Remarks on Color Confinement

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Clay Millenium Problems

Yang-Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Status: Unsolved

STRINGS MICHIGAN THEORY

July 10-15, 2000 University of Michigan Ann Arbor

"Millennium Madness"

Physics Problems for the Next Millennium

The best 10 problems were selected at the end of the conference by a selection panel consisting of:

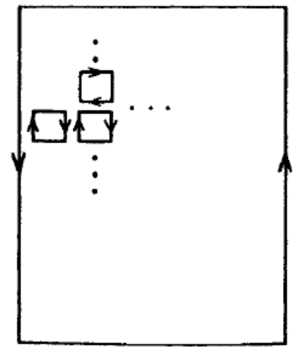
- Michael Duff (University of Michigan)
- David Gross (Institute for Theoretical Physics, Santa Barbara)
- Edward Witten (Caltech & Institute for Advanced Studies)

10. Can we quantitatively understand quark and gluon confinement in Quantum Chromodynamics and the existence of a mass gap?
    Igor Klebanov, Princeton University
    Oyvind Tafjord, McGill University

Lattice SU(N) Gauge Theory

- The gauge field kinetic term is encoded in the plaquette terms.

$$S = -(1/2g^2) \sum_{n,\,\mu\nu} \mathrm{tr}\, U_\mu(n) U_\nu(n+\mu)\, U_{-\mu}(n+\mu+\nu) U_{-\nu}(n+\nu) + \mathrm{h.c.}$$



- In the strong coupling expansion where these terms are treated as perturbations, the Area Law of the Wilson loop is obvious.

$$\left\langle \prod_C \exp[iB_\mu(n)] \right\rangle \approx \exp[-F(g^{-2})A]$$

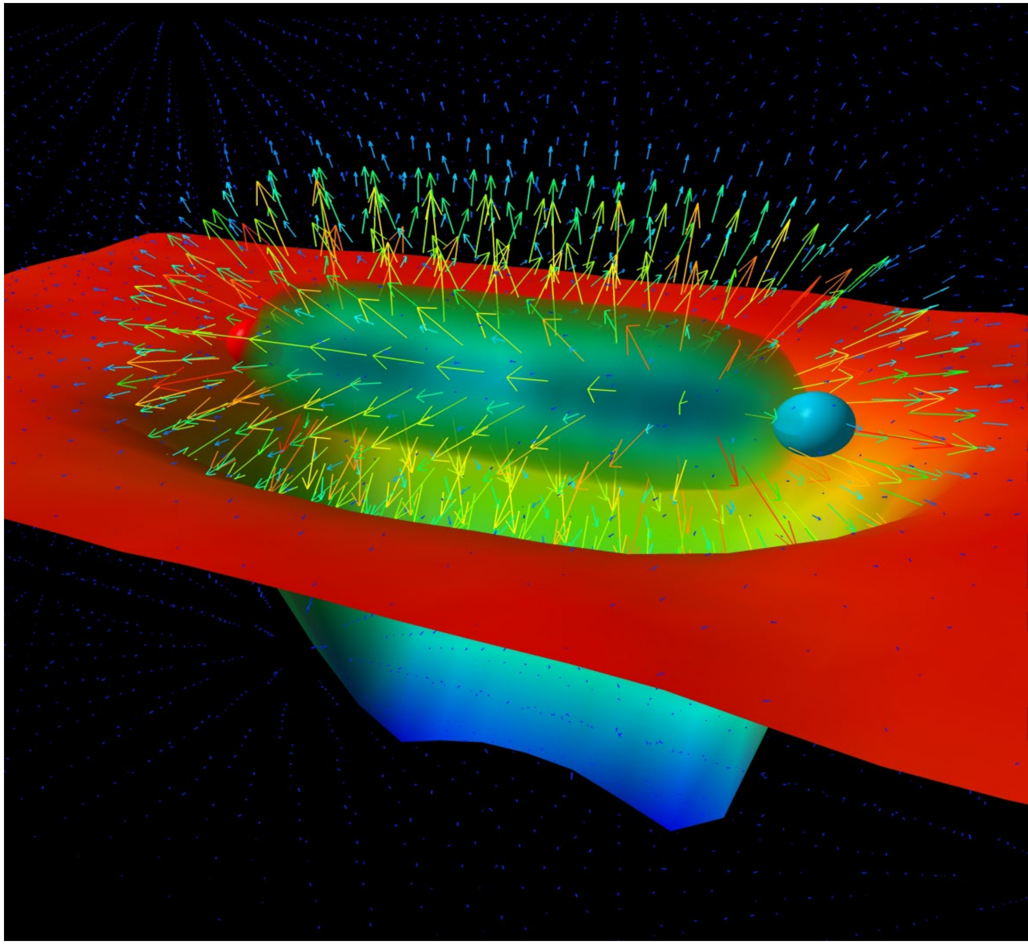
- To obtain the continuum limit, one needs to interpolate to the weak coupling limit on lattice scale due to Asymptotic Freedom

$$g^2(a) = \frac{g_0^2}{1 + (Cg_0^2/2\pi)\ln(a_0/a)}$$

- Can color confinement disappear in this limit? Numerical simulations strongly suggest that the answer is "No." Lattice sizes up to ~100^4 now.

QCD and Strings

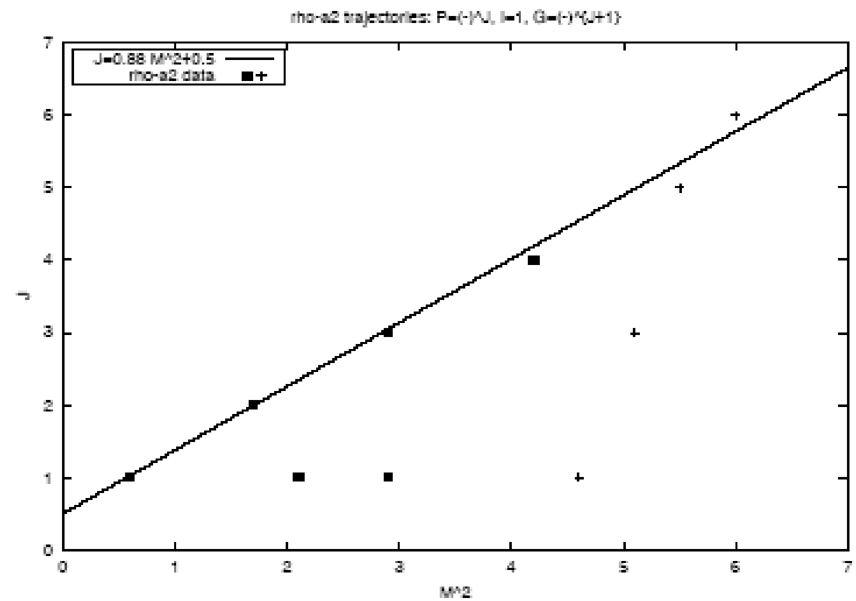
- At distances much smaller than 1 fm, the quark-antiquark potential is nearly Coulombic.
- At larger distances the potential should be linear (Wilson) due to formation of confining flux tubes. Their dynamics is described by the Nambu-Goto area action with corrections (Aharony's review talk; remarks by Dubovsky).
- So, strings have been observed in numerical simulations of Yang-Mills theory!



Open String Picture of Mesons

$$J = \alpha' m^2 + \alpha(0)$$

- Mesons are identified with excitations (rotational and vibrational) of a relativistic string of energy density ~ 1 GeV/fm, which is around 1.6 kiloJoules/cm.
- Regge trajectory starting with J=1 ρ meson.



Large N

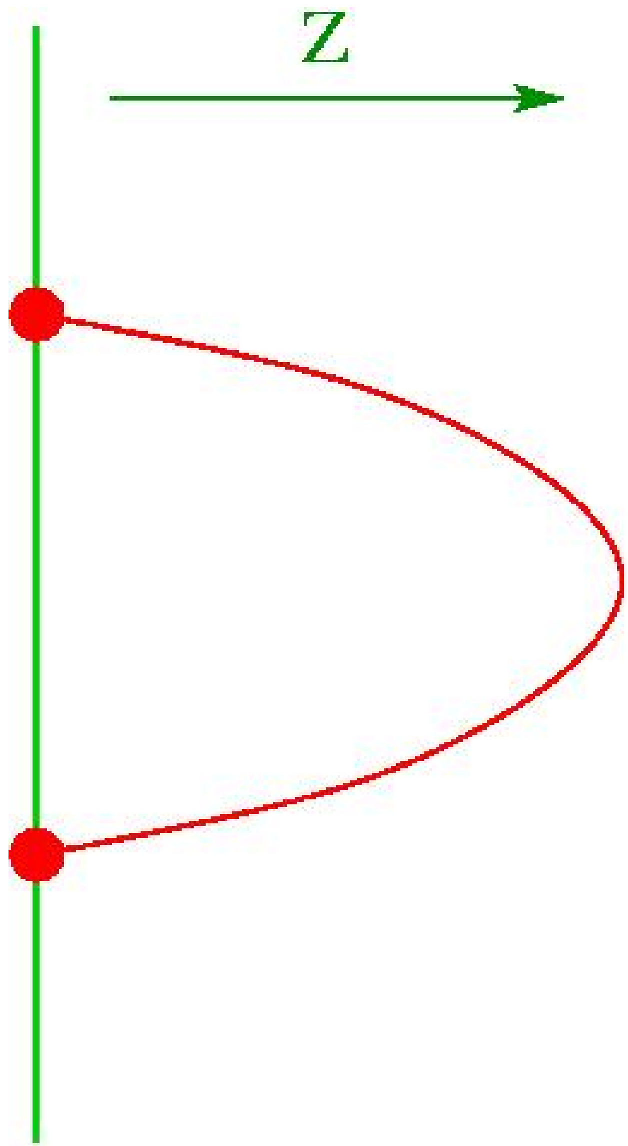
- Large N gauge theory should define string theory with $g_{st} \sim 1/N$. 't Hooft
- For example, SU(N) gauge theory in d=4 should define a stable, non-supersymmetric theory of closed strings (glueballs) and open strings (mesons).
- Adding maximal supersymmetry makes this theory conformal. Then we know what the string theory is: type IIB in $AdS_5 \times S^5$. Maldacena; Gubser, IRK, Polyakov; Witten
- In this case, no color confinement, but there are still strings!
- It is possible to find more general backgrounds that describe confining theories. Witten; Polchinski, Strassler; IRK, Strassler; …

The quark anti-quark potential

- What has gauge/string duality taught us?

- The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential Maldacena; Rey, Yee

$$V(r) = -\frac{4\pi^2\sqrt{\lambda}}{\Gamma\left(\frac{1}{4}\right)^4 r}$$



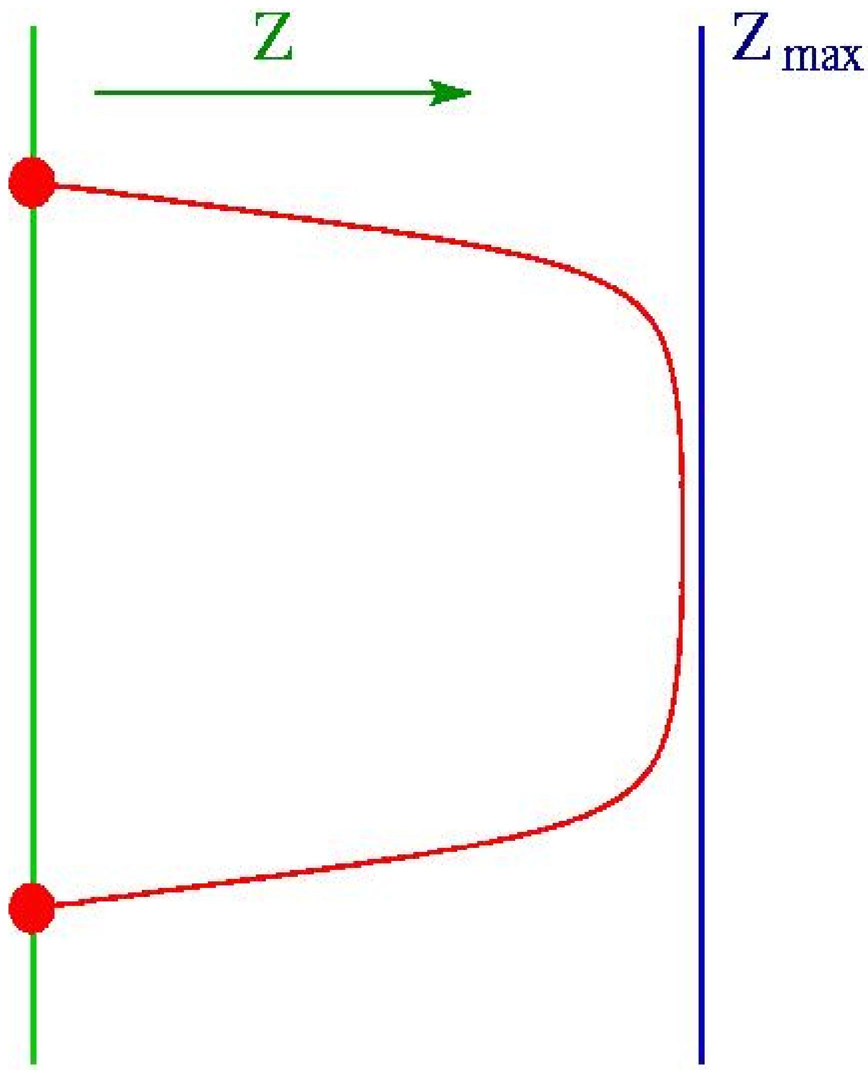
Confining = Fundamental

- The quark anti-quark potential is linear at large distances but nearly Coulombic at small distances.
- The 5-d metric should have a warped form Polyakov

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

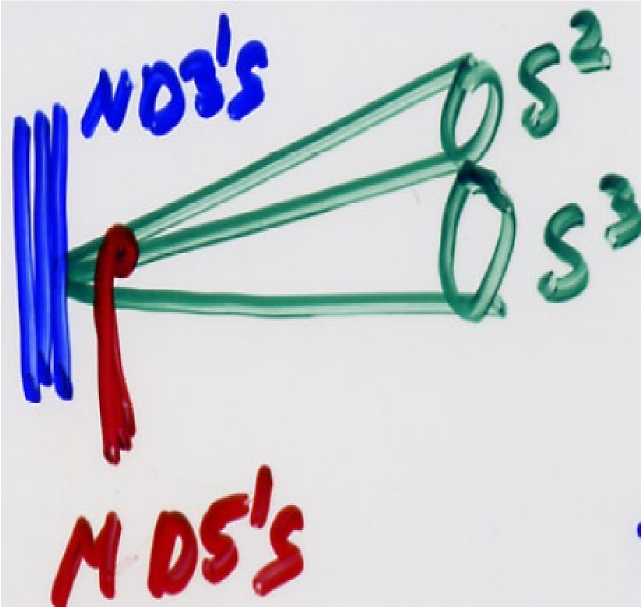
- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is

$$\frac{a^2(z_{\max})}{2\pi\alpha'}$$



Confinement and Warped Throat

- To break conformal invariance, add to the N D3-branes M D5-branes wrapped over the sphere at the tip of the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold IRK, Strassler



$$ds_{10}^2 = h^{-1/2}(y)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(y)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- The quark anti-quark potential is qualitatively similar to that found in numerical simulations of QCD (graph shows lattice QCD results by G. Bali et al with $r_0 \sim 0.5$ fm).

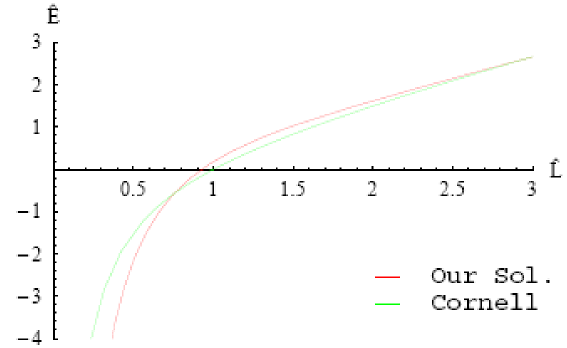
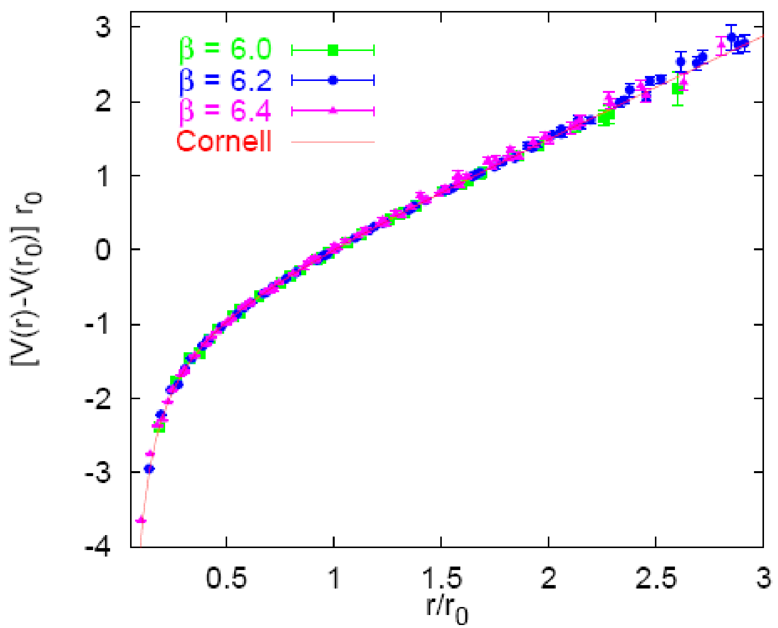


Figure 11: Comparison to the Cornell model



- Normal modes of the warped throat correspond to glueball-like bound states in the gauge theory.
- Their spectra have been calculated using standard methods of (super)gravity.
- Incorporates Dimensional Transmutation.

- Perhaps the gauge/string duality has provided us with a "physicist's proof of confinement" in some exotic gauge theories like the one described by the warped deformed conifold.
- Yet, we still don't have a quantitative handle on the Asymptotically Free theories in 3+1 dimensions.
- As a modest step, "drop back" to 1+1 dimensional gauge theories, which can hopefully provide some intuition about aspects of the higher dimensional dynamics.

2D QCD with Adjoint Matter

- Not exactly solvable at large N, but numerically tractable using Discretized Light-Cone Quantization (DLCQ). Dalley, IRK; Gross, Hashimoto, IRK
- The model with an adjoint Majorana fermion (a toy gluino) has particularly nice properties.
- Super-renormalizable, has non-trivial vacua and a mass gap. Witten; Kutasov; Demeterfi, IRK, Bhanot; Smilga

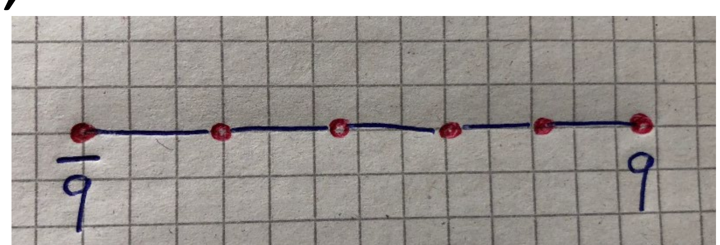
$$S_{\rm f} = \int d^2x \, {\rm Tr} \left[i\Psi^T \gamma^0 \gamma^\alpha D_\alpha \Psi - m\Psi^T \gamma^0 \Psi - \frac{1}{4g^2} F_{\alpha\beta} F^{\alpha\beta} \right]$$

A New 2D Model for Mesons

- If we add N_f fundamental Dirac fermions to the adjoint Majorana, we find a model which contains both gluinoballs and mesons: Dempsey, IRK, Pufu

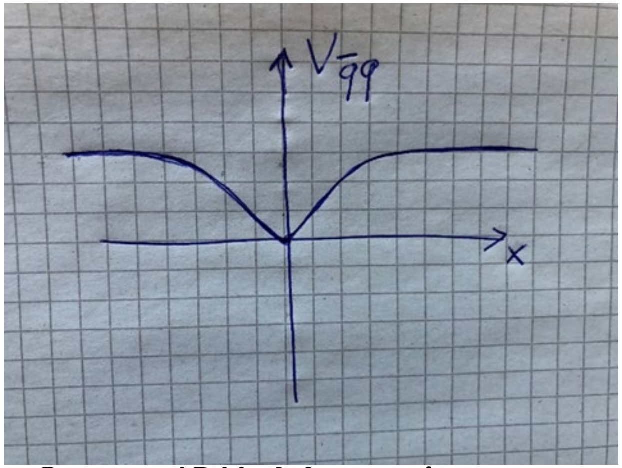
$$S = \int d^2x \left[\mathrm{tr} \left(-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \overline{\Psi} \not{D} \Psi - \frac{m_{\mathrm{adj}}}{2} \overline{\Psi} \Psi \right) + i \sum_{\alpha=1}^{N_f} \left(\overline{q}_\alpha \not{D} q_\alpha - m_{\mathrm{fund}} \overline{q}_\alpha q_\alpha \right) \right]$$

- The mesons are more complicated than in the ‘t Hooft model, since they also contain the adjoint quanta. There are now multiple Regge trajectories of mesons, which can be bosonic or fermionic.



Liberation

- For heavy fundamentals and massless adjoints the spectrum of mesons is continuous above a certain threshold. Dempsey, IRK, Pufu
- This implies that the q-qbar potential flattens at infinity!



- The massless adjoints renormalize string tension to 0. Gross, IRK, Matytsin, Smilga; Komargodski, Ohmori, Roumpedakis, Seifnashri
- For a small mass, it becomes non-zero giving a model of weak confinement.

Discussion

- Don't take confinement for granted, even in 1+1 dimensions where it seems obvious.
- Proof of Color Confinement in 2+1 and 3+1 dimensions would be very important.
- Improve connections between QCD Strings and Fundamental Strings.
- Need new analytical insights and approximations to the spectra of hadrons and k-string tensions.
- Guided by the improving numerical results on SU(N) gauge theories for a range of N. Athenodorou, Teper