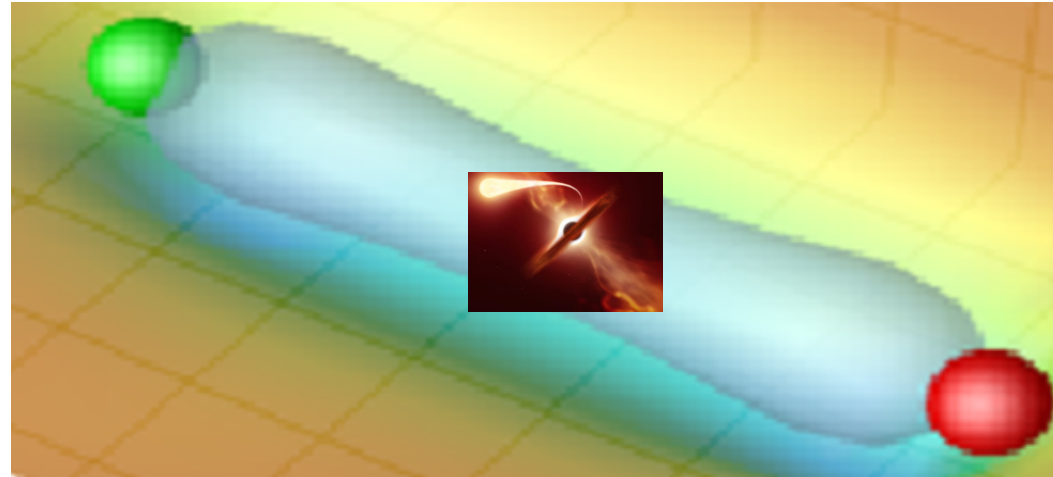


Comments on (mostly long) QCD Strings



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STRINGS'2021 Discussion Session

Let's take a complementary approach to Igor's remarks:

Let's assume confinement. What is the resulting string theory?

Two (related) questions I hope we will discuss:

1. It feels that relatively few people are actively thinking about this problem. Why?
2. What are the right questions to ask in this field?

Answers I heard to the first one:

- * Gravity is sexy, QCD is boring
- * There is no sharply formulated problem
- * The problem is too hard
- * The problem is morally solved (holography)

Will be interesting to hear what people think. Will try to convince you that the ones above are misconceptions.

A (limited) sharp version of the problem:

Consider an infinitely long string in the strict 't Hooft limit. It defines a microscopic relativistic theory of fluctuating 2d geometry (asymptotically flat 2d gravity).

What is the S-matrix?

As reviewed by Ofer, IR is largely fixed by symmetries: this is a system of $ISO(1, D - 1) / ISO(1, 1) \times O(D - 2)$ Goldstones + maybe some massive modes (for pure glue).

What happens in the UV?

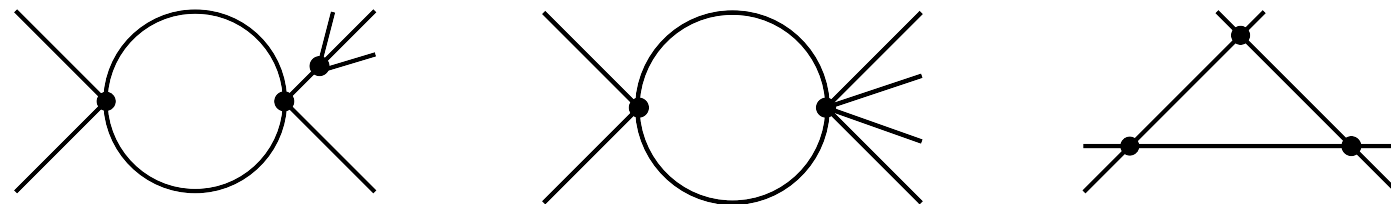
Sharp formulation of the problem tells us what are **not** good questions to ask.

A goal which looked reasonable early on:

$1/N_c$ expansion has a form of genus expansion. Let's solve the free planar theory, and develop string perturbation theory. Works in $N = 4$ SYM.

Will never work in pure glue, worldsheet theory is not integrable.

$$X^i(p_+ + q_+) X^i(p_- + q_-) \rightarrow X^j(p_+) X^j(q_+) X^k(p_-) X^k(q_-)$$

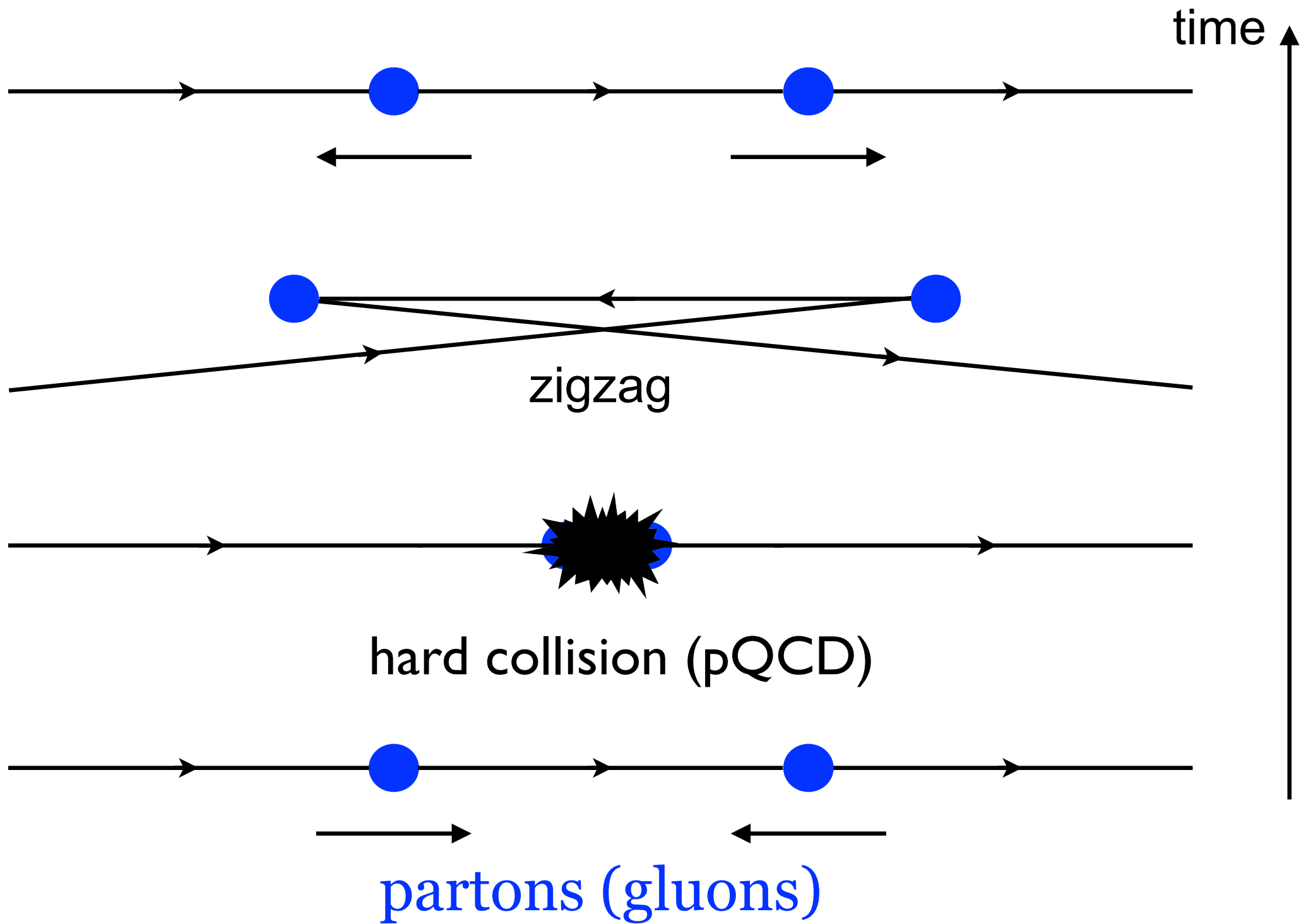


$$-\ell_s^6 \frac{D-26}{24\pi} p_+ q_+ p_- q_- (p_+ + q_+)^2 (p_- + q_-)^2$$

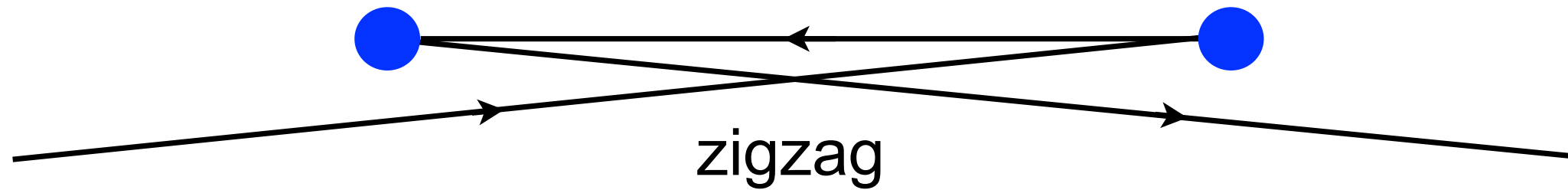
What can we do instead?

- * Find a confining theory with integrable dynamics on the worldsheet (looks hard, perhaps impossible, $D=2$ adjoint QCD-like theories may be the best place to look for examples)
- * Bootstrap
- * Develop high energy perturbative expansion for a worldsheet theory
- * ...

High-Energy Worldsheet Scattering



Asymptotic Freedom+Confinement=(Approximate) Integrability

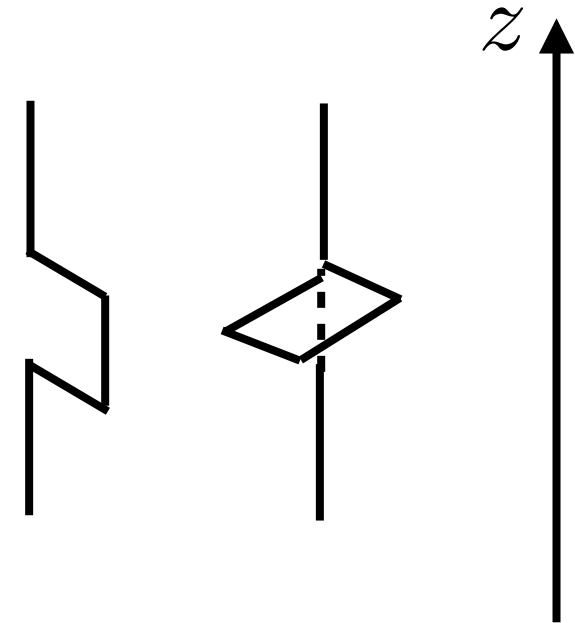


- * Can be studied quantitatively at least in 2d adjoint QCD
- * Leads to time delays proportional to collision energy (c.f. $T\bar{T}$)
- * Expected to lead to copious soft particle production (no-go for exact integrability at $D > 2$?)

Expected Degrees of Freedom

*in D=4 YM one expects to find

$$X^i \sim \text{Tr} P e^{i \oint_z A} F_{zi} \quad a \sim \text{Tr} P e^{i \oint_z A} F_{xy}$$



*in D=3 YM one expects to find

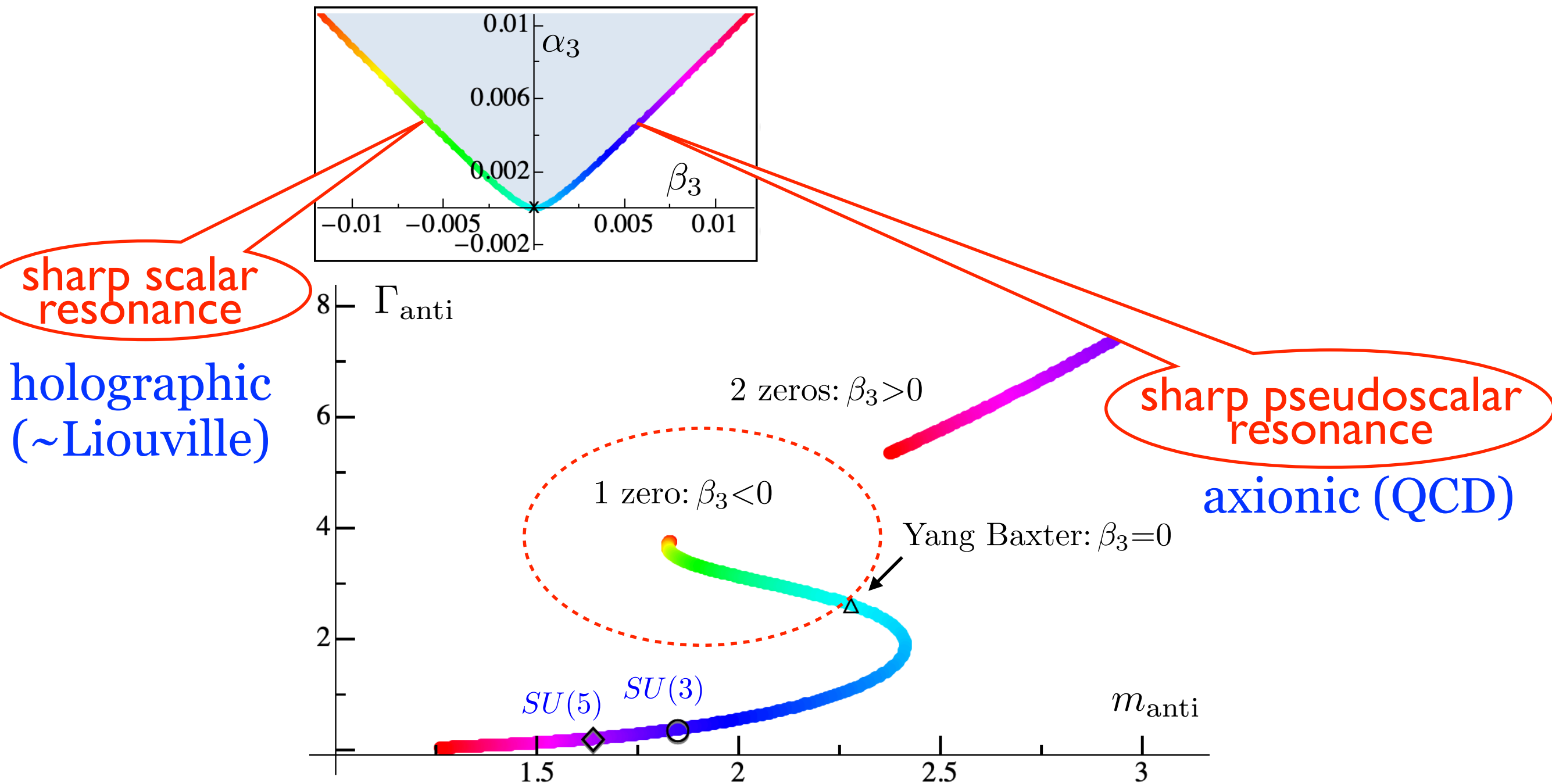
$$X \sim \text{Tr} P e^{i \oint_z A} F_{zx}$$

Would be nice to make this more rigorous by deriving
“worldsheet LSZ”

Exactly what one finds on a lattice!
Axion coupling takes an integrable value there.

Fluxtube S-matrix Bootstrap

Miro, Guerrieri, Hebbar, Penedones, Vieira '19



Is there a hope to ever say anything about short strings (glueballs)?

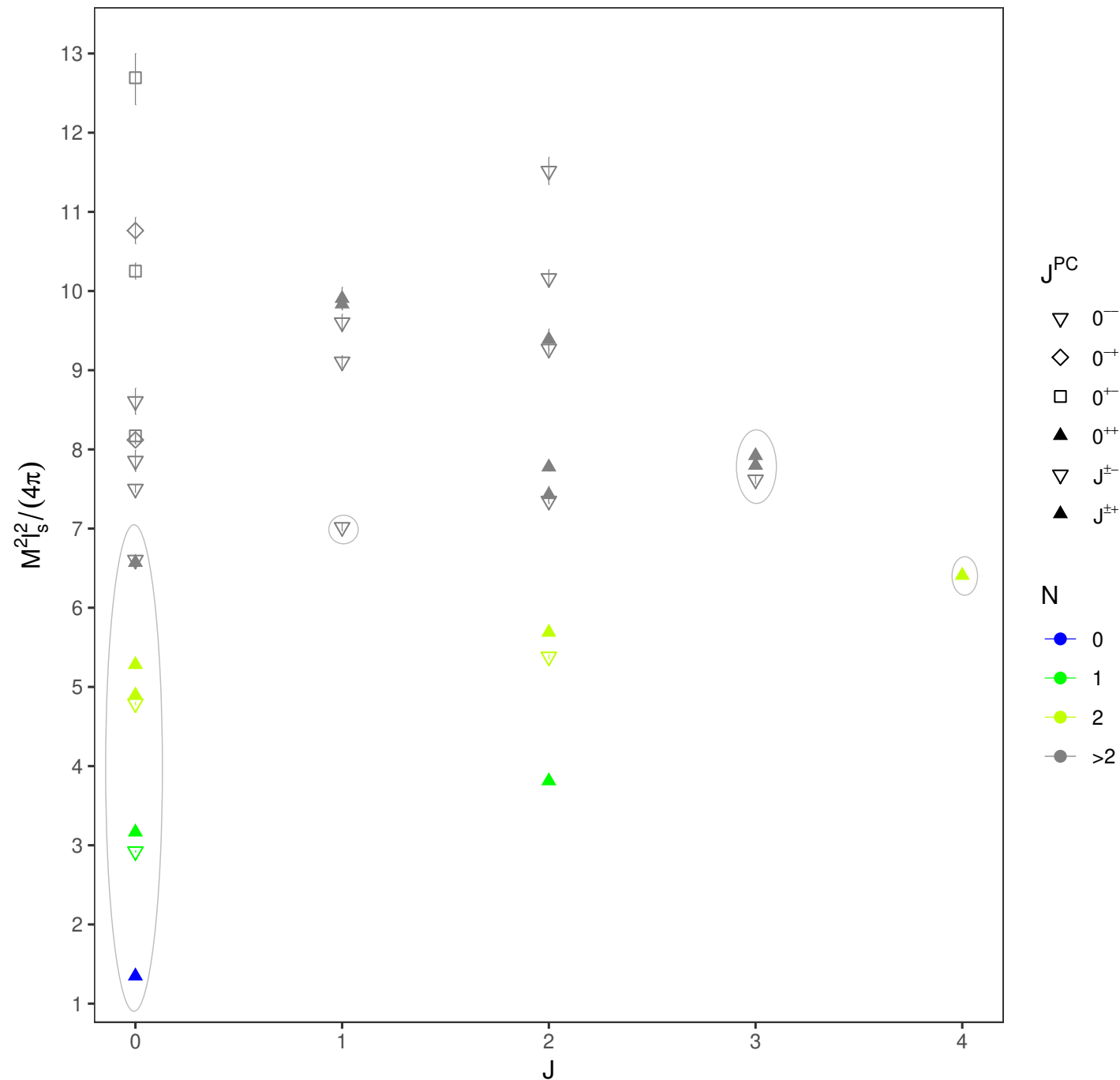
semiclassical prediction for D=3 glueball quantum numbers

SD, Hernandez-Chifflet '16

N	Glueball states	# of states
0	$0 \otimes 0 = 0^{++}$	1
1	$1 \otimes 1 = 0^{++} + 0^{--} + 2^+$	4
2	$(0 + 2) \otimes (0 + 2) = 2 \cdot 0^{++} + 0^{--} + 2^+ + 2^- + 4^+$	9
3	$(0 + 1 + 3) \otimes (0 + 1 + 3) =$ $3 \cdot 0^{++} + 2 \cdot 0^{--} + 1^+ + 1^- + 2 \cdot 2^+ + 2^- + 3^+ + 3^- + 4^+ + 4^- + 6^+$	25
4	$(0^{P_1} + 0^{P_2} + 1 + 2 + 4) \otimes (0^{P_1} + 0^{P_2} + 1 + 2 + 4) =$ $5 \cdot 0^{++} + 3 \cdot 0^{--} + 0^{P_1 P_2 +} + 0^{P_1 P_2 -} + 3 \cdot (1^+ + 1^-) + 4 \cdot 2^+ + 3 \cdot 2^- +$ $2 \cdot (3^+ + 3^-) + 3 \cdot 4^+ + 2 \cdot 4^- + 5^+ + 5^- + 6^+ + 6^- + 8^+$	64

Lattice Data circa 2016

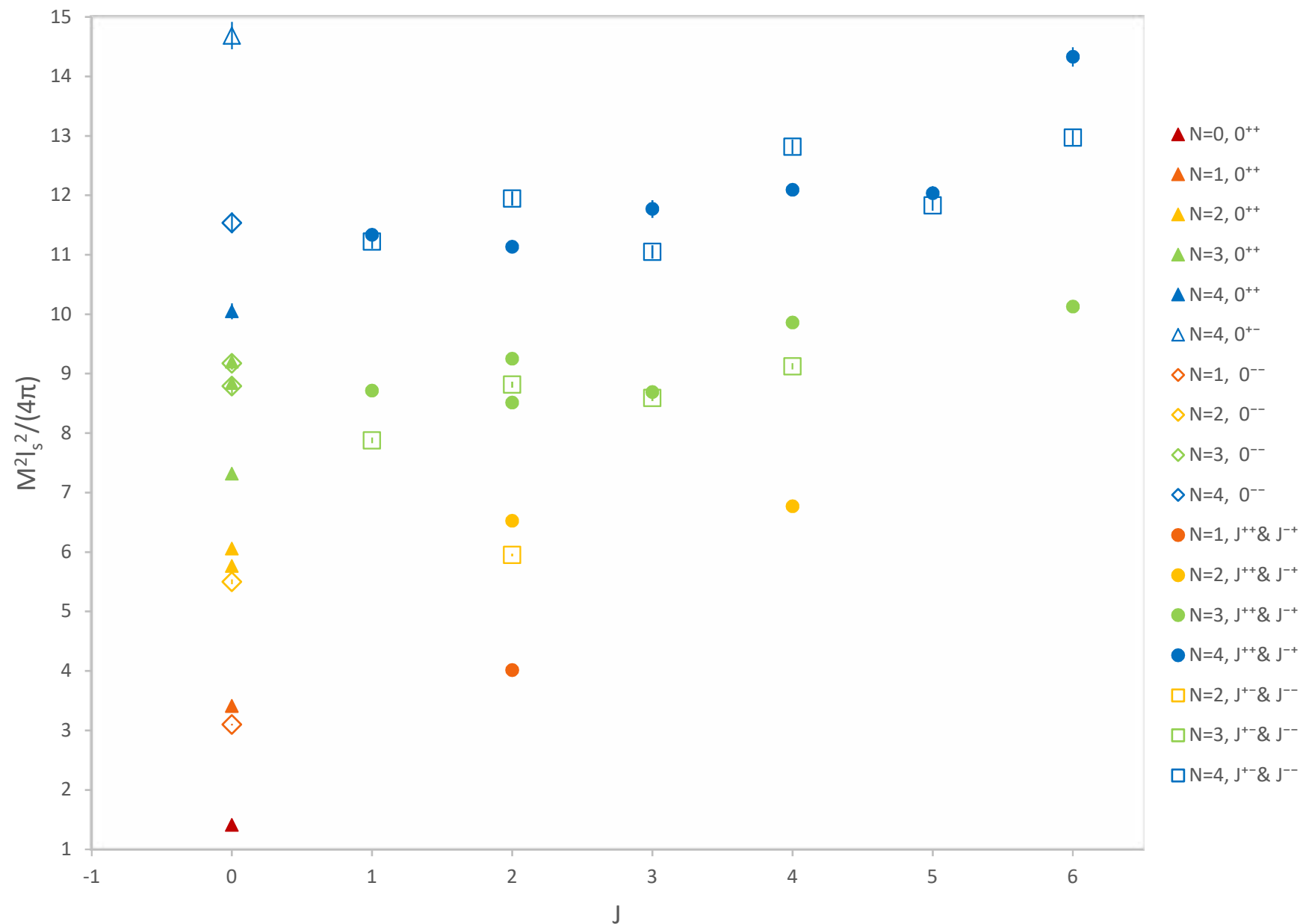
Athenodorou, Teper '16



ansatz worked for $N=0,1,2$ (14 states)
tension at $N=3$

Current Lattice Data

Conkey, SD, Teper '19



works for $N=0,1,2,3$ (39 states)

glueballs definitely look like oscillating strings!

Often people say that string theory is beautiful, but we may never know whether strings are realized in Nature. This is definitely wrong, QCD is a theory of strings. We don't understand them yet but have all reasons to expect that they are not disconnected from the rest of the string theory.

Furthermore, confining strings provide perhaps the most basic example of a connection between gauge theories and fluctuating geometry. This may be the closest we will ever get to doing experimental quantum gravity.

With modern lattice tools and understanding of QFT/string theory we are in a good position to solve this problem. It may take another 30 years to build a new collider and get new data in particle physics. But we are lucky to have a tractable and fundamental problem to solve while waiting. Participants of STRINGS 2051 will be puzzled if we don't make an effort now and leave it to them.