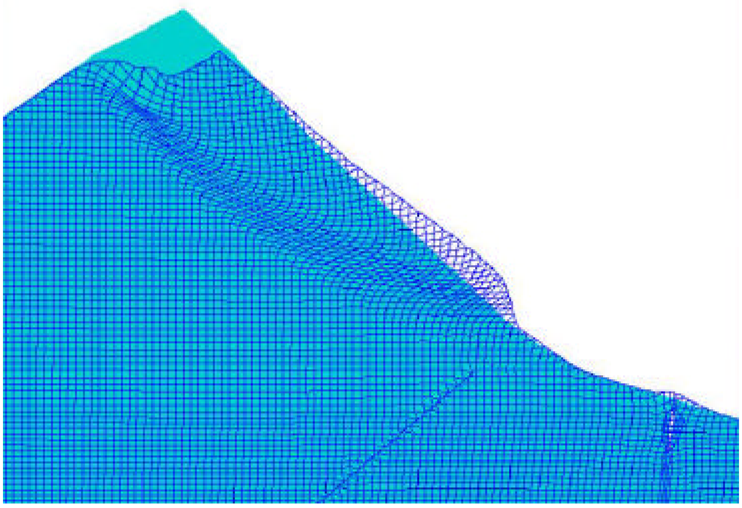


Figure 15. Finite-difference model showing large-strain failure of a rock slope as modelled through an elasto-plastic constitutive model based on a Mohr-Coulomb yield criterion (after Stead *et al.* 2000).

Two-dimensional continuum codes assume plane strain conditions, which are frequently not valid in inhomogeneous rock slopes with varying structure, lithology and topography. The recent advent of 3-D continuum codes such as FLAC3D (Itasca 1997) and VISAGE (VIPS 2001) enables the engineer to undertake 3-D analyses of rock slopes on a desktop computer. Three-dimensional codes make it possible to explore three-dimensional influences on slope stability, including slope geometry in plan and section, geology, pore water pressures, *in situ* stress, material properties and seismic loading due to earthquakes. An example of a FLAC3D analysis of a china clay slope, which incorporated distinct zones of alteration along strike, is shown in Figure 16.

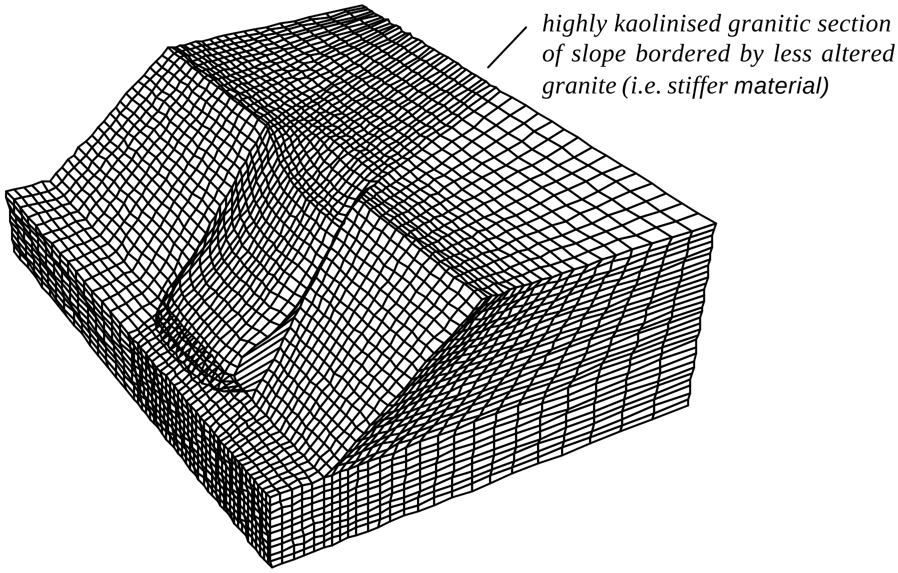
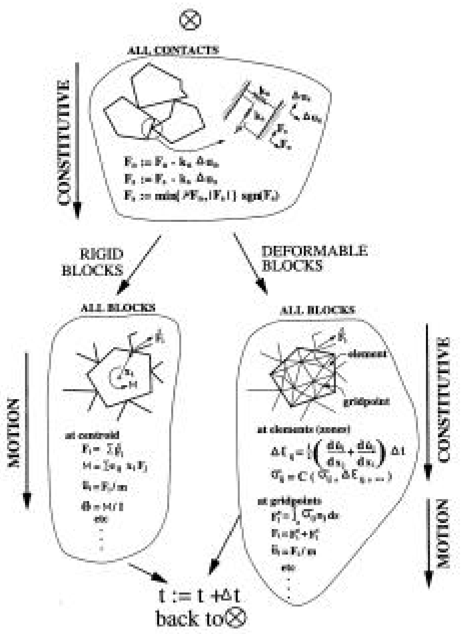


Figure 16. Three-dimensional finite-difference model showing rock slope displacements along a China clay slope in the U.K. (after Stead *et al*. 2001).

3.2 Discontinuum approach

Although 2-D and 3-D continuum codes are extremely useful in characterizing rock slope failure mechanisms it is important to recognize their limitations, especially with regards to whether they are representative of the rock mass under consideration. Where a rock slope comprises multiple joint sets, which control the mechanism of failure, then a discontinuum modelling approach may be considered more appropriate. Discontinuum methods treat the problem domain as an assemblage of distinct, interacting bodies or blocks that are subjected to external loads and are expected to undergo significant motion with time. This methodology is collectively referred to as the discrete-element method (DEM).

The development of discrete-element procedures represents an important step in the modelling and understanding of the mechanical behaviour of jointed rock masses. Although continuum codes can be modified to accommodate discontinuities, this procedure is often difficult and time consuming. In addition, any modelled inelastic displacements are further limited to elastic orders of magnitude by the analytical principles exploited in developing the solution procedures. In contrast, discontinuum analysis permits sliding along and opening/closure between blocks or particles. The underlying basis of the discrete-element method is that the dynamic equation of equilibrium for each block in the system is formulated and repeatedly solved until the boundary conditions and laws of contact and motion are satisfied (Fig. 17). The method thus accounts for complex non-linear interaction phenomena between blocks.



Contacts	
F_n, F_s	normal and shear forces
Δu_n, Δu_s	increment normal and shear displacements
k_n, k_s	normal, shear stiffness
μ	friction coefficient
Rigid Blocks	
F_i	block force vector
F_i^c	contact force vector
I	moment of inertia
M	block moment
$\ddot{u}_i$	translational acceleration
x_i	position vector
$\ddot{\theta}$	angular acceleration
e_{ij}	permutation tensor
Deformable Blocks	
$C(\ )$	functional form of constitutive law
F_i^c	contact force vector
F_i^e	gridpoint force vector
$\dot{u}_i$	gridpoint velocity vector
$\Delta\varepsilon_{ij}$	incremental strain in zone
σ_{ij}	stress tensor in zone
Δt	timestep
m	zone mass
n_j	unit outward normal vector
ds	incremental segment

Figure 17. Example calculation cycle used in discrete-element methodologies (from Hart 1993).

Discontinuum modelling constitutes the most commonly applied numerical approach to rock slope analysis. Several variations of the discrete-element methodology exist:

- distinct-element method;
- discontinuous deformation analysis
- particle flow codes.

3.2.1 Distinct-element method

The distinct-element method, developed by Cundall (1971) and described in detail by Hart (1993), was the first to treat a discontinuous rock mass as an assembly of quasi-rigid, and later deformable, blocks interacting through deformable joints of definable stiffness. As such, the numerical model must represent two types of mechanical behaviour: that of the discontinuities and that of the solid material.

In the distinct-element approach, the algorithm is based on a force-displacement law specifying the interaction between the deformable intact rock units and a law of motion, which determines displacements induced in the blocks by out-of-balance forces. Joints are viewed as interfaces between the blocks and are treated as a boundary condition rather than a special element in the model (Fig. 18). Block deformability is introduced through the discretization of the blocks into internal constant-strain elements in order to increase the number of degrees-of-freedom (Fig. 19).

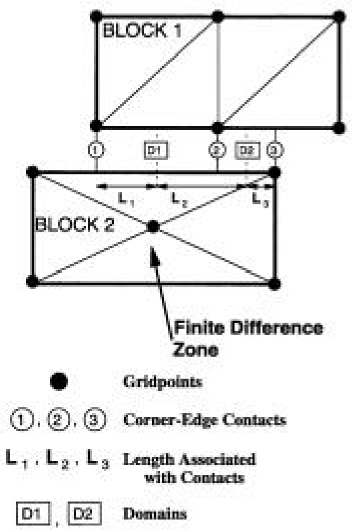


Figure 18. Representation of contact domains between two deformable blocks as formulated in the distinct-element method (from Hart 1993).

The dual nature of distinct-element codes, for example UDEC (Itasca 2000), make them particularly well suited to problems that involve jointed rock slopes. On the one hand, they are highly applicable to the modelling of discontinuity-controlled instabilities, allowing two-dimensional analysis of translational mechanisms of slope failure (Fig. 20) and are capable of simulating large displacements due to slip, or opening, along discontinuities. On the other hand, they are also capable of modelling the deformation and material yielding of the joint-bounded intact rock blocks. This becomes highly relevant for high slopes in weak rock, flexural-topples (Fig. 21) and other complex modes of rock slope failure (Fig. 22).

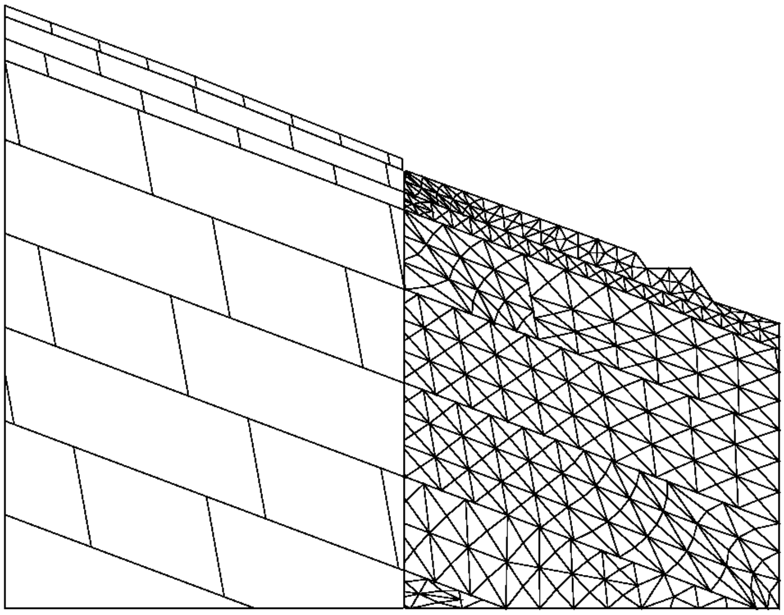


Figure 19. Rock slope distinct-element model showing discretization of geometry blocks into finite-difference elements.

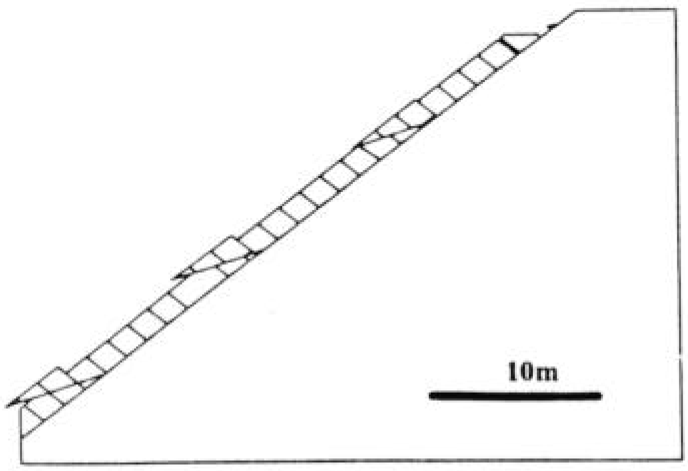


Figure 20. Distinct-element model of a translational bi-linear slab failure (after Stead & Eberhardt 1997).

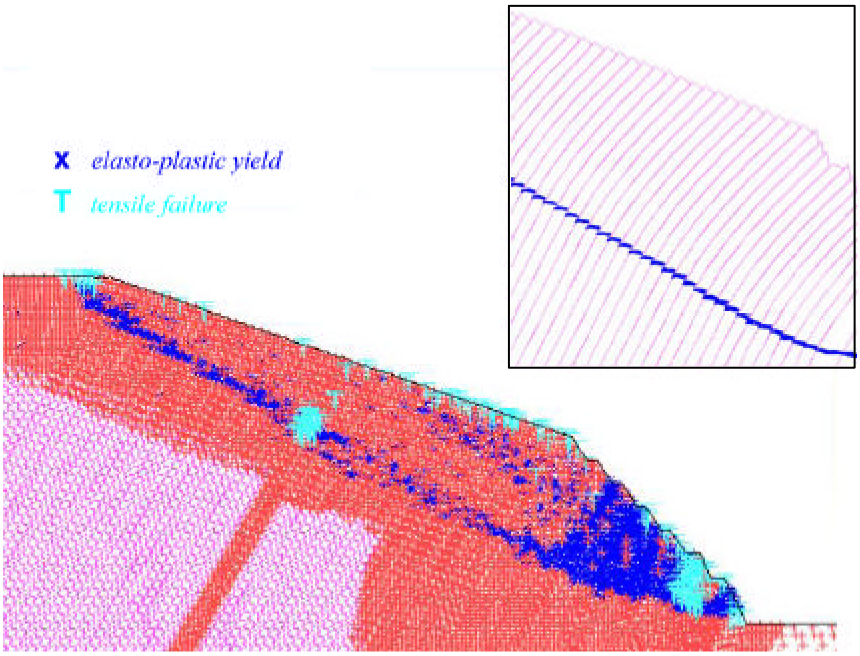


Figure 21. Distinct-element model of a flexural toppling failure showing the development of an underlying shear plane through intact material yield (after Benko 1997).

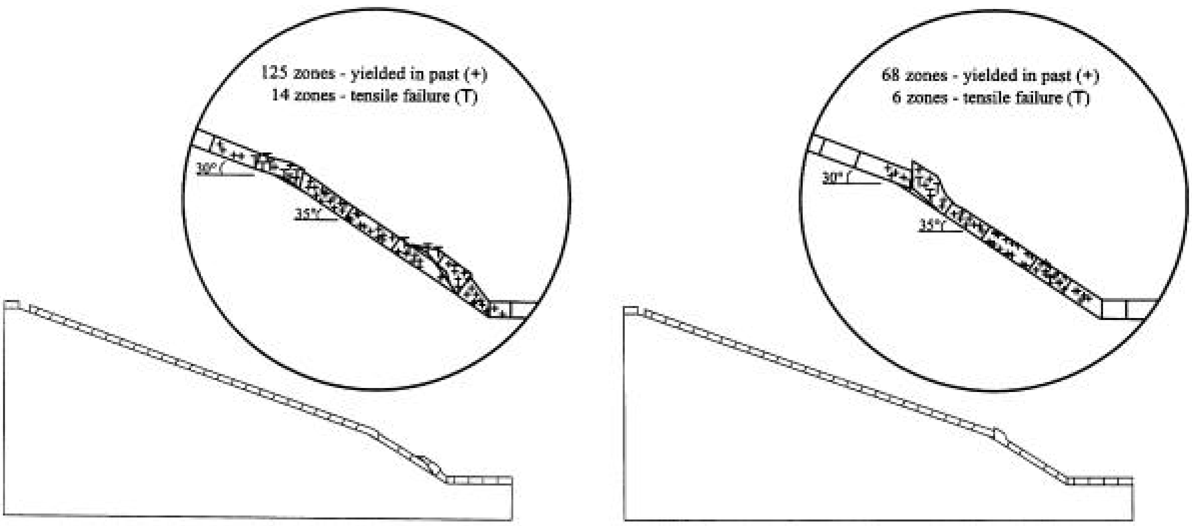


Figure 22. Complex slope failure modes including ploughing (LEFT) and three-hinge buckling (RIGHT) as modelled using the distinct-element method (after Stead & Eberhardt 1997).

The influence of external factors such as groundwater pore pressures and seismic activity on block sliding and deformation can also be simulated using the distinct-element formulation. Fluid flow is simulated through a series of interconnected discontinuities, whereby the intact blocks are assumed to be impermeable. A coupled hydro-mechanical analysis is performed in which fracture conductivity is dependent on mechanical deformation and, conversely, joint water pressures affect the mechanical behaviour (Fig. 23).

Fluid flow along planar contacts is idealized as laminar viscous flow where the rate of flow is assumed to be dependent upon the cubic power of the joint aperture (i.e. cubic flow law). Fluid flow is then determined by the pressure difference between adjacent joint domains. An example of a coupled hydro-mechanical analysis showing the effects of rock mass drainage on slope stabilization, through the construction of a drainage adit at depth, is presented in Figure 24.

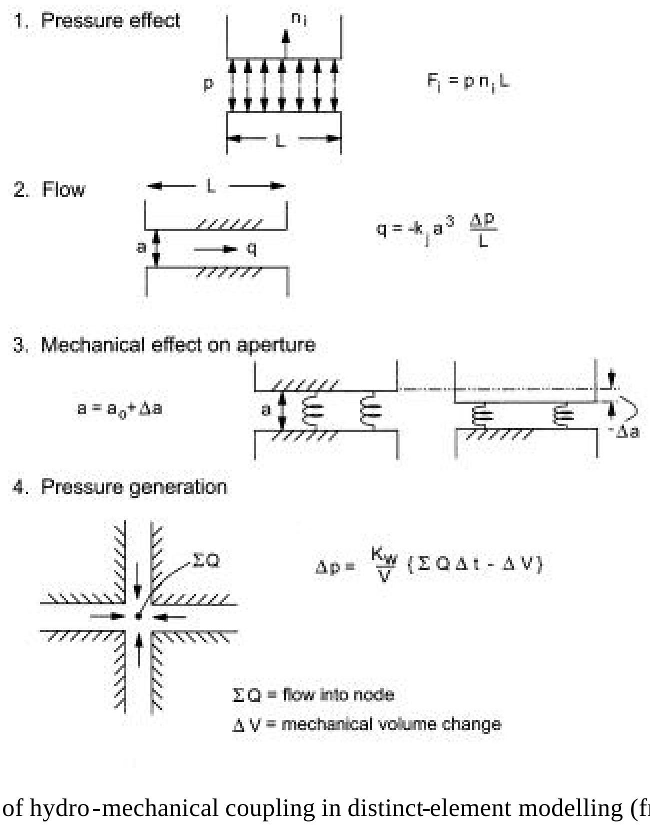


Figure 23. Formulation of hydro-mechanical coupling in distinct-element modelling (from Itasca 2000).

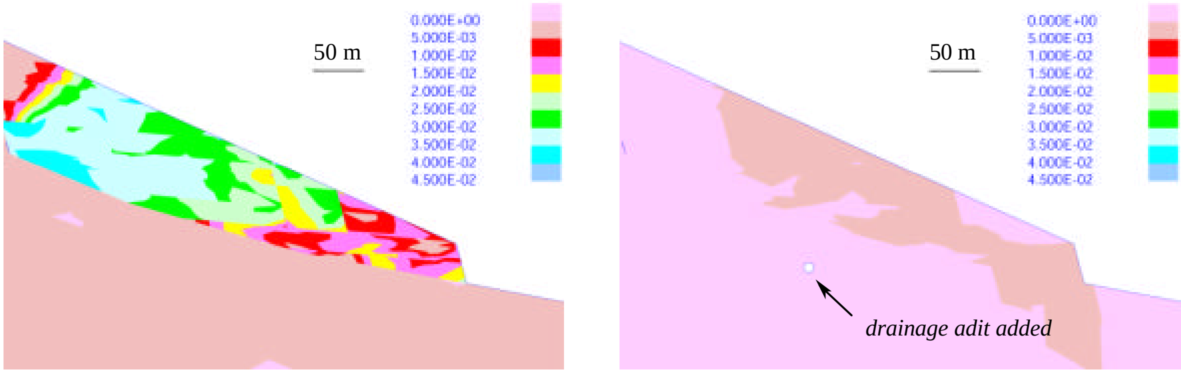


Figure 24. Coupled hydro-mechanical distinct-element model showing horizontal velocities before (LEFT) and after (RIGHT) introduction of drainage adit (after Bonzanigo *et al.* 2001).

The distinct-element method is also a powerful tool for modelling rock slope susceptibility to seismic events relating to earthquakes or blasting. In this respect, the explicit solution in the time domain used by the method is ideal for following the time propagation of a stress wave. The construction of a dynamic model consists of three main components: boundary conditions, mechanical damping and dynamic loading (Fig. 25). Boundaries for the problem domain can be chosen to permit energy radiation and to limit reflection of outward propagating waves through the use of dashpots as viscous damping elements placed around the problem boundary. To account for the natural damping of vibrational energy and energy losses that exists in a real system, mechanical damping (e.g. Raleigh damping consisting of both a mass- and stiffness-proportional component) is then added to the model. Lastly, dynamic loading is added to the model in the form of an upward propagating stress wave originating from the bottom boundary of the model.

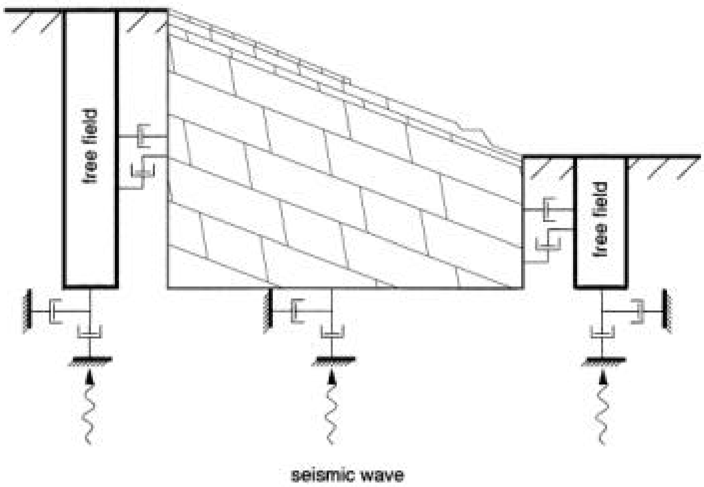


Figure 25. Distinct-element modelling of free-field boundary conditions and seismic input.

Figure 26 provides an example of a modelled stress wave used in a distinct-element analysis of a natural rock slope. The model shows an initially stable slope subjected to an earthquake, resulting in yielding and tensile failure of intact rock at the slope's toe. Toe failures of this type may then lead to planar failure of the upper slope (Fig. 26). In addition to material yielding, the oscillating nature of the dynamic load results in rotational type movements, which in turn could induce falls of loose rock.

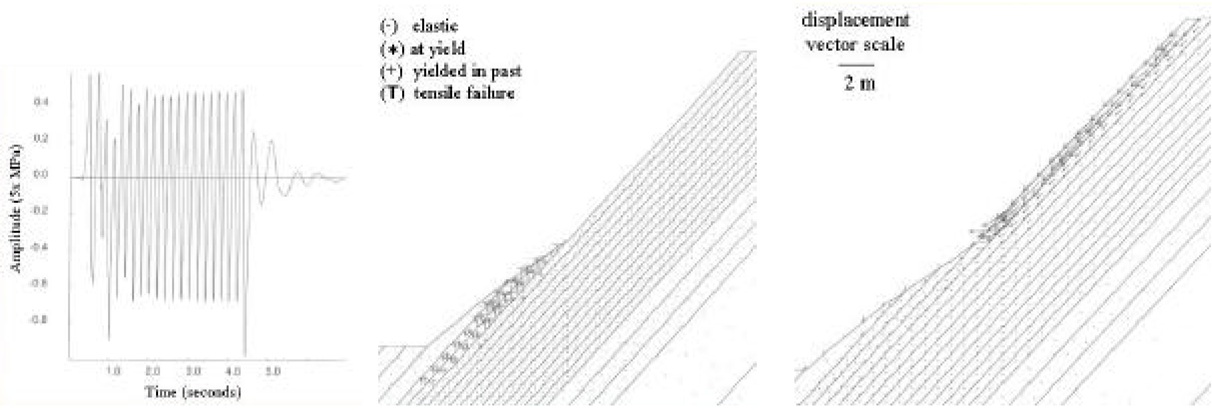


Figure 26. Dynamic modelling of a natural rock slope using the distinct-element method (after Eberhardt & Stead 1998): the generated seismic wave (LEFT); the subsequent yielding of the slope's toe material (CENTRE); and resulting displacement vectors indicating planar sliding failure (RIGHT).

Although the distinct-element method is ideally suited to rock slope stability problems, caution must be taken that the structural input into the analysis is representative. Hencher *et al.* (1996) illustrate the importance of discontinuity spacing and Stead & Eberhardt (1997) show the importance of discontinuity orientation on predicted failure mechanisms. It must therefore be stressed that tailoring the structure of the model to accommodate computing power and solution times, for example by using unrepresentative discontinuity spacing, may lead to unrepresentative results. Simulations must always be verified with field observations and wherever possible instrumented slope data. This becomes even truer with the development of 3-D discontinuum codes such as 3DEC (Itasca 1998).

To date, use of the three-dimensional extension of the distinct-element method has been somewhat limited for both practical and economic reasons. The software enables 3-D simulation of slope failures by representing the rock mass as a series of polyhedra (Fig. 27). The code is designed specifically for modelling the response of rock masses that contain multiple intersecting discontinuities and, hence, is well suited to the analysis of most rock slope failure mechanisms - in particular, to the analysis of wedge instabilities and the influence of rock support (e.g. rockbolts and cables). However, although further improvement in understanding rock slope failure mechanisms is made possible in 3-D, realistic characterization of the slope becomes even more critical. Three-dimensional variations in material properties, geology, geometry and loading conditions will have a fundamental effect on the modelled instability. Only when a confident, realistic portrayal of the 3-D characteristics of a slope has been obtained, which requires considerable site investigation, can the results of the analyses be considered representative.

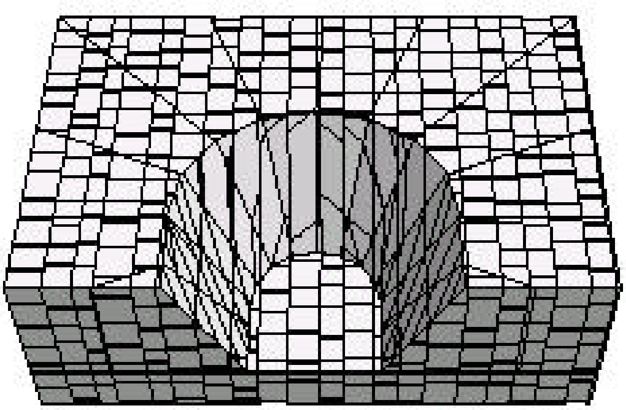


Figure 27. Three-dimensional distinct-element model of an open pit slope (from Itasca 1998).

3.2.2 Discontinuous deformation analysis

The discontinuous deformation analysis (DDA) method developed by Shi (1989; 1993) has also been used with some success in the modelling of discontinuous rock masses, both in terms of rockslides (Sitar & MacLaughlin 1997) and rockfalls (Chen & Ohnishi 1999). The method differs from the distinct-element method in that the unknowns in the equilibrium equations are displacements as opposed to forces. By using the displacements as unknowns, the equilibrium equations can be solved in the same manner as the matrix analysis used in finite-element formulations.

As such, the DDA method parallels the finite-element method (whereas the distinct-element method incorporates aspects of the finite-difference method). The formulation solves a finite-element type mesh where each element represents an isolated block bounded by discontinuities. These elements, or blocks, can be of any convex or concave shape, or can be joined to form more complex multi-connected polygons. Displacement functions (analogous to shape functions in the finite-element method) provide the complete first order approximations of the block displacements, the advantage being that the energy formulas become very simple and lead to very simple stiffness, contact and loading sub-matrices.

With respect to slope stability analysis, the method has the advantage of being able to model large deformations and rigid body movements, and can simulate coupling or failure states between contacted blocks. For example, if the separating forces between two blocks exceed the tensile strength prescribed along the discontinuity, then the stiffness between the blocks is removed and the separating motion is allowed (Fig. 28). The same principals apply to sliding and shear motions between neighbouring blocks. As such, these algorithms can be further extended to include the simulation of block fracturing based on shear (Mohr-Coulomb) or tensile fracture propagation criterion (Amadei *et al.* 1994). Figure 29 demonstrates a DDA analysis that simulates the breakdown of a falling rock block into smaller pieces during ground impact.

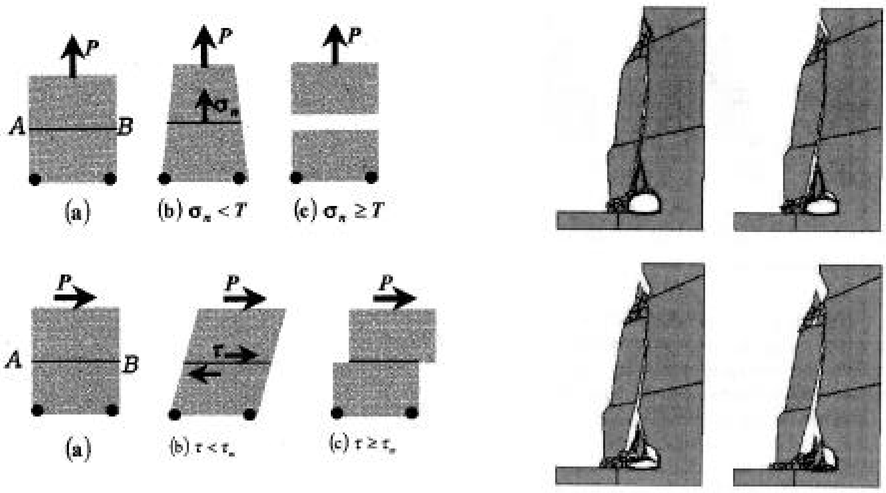


Figure 28. Deformation and failure of two blocks in contact under tensile and shear loading (LEFT) and an example of a discontinuous deformation analysis applied to a rock slope failure in Japan (RIGHT – from Chen & Ohnishi 1999).

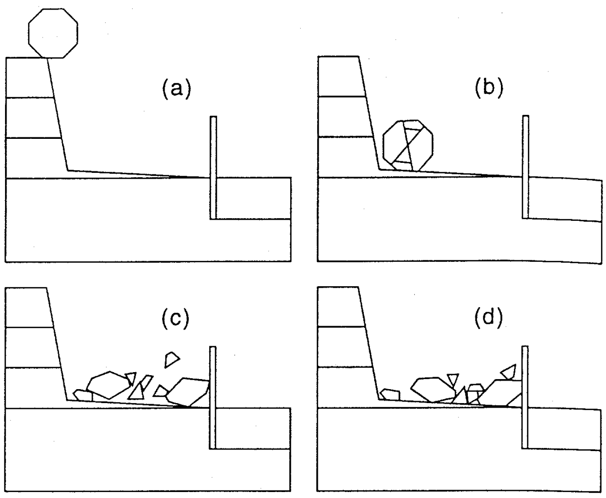
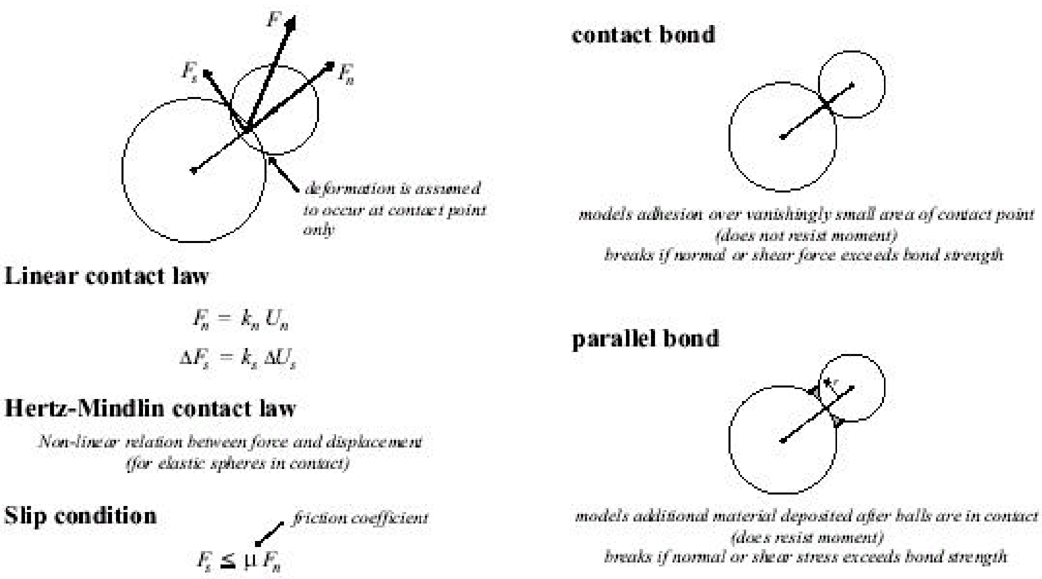


Figure 29. Discontinuous deformation analysis showing internal fracturing and breakdown of a rockfall block during ground impact (from Amadei *et al.* 1994).

3.2.3 Particle flow codes

A more recent development in discontinuum modelling techniques is the application of distinct-element methodologies in the form of particle flow codes, e.g. PFC2D/3D (Itasca 1999a). This code allows the rock mass to be represented as a series of spherical particles that interact through frictional sliding contacts (Fig. 30). Clusters of particles may also be bonded together through specified bond strengths in order to simulate joint bounded blocks. The calculation cycle then involves the repeated application of the law of motion to each particle and a force-displacement law to each contact (Fig. 31).



$$F_n = k_n U_n$$
$$\Delta F_s = k_s \Delta U_s$$
$$F_s \leq \mu F_n$$

Figure 30. Contact and bonding logic of interacting particles used in the discrete-element program PFC2D (from Itasca 1999a).

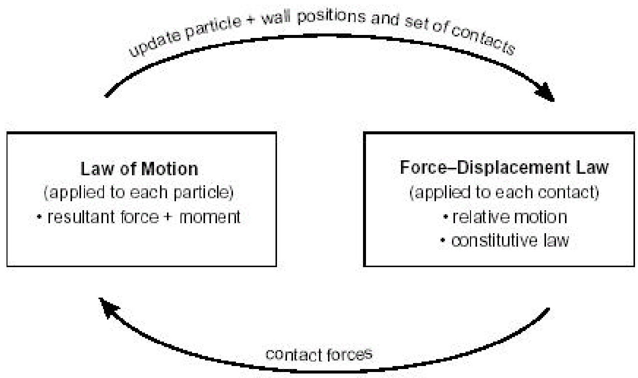


Figure 31. Calculation cycle used in PFC2D (from Itasca 1999).

With these codes it is possible to model granular flow, translational movement of blocks, fracture of intact rock and dynamic response to blasting or seismicity. The breaking of bonds between circular particles roughly simulates intact rock fracture and failure (although not fracture propagation). Deformation between particles due to shear or tensile forces can also be included, where slip between adjacent particles is prescribed in terms of frictional coefficients that limit the contact shear force.

Particle flow codes are thus able to simulate material from the macro level of fault- or joint-bounded blocks to the micro scale of grain-to-grain contact, the main limiting factors being computing time and memory requirements. In this sense, it becomes possible to model a number of rock slope failure processes, and subsequently, the runout of the failed material down the slope and into an underlying valley. Figure 32 demonstrates a 3-D example of a rock fall simulation whereby several particles are bonded together to model the breaking apart of a falling block upon impact with the slope face. At present, these codes are predominantly a research tool, but its potential is being widely recognized in mining, petroleum and civil engineering.

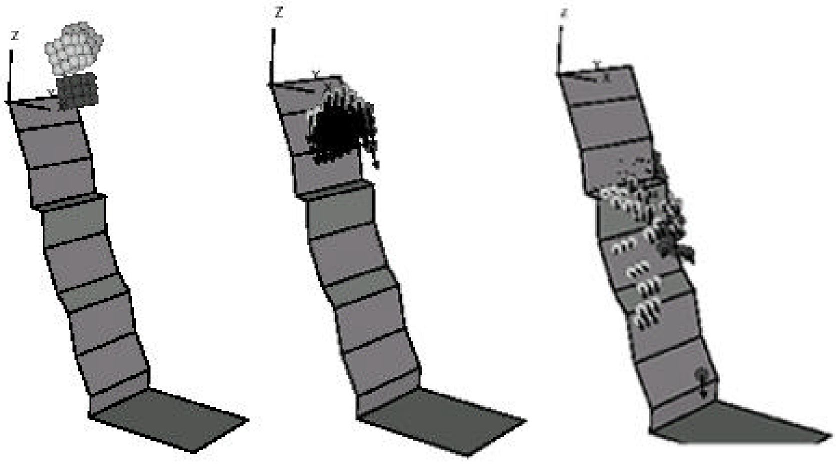


Figure 32. Discrete-element model of a rockfall using bonded elements to represent larger, destructible blocks (after Itasca 1999a).

3.3 Hybrid approach

Hybrid approaches are increasingly being adopted in rock slope analysis. This may include combined analyses using limit equilibrium stability analysis and finite-element groundwater flow and stress analysis such as adopted in the GEO-SLOPE suite of software (Geo-Slope 2000). Hybrid numerical models have been used for a considerable time in underground rock engineering including coupled boundary-/finite-element and coupled boundary-/distinct-element solutions. Recent advances include coupled particle flow and finite-difference analyses using PFC3D and FLAC3D (Itasca 1999b). These hybrid techniques already show significant potential in the investigation of such phenomena as piping slope failures, and the influence of high groundwater pressures on the failure of weak rock slopes.

Coupled finite-/distinct-element codes are now available which incorporate adaptive remeshing. Although separately continuum and discontinuum analyses provide a useful means to analyze rock slope stability problems, complex failures often involve mechanisms related to both pre-existing discontinuities and the brittle fracturing of intact rock. The coupling of finite-/distinct-element codes, for example in ELFEN (Rockfield 2001), allow for the modelling of both intact rock behaviour and the development and behaviour of fractures. These methods use a finite-element mesh to represent either the rock slope or joint bounded blocks coupled together with discrete elements able to model deformation involving joints. If the stresses within the rock slope exceed the failure criteria within the finite-element continuum a discrete fracture is initiated. Adaptive remeshing allows the propagation of the cracks through the finite-element mesh to be simulated.

Figure 33 illustrates a two-dimensional finite-/distinct-element hybrid analysis of the 1991 Randa rockslide in Switzerland. Such studies represent the future direction of numerical modelling, in which ideas as to how existing discontinuities and stress-induced brittle fracturing work together to promote rock slope instabilities are being forwarded (Eberhardt *et al.* 2002). Through such hybrid techniques, modelling will be extended towards modelling the complete failure process from initiation, through transport to deposition.

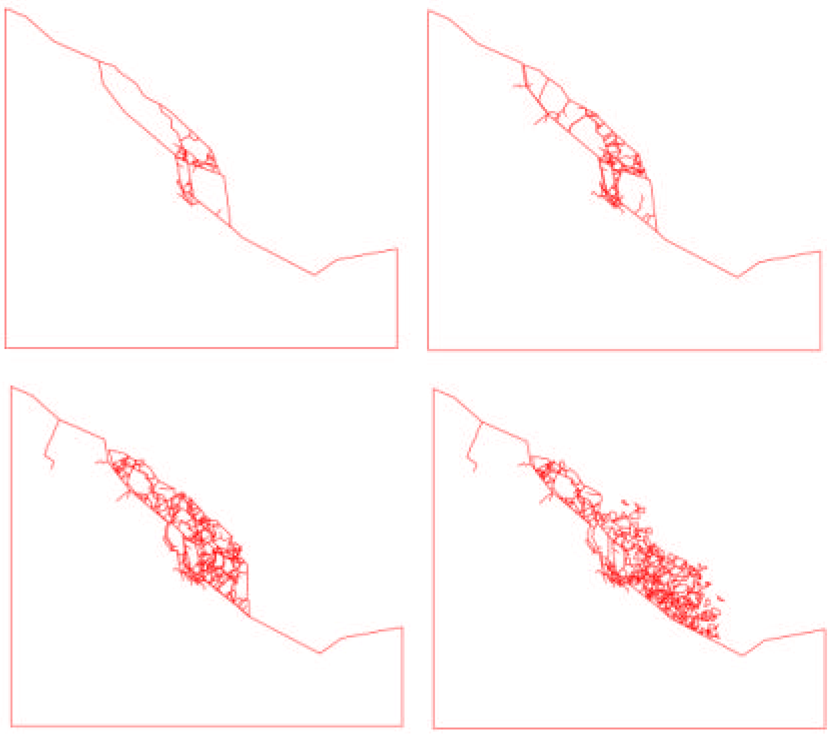


Figure 33. Hybrid finite-/discrete-element rockslide analysis showing several progressive stages of brittle failure (from Eberhardt *et al*. 2002).

4. Numerical Model Development and Application

"Numerical modelling should not be used as a substitute for thinking, but as an aid to thought".

Numerical modelling is a powerful tool, and as with any tool, it must be applied in the manner in which it was designed for. This involves following proper modelling practices, for example as forwarded by Coggan *et al.* (1998). As such, a critical definition of the problem is essential so that the decision as to whether a detailed analysis is necessary or the level of detail required can be assessed. The nature of the problem (soil or rock, mode of failure, 2-D or 3-D, etc.), the type of results required (deterministic or probabilistic, Class A or Class C prediction, evaluation of different stabilization schemes, etc.), and/or user experience may play a controlling factor as to which analysis method or combination of methods are required.

It must also be emphasized that unlike the application of numerical methods to design problems involving fabricated materials (e.g. steel, concrete, etc.), earth materials (i.e. rock and soil) require special considerations. Furthermore, slope stability problems involve a complex relationship between cause and effect linked by a triggering mechanism (Fig. 34), thus requiring insight into the potential coupling between processes and triggers (e.g. hydro-mechanical coupling). In general, analyses of rock slope stability problems must be achieved with relatively limited site-specific data and knowledge of the rock mass deformation, strength and hydrogeologic properties. These limitations may be offset by a detailed site investigation, so that in practice a continuous spectrum of situations exists with respect to the amount of data that may be available for a particular analysis (Fig. 35).

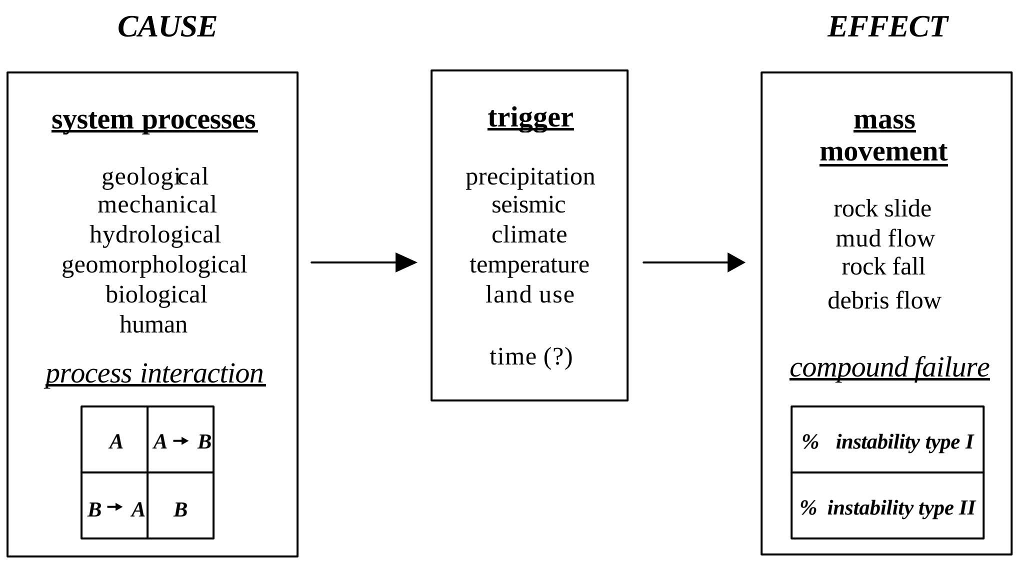


Figure 34. Representation of cause and effect relationships with respect to slope mass movements.

Typical Situation	• **complicated geology** • **inaccessible** • **no testing budget**	• **relatively simple geology/structure** • **$$$ spent on site investigation**
Data	**none**	**complete (?)**
Approach	**investigation of failure mechanism(s)**	**predictive (design use)**

Figure 35. Spectrum of modelling situations (after Itasca 2000).

Figure 35 also demonstrates that the objective of a numerical slope analysis may take the form of being fully predictive (i.e. forward modelling of a potential instability), or in cases where the data is limited, as a means to establish and understand the dominant mechanisms affecting the behaviour of the system. In the latter instance, the numerical model provides a means to test several hypotheses to gain an understanding of the problem. Figure 36 lists the key steps that should be followed to perform a success numerical slope analysis.

Once the objective has been defined, a conceptual picture of the physical system has been developed and a choice of program(s) has been made, it is important to determine the representative geometry of the slope and to define realistic input values. Appropriate constitutive criteria must be used for both material and discontinuity behaviour in the case of discontinuum modelling.

During modelling, the most appropriate methodology is to start with a simple model and to gradually build up its complexity as the problem dictates. The approach followed should incorporate sensitivity analyses on key input parameters. Probabilistic analysis may also be performed to assess the influence of data variability on the modelled stability. The potential influences of changes in model geometry (e.g. mesh size, element aspect ratio, mesh grading and symmetry), boundary conditions, *in situ* stress, discontinuity spacing and persistence should all be assessed and be part of the model evaluation. Figure 37 provides an example of how *in situ* stresses, a parameter commonly overlooked in slope stability investigations, can influence the modelled outcome of an analysis.

Figure 36. Flowchart showing the components of a proper numerical modelling study (after Coggan *et al*. 1998).

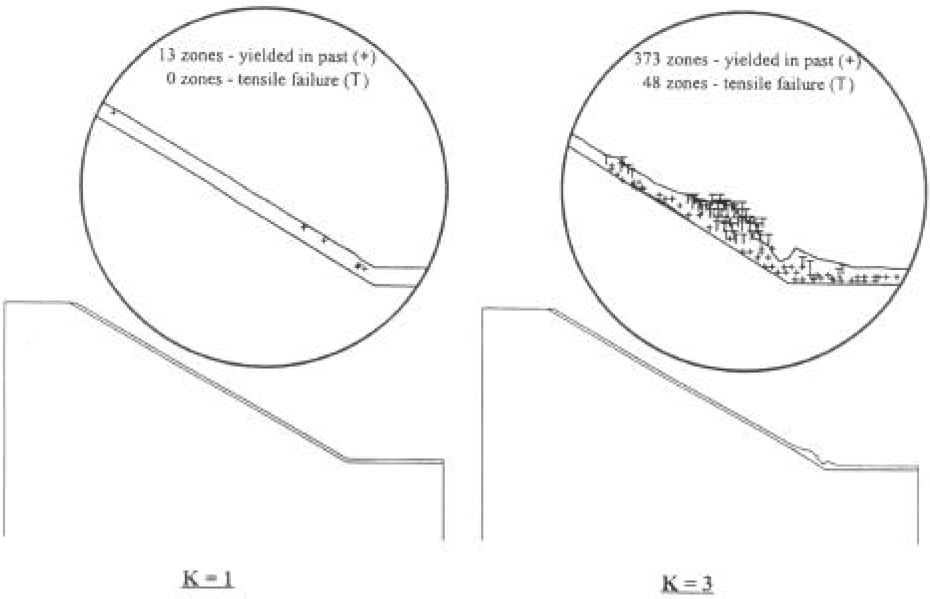


Figure 37. Distinct-element model of continuum buckling failure in weak bedded rock assuming an *in situ* horizontal to vertical stress ratio, K, of 1 (LEFT) and 3 (RIGHT; after Stead *et al*. 1995).

The model output should be subjected to validation in which the computed results are compared with those derived from *in situ* observation and instrumented measurements. This is especially important with respect to constraining and validating the input data used and to avoid incorrect, or in some cases, unrealistic or impossible results (Figure 38).

In this sense:

- numerical models *should* be constrained by high-quality input data;
- sensitivity analyses *should* be performed on critical input parameters; and
- rigorous validation *should* be provided where possible against instrumentation records.

Good modelling practice should also include, where possible, independent checks on proposed conceptual models or failure mechanisms and on the numerical results.

Data type	Properties	Examples
Impossible	Defy laws of nature Exceed theoretical limits Physically meaningless	γ, ϕ', s_u, E<0, jk_n and jk_s<0 ϕ'>90, υ>0.5
Implausible	Exceed all known values Disrupt established pecking orders Outside practical limits	ϕ'>60°, γ>3 500 kg/m^3 E_{soil}>E_{rock}

γ, unit weight; ϕ', effective angle of friction; E, modulus of elasticity; s_u, undrained shear strength.

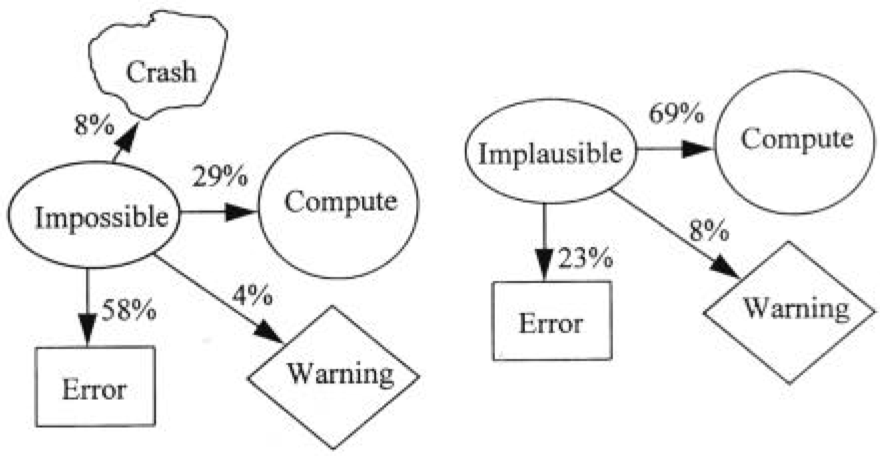


Figure 38. Results of a survey of nine commonly used geotechnical modelling programs and their response to impossible and implausible input data (after Crilly 1993).

5. Future Developments

Today, the analysis of complex landslides can be undertaken routinely using state-of-the-art numerical modelling codes on desktop computers. If the benefits of these methods are to be maximized then it is essential that field data collection techniques are more responsive to advances in design capabilities. Much of current data collection methodology has changed little over the last decade and is aimed towards limit equilibrium analysis. Data including rock mass characteristics, *in* situ deformation and stress, and pore pressures must be collected in order to allow more realistic modelling of rock slope failure mechanisms.

The next decade holds enormous potential in our ability to model the complete failure process from initiation, through transport to deposition. This will provide a far more rigorous understanding on which to base risk assessment. Practitioners and researchers must make the effort to think beyond the use of stand-alone computers and embrace the rapidly developing technology of parallel computing. The advent of virtual reality programming will allow the engineer to convey the results of simulations in a powerful and graphically efficient manner. It is essential however that quality/quantity of both input data and instrumentation data for modelling purposes be improved concomitantly in order to provide the requisite validation.

Through this lecture series, the overview and results presented demonstrate the benefits of integrating conventional and numerical modelling techniques in order to efficiently capitalize on the strengths of the different methodologies available for slope stability analysis. As such it is vital that good modelling practice be observed and followed. This then means that not only must consideration be given to integrating different numerical techniques, but integrating numerical modelling with site investigation, laboratory testing and in situ monitoring campaigns as well (e.g. Table 4).

Table 4. Integration of slope instability investigation methods.

Investigation Method	Parameters Investigated
Desk Study	Previous investigations, literature review, available data.
Site Investigation	Field mapping, scanline surveys, observations of instability, hydrogeological observations.
Laboratory Testing	Determination of rock mass strength and material behaviour including discontinuity shear strength evaluation.
Conventional Stability Analysis	Kinematic feasibility, deterministic limit equilibrium (i.e. Factor of Safety), probabilistic sensitivity analysis.
Numerical Modelling	Simulation of slope deformation and stability, analysis of progressive failure and shear surface development.
Field Monitoring	Monitoring of 3-D deformations, groundwater and microseismicity.

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